KEYNESIAN DSGE

Background for Clarida, Gali & Gertler (1999)

Some ideas about monetary policy originally developed using old-Keynesian models

1) Set i not m (Poole)

2) Dynamic inconsistency

3) Inflation targeting

A set of policies or practices for a central bank

— Have an explicit, numerical inflation target π^T

— Set i_t to set r_t so that $E_t \pi_{t+j} = \pi^T$ at some horizon j ("inflation-forecast targeting")

— Tell public you have a target, it's π^T, & explain how your setting of i_t will drive π to π^T so that public's $\pi^e = \pi^T$

$$\pi_t = \pi^e + \beta Y_t$$

if $\pi^e > \pi^T$, c.b. must cause recession — bad

KEYNESIAN DSGE

Background (cont.)

3) Taylor rules (Romer 11.6)

In early 1990s John Taylor observed Fed's target i had approximately followed:

$$i_t = \text{Constant} + \phi_\pi \pi_t + \phi_y \underbrace{(\ln Y - \text{trend } Y)}_{\text{estimate of output gap}}$$

where $\phi_\pi > 1$: central bank raises i more than one-for-one with inflation.

If constant $= \bar{r} + \pi^* - \phi_\pi \pi^*$ this is

$$i_t = \bar{r} + \pi^* + \phi_\pi (\pi_t - \pi^*) + \phi_y y_t \quad \text{(see CGG 7.1)}$$

subtract π_t from both sides

$$i_t - \pi_t = \bar{r} + (\phi_\pi - 1)(\pi_t - \pi^*) + \phi_y y_t$$

If $\pi_t \approx \pi^e$, this means Fed was raising r above $\bar{r}$ when $\pi^e > \pi^*$, etc. $\rightarrow$ satisfies condition for stable inflation.

"Taylor principle": make sure central bank raises i more than one-for-one with inflation.

NK IS-LM & Monetary Policy

Clarida, Gali & Gertler (1999)

What this paper does:

1) NK IS/LM with central bank loss fn, not $v(\pi, y)$
2) How does this change ideas about monetary policy developed from OK models?

<u>Biggest change:</u>

OK models say that "discretion," period-by-period policymaking is fine as long as c.b. acts "as if" desired output = natural rate; no advantage from binding commitment to a policy rule.

NK model says, even if c.b. aims at natural rate of output, there is a gain from binding commitment in response to AS shocks.

<u>but</u> you can get most of these gains from another layer of "as if" behavior.

Monetary Policy

CGG

Notation & assumptions

x: output gap

r_t $\quad i_t - \pi^e_{t+1} - \bar{r}$ ("deviation from long-run level")

$$x_t = -\varphi r_t + {}_t x^e_{t+1} + g_t$$

$$g_t = \mu g_{t-1} + \hat{g}_t$$

serial correlation

i.i.d. shock

$$\pi_t = \lambda x_t + \beta_t \pi^e_{t+1} + u_t$$

"cost-push"

$$u_t = \rho u_{t-1} + \hat{u}_t$$

Central bank objective:

$$\max -\tfrac{1}{2} E_t \left[\sum_{i=0}^{\infty} \beta^i \left[\alpha (x_{t+i} - k)^2 + \pi^2_{t+i} \right] \right] \qquad (2.7, 4.1)$$

— if $k = 0$, c.b. is aiming for natural rate

— if $k > 0$, ... dynamic inconsistency problem?

g_t, u_t & distn's of $\hat{g}$, $\hat{u}$ known to c.b. & public.

Monetary Policy

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Discretion vs. commitment

"Commitment" means c.b. can define a rule that will determine future r's as a function of future values of observable variables, and public believes c.b. will follow rule, so form/parameters of rule affect $_t\pi^e_{t+i}$, $_t x^e_{t+i}$.

At time t, choice of rule can affect π^e_{t+1} etc.

C.B.'s problem:

at time t, choose rule (form & parameters) that minimize $\frac{1}{2} E_t \sum_{i=0}^{\infty}$ ---

accounting for effect of rule on π^e_{t+i}, x^e_{t+i}.

Then rule determines r_t. (Can't choose r_t.)

You don't need to worry that the rule that looks optimal today will prove to be less-than-opt. in future — structure of economy is known & unchanging.

CGG

Discretion vs. commitment (cont.)

Discretion means c.b. chooses r_t to minimize

$$\frac{1}{2}E_t\sum_{i=0}^{\infty} \dots = \frac{1}{2}\Big[\alpha(-\varphi r_t + {}_t x^e_{t+1} + g_t - k)^2 + (-\lambda\varphi r_t + \lambda\, {}_t x^e_{t+1} + \lambda g_t + \dots)^2$$

$$+ E_t\beta(\alpha(-\varphi r_{t+1} + {}_t x^e_{t+2} + \dots) + E_t\beta^2(\qquad) + \dots\Big]$$

Take f.o.c., solve for r^*_t. It's easy because we know

$$\frac{\partial r_{t+i}}{\partial r_t} = \frac{\partial x^e_{t+i}}{\partial r_t} = \frac{\partial \pi^e_{t+i}}{\partial r_t} = \dots = 0$$

so just minimize $E_t[\alpha(x_t-k)^2 + \pi_t^2]$.

