KEYNESIAN DSGE

NKIS/LM

No persistence, monetary policy loss function

C.B. sets r_t to minimize

$$L = E\left[\tfrac{1}{2}(\pi - \pi^*)^2 + \tfrac{1}{2}(y - y^*)^2\right]$$

given info available to C.B.

To simplify, we'll say $\pi^* = 0$ ← desired π

If $y^* > 0$, C.B. is aiming to keep output <u>above</u> natural rate.

If $y^* = 0$, ----

Recall that in microeconomic model we used to derive these equations, natural rate is too low due to monopoly — higher y would boost utility of representative agent.

So $y^* > 0$ makes sense.

We'll consider various cases.

KEYNESIAN DSGE

loss fn (cont.)

1) $y^* = 0$, C.B. can't see ε_t's when it sets r_t

As before, conjecture public's $y^e_{t+1} = \pi^e_{t+1} = 0$

then verify.

$$y_t = -s r_t + \varepsilon^{IS}_t$$

$$\pi_t = k y_t + \varepsilon^{\pi}_t = -sk r_t + k\varepsilon^{IS}_t + \varepsilon^{\pi}_t$$

$$\min_{r_t} E\left[\tfrac{1}{2}\left(-sk r_t + k\varepsilon^{IS}_t + \varepsilon^{\pi}_t\right)^2 + \tfrac{1}{2}\left(-s r_t + \varepsilon^{IS}_t\right)^2\right]$$

Recall $E[X^2] = (E[X])^2 + \sigma_X^2$

When C.B. sets r_t, $E[\varepsilon^{IS}_t] = E[\varepsilon^{\pi}_t] = 0$

it knows variances σ^2_{IS} σ^2_{π}

$$\min_{r_t} \tfrac{1}{2}\Big[\underbrace{(-sk r_t)^2}_{(E[\pi])^2} + \underbrace{k^2\sigma^2_{IS} + \sigma^2_{\pi}}_{\sigma^2_{\pi}} + \underbrace{(-s r_t)^2}_{(E[y])^2} + \underbrace{\sigma^2_{IS}}_{\sigma^2_y}\Big]$$

Take F.O.C.: $\dfrac{\partial L}{\partial r} = 0$

$$0 = (-sk r_t^*)(-sk) + (-s r_t^*)(-s) \quad \text{so } r^* = 0.$$

KEYNESIAN DSGE

1) cont.

Result: $y_t = \varepsilon_t^{IS}$

$\pi_t = \kappa \varepsilon_t^{IS} + \varepsilon_t^{\pi}$

Check: is $E_t[y_{t+1}] = E_t[\pi_{t+1}] = 0$? Yes!

Regress π on y: $\beta = \kappa$ (ε_t^{π} is residual)

Regress y on r: can't do variation in r.

What if you regress y on real interest rate, rather than real interest rate minus natural rate? As natural rate varies, real rate does too, so you can dry this.

But coefficient is zero.

So is coefficient from regression of π on real interest rate.

KEYNESIAN DSGE

2) $y^* = 0$, C.B. can see ε_t's when it sets r_t

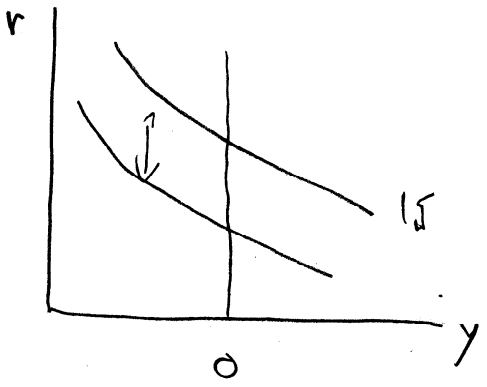
$$\min_{r_t} \tfrac{1}{2}\left(-s k r_t + k\varepsilon_t^{IS} + \varepsilon_t^{\pi}\right)^2 + \tfrac{1}{2}\left(-s r_t + \varepsilon_t^{IS}\right)^2$$

F.o.c.: $\frac{\partial L}{\partial r} = 0 \rightarrow r_t^* = \frac{1}{s}\varepsilon_t^{IS} + \frac{k}{s(1+k^2)}\varepsilon_t^{\pi}$

Resulting $y_t = -s r_t^* + \varepsilon_t^{IS} = -\frac{k}{1+k^2}\varepsilon_t^{\pi}$

$$\pi_t = k y_t + \varepsilon_t^{\pi} = \frac{1}{1+k^2}\varepsilon_t^{\pi}$$

What's going on here?
C.B. is adjusting r to totally counteract IS shocks because this keeps both π & y at desired levels.



C.B. is only <u>partly</u> counteracting effect of ε^{π}, because here it must trade off stabilizing π against <u>destabilizing</u> y.