1) Choice of r_t today doesn't bind future c.b. to any path for r_{t+i}. Future c.b. will choose r_{t+i} to minimize loss function looking forward from $t+i$.

2) Public knows c.b. will do this, so x^e_{t+i}, π^e_{t+i} come from working out what c.b. will do looking forward from $t+i$. Today's r_t doesn't affect that.

Today's r_t does not change any aspect of environment c.b. will be facing in future.

Note: things would be different if there were capital in the model.

NK IS-LM &...

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Discretion & $k=0$ ("as if" behavior)

Because c.b. knows g_t (position of IS curve), we can describe it as choosing x_t directly. (It's really setting r_t, but it knows exactly what x_t results. From x_t it chooses, we can back out the corresponding r_t.)

$$\underset{x_t}{\text{Min}}\ \tfrac{1}{2}\left[\alpha x_t^2 + (\lambda x_t + \beta \pi^e_{t+1} + u_t)^2\right]$$

$$\frac{\partial(\ )}{\partial x_t} = 0 \rightarrow x_t = -\frac{\lambda\beta}{\alpha+\lambda^2}\pi^e_{t+1} - \frac{\lambda}{\alpha+\lambda^2}u_t$$

stick into Phillips curve,

$$\pi_t = \frac{\alpha\beta}{\alpha+\lambda^2}\pi^e_{t+1} + \frac{\alpha}{\alpha+\lambda^2}u_t$$

apply math trick (assume LRSS w/stable π):

$$\pi_t = \frac{\alpha}{\alpha+\lambda^2}\,\frac{1}{1-\frac{\alpha\beta}{\alpha+\lambda^2}\rho}\,u_t = \frac{\alpha}{\alpha+\lambda^2-\alpha\beta\rho}u_t$$

$$= \alpha\,\underbrace{\frac{1}{\lambda^2+\alpha(1-\beta\rho)}}_{\text{"}q\text{"}}\,u_t \qquad (3.5)$$

From this, get x_t etc. For x_t, need π^e_{t+1}.

monetary policy

CGG

k=0 (cond.)

Recall $\pi_t = \alpha q u_t$

so $\pi^e_{t+1} = \alpha q \rho u_t$

so $x_t = -\frac{\lambda\beta}{\alpha+\lambda^2}\alpha q \rho u_t - \frac{\lambda}{\alpha+\lambda^2} u_t$

$= -\lambda q u_t$ (3.4)

Now back out r_t & i_t

$-\lambda q u_t = x_t = -\varphi r_t + {}_t x^e_{t+1} + g_t$

${}_t x^e_{t+1} = -\lambda q \, {}_t u^e_{t+1} = -\lambda q \rho u_t$ so

$-\lambda q u_t = -\varphi r_t - \lambda q \rho u_t + g_t$

$\Rightarrow r_t = \frac{1}{\varphi} g_t + \frac{(1-\rho)\lambda q}{\varphi} u_t$

$i_t = r_t + {}_t \pi^e_{t+1} = r_t + \rho \alpha q u_t$

so...

monetary policy

CGG

k=0 (cond.)

$$i_t = \frac{1}{\varphi} g_t + \frac{(1-\rho)\lambda q}{\varphi} u_t + \rho \alpha q u_t$$

$$i_t = \frac{1}{\varphi} g_t + \underbrace{\left(1 + \frac{(1-\rho)\lambda}{\rho \varphi \alpha}\right)}_{\text{"}\gamma_\pi\text{"}} \underbrace{(\rho \alpha q u_t)}_{+\pi^e_{t+1}} \quad (3.6)$$

& recall $x_t = -\lambda q u_t$

so:

IS   cost-push

c.b. raises r & i in response to g_t & u_t

Completely counteract g_t, so no effect on y or π

Partly counteract u, so some effect on y, π

To get all this, we assumed LRSS with stable π.

But does this behavior by c.b. satisfy necessary condition for that?

(if $\pi^e > 0$, $r > 0$....)

monetary policy

CGG

k=0 (cont.)

Recall before we invoked LRSS ("math trick"),

F.O.C. gave $x_t = -\frac{\lambda\beta}{\alpha+\lambda^2}\pi^e_{t+1} - \frac{\lambda}{\alpha+\lambda^2}u_t$

Set $u_t = 0$, $g_t = 0$ (on average)

$$x_t = \frac{-\lambda\beta}{\alpha+\lambda^2}\pi^e = -\varphi r_t + {}_t x^e_{t+1}$$

In LRSS, $x_t = x^e_{t+1}$ $\qquad \pi^e = \hat{\pi}$

See: you can't have LRSS with $\hat{\pi} \neq 0$ — desired π in loss fn.

because $\pi^e > 0 \rightarrow x_t < 0 \rightarrow r_t > 0$,

$\pi_t < \pi^e$

satisfies condition.