DYNAMIC INCONSISTENCY OF LOW-INFLATION MONETARY POLICY (see also Romer 11.7)

$$y_t = \gamma_{t-1} y^e_{t+1} - \delta r_t$$

$$\pi_t = \pi^e_{t+1} + \kappa y_t$$

You know any LRSS requires $y = 0$ (to keep π stable).

public's preferences:

$$L = \tfrac{1}{2}(\pi_t - \pi^*)^2 + \tfrac{1}{2} a (y_t - y^*)^2$$

where $y^* > 0$ (recall why).

For simplicity, set $\pi^* = 0$.

We'll see:

- if public sets r period-by-period to minimize public's own loss function, LRSS with rational expectations is bad: $y = 0$ (of course)
  $\pi > \pi^*$ ← (bad

- get better outcome, with π closer to π^*, if central banker is "conservative" ← (has his own "a" smaller than public's

- get $y = 0$ (of course)
  $\pi = \pi^*$ ← (good!

  if central bankers act "as if" $y^* = 0$

DYNAMIC INCONSISTENCY (cont.)

Central bank uses public's loss function

$\pi^e_{t+1} = \pi = 0$ not a rational expectations equilibrium

Assume $y^* = 0$, $\pi^e = 0$

See that c.b. <u>won't</u> choose r to make $\pi = \pi^e$

Hence not equilibrium

$$L = \tfrac{1}{2}\pi_t^2 + \tfrac{1}{2}a(y_t - y^*)^2$$

For $\pi^e = 0$, $y^e = 0$

$$L = \tfrac{1}{2}(-k s r_t)^2 + \tfrac{1}{2}a(-s r_t - y^*)^2$$

We <u>could</u> calculate $\frac{\partial L}{\partial r_t}$, solve for r, then substitute r^* into PC & IS.

But a shortcut:

$$L = \tfrac{1}{2}(k y)^2 + \tfrac{1}{2}a(y - y^*)^2$$

Calculate $\frac{\partial L}{\partial y}$, solve for y. C.B. chose r_t to hit this y. Results

$$y = \frac{a}{a+k^2} y^* \qquad \pi = k\frac{a}{a+k^2} y^*$$

Not an equilibrium!

DYNAMIC INCONSISTENCY

Central bank uses public's loss function

What is the rational expectations equilibrium?

Figure out what value of π^e will be confirmed by experience.

Assume $y^e = 0$, $\pi^e = \hat{\pi}^e$,

Central bank's choice of π results in a value of π;

$\pi_t = \dots \hat{\pi}^e \dots$ ← determinant of π

Set $\pi_t = \hat{\pi}^e$, solve. That's an equilibrium.

$$L = \frac{1}{2}\left(\hat{\pi}^e + k y_t\right)^2 + \frac{1}{2} a \left(y - y^*\right)^2$$

$$\frac{\partial L}{\partial y} = 0 \rightarrow \quad \pi_t = \frac{a}{a+k^2}\hat{\pi}^e + \frac{ak}{a+k^2} y^*$$

in equilibrium $\pi = \frac{a}{a+k^2}\pi + \frac{ak}{a+k^2} y^*$

$$\Rightarrow \pi = \frac{a}{k} y^*, \quad y = 0$$

See the move the public/c.b. cares about y, the higher is inflation ($a \uparrow \rightarrow \pi \uparrow$) but output isn't any higher (of course).

DYNAMIC INCONSISTENCY

Conservative central banker

Say a in central bank's loss fn. is $\hat{a} < a$

then outcome is

$$\pi = \frac{\hat{a}}{k} y^* \quad , \quad y = 0$$

This is better for public!

Loss with normal central banker:

$$L = \frac{1}{2}\left(\overbrace{\frac{a}{k} y^*}^{\pi}\right)^2 + \frac{1}{2} a (y^*)^2$$

Loss with conservative central banker:

$$L = \frac{1}{2}\left(\frac{\hat{a}}{k} y^*\right)^2 + \frac{1}{2} a (y^*)^2$$

"As if" central banker

Result will be $\pi = 0, y = 0$

$$L = \frac{1}{2}(0)^2 + \frac{1}{2} a (y^*)^2$$

Dynamic Inconsistency (cont.)

Why is this issue referred to as "DI"?

Originally (Kydland & Prescott, 1977), it was viewed this way:

if central bank has "discretion" (can choose setting of r or m^s period-by-period, with constraint), a promise to deliver low inflation is not credible because c.b.'s choices in future (moving through time) will not be consistent with orginal promise. C.B. will renege on commitment, because expectations that it creates with promise in turn create incentives for c.b. to break promise.

So people originally concluded, c.b. must be bound by rules.

"Interest rate rules" (e.g. $r(\pi, y)$) at first viewed this way.