Inflation targeting

Target is zero. Is $\pi^e_{t+i} = 0$ at some horizon?

Recall $\pi_t = \alpha q\, u_t$

$${}_t\pi^e_{t+i} = \alpha q\, {}_t u^e_{t+i} = \alpha q \rho^i u_t$$

Yes, but if there are cost-push shocks, horizon may be distant.

NK IS-LM & MONETARY...

CGG

"Classic inflationary bias problem"

What if $k > 0$ (C.B. aims at output greater than natural rate)?

Old idea says, result is $\pi > \pi^*$ ← here zero

That's true here, too.

For simplicity, no cost-push shocks

$u = 0$ ← no cost-push shocks

$$\min_{x_t} \tfrac{1}{2}\left[\alpha(x_t - k)^2 + (\lambda x_t + \beta\pi^e + u_t)^2\right]$$

$$\frac{\partial(\ )}{\partial x_t} = 0 = \alpha(x_t - k) + (\lambda x_t + \beta\pi^e + u_t)\lambda$$

$$\Rightarrow x_t = \frac{\alpha}{\alpha+\lambda^2}k - \frac{\lambda\beta}{\alpha+\lambda^2}\pi^e$$

put into AS:

$$\pi_t = \frac{\lambda\alpha}{\alpha+\lambda^2}k + \frac{\alpha\beta}{\alpha+\lambda^2}\pi^e_{t+1}$$

divide both sides by $(-\lambda/\alpha)$ and get...

$$-\frac{\lambda}{\alpha}\pi_t = -\frac{\lambda^2}{\alpha+\lambda^2}k - \frac{\lambda\beta}{\alpha+\lambda^2}\pi^e$$

zero

$$= -\frac{\lambda^2}{\alpha+\lambda^2}k - \frac{\alpha}{\alpha+\lambda^2}k + \frac{\alpha}{\alpha+\lambda^2}k - \frac{\lambda\beta}{\alpha+\lambda^2}\pi^e$$

$$= -\frac{\alpha+\lambda^2}{\alpha+\lambda^2}k + \underbrace{\frac{\alpha}{\alpha+\lambda^2}k - \frac{\lambda\beta}{\alpha+\lambda^2}\pi^e}_{x_t}$$

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Classic inflationary bias (cont.)

$$\Rightarrow x_t = -\frac{\lambda}{\alpha}\pi_t + k \qquad (4.2)$$

Substitute in AS equation

$$\pi_t = \lambda\left(-\frac{\lambda}{\alpha}\pi + k\right) + \beta\pi^e$$

$$\Rightarrow \pi_t = \frac{\alpha\beta}{\alpha+\lambda^2}\pi^e_{t+1} + \frac{\alpha\lambda}{\alpha+\lambda^2}k$$

To have a LRSS with a stable inflation rate, must be true that $\pi^e_{t+1} = \pi_t$ which gives

$$\pi^e = \pi_t = \frac{\alpha\lambda}{\alpha(1-\beta)+\lambda^2} \quad \text{and } x = 0$$

If $\beta = 1$ as CGG assume for this section,

$$\pi = \frac{\alpha}{\lambda}k \quad \text{and } x = 0$$

Recall $\pi^* = 0$ so this is Kydland-Prescott dynamic inconsistency.

desired

Discretion with $k > 0$ is bad. You get smaller loss (from loss fn with $k > 0$) if you force C.B. to act as if $k = 0$.

CGG

Commitment with $k=0$

If C.B. acts as if $k=0$, does discretion achieve best possible outcome?

Is there a commitment policy that can achieve smaller loss than (discretion + $(k=0)$)?

Old K models say no.

In NK model, answer is yes (surprise)! as long as there are cost push shocks

To show this, CGG present

1) A commitment policy which is easy to derive & achieves outcome better than (discretion + $(k=0)$)

2) The optimal commitment policy, which is hard to derive.

Easy-to-derive commitment policy

Recall $k=0$ discretion policy was: (& figure out corresponding r_t, i_t)

$$x_t = -\lambda q u_t$$

Now suppose c.b. can commit to rule of form: where you pre-commit to ω value.

$$x_t^c = -\omega u_t$$

If ω that minimizes loss isn't equal to $-\lambda q$, discretion isn't best.

CGG

Easy commitment policy

Note: there's possible policy rules other than $x_t^c = \omega u_t$ e.g. $x_t^c = \omega_1 u_t + \omega_2 u_{t-2}$ etc.

$x_t^c = \omega u_t^2$

so best ω is not necessarily best possible commitment policy.

Determination of π_t under easy commitment policy

Recall that iterating back from LRSS we had

$$\pi_t = E_t \sum_{i=0}^{\infty} \beta^i [\lambda x_{t+i} + u_{t+i}] \qquad (2.6)$$

If $x = -\omega u$ every period,

$$\pi_t = E_t \sum_{i=0}^{\infty} \beta^i [-\lambda \omega u_{t+i} + u_{t+i}]$$

$$= \qquad [(1-\lambda\omega) u_{t+i}]$$

$$= (1-\lambda\omega) E_t [\sum \beta^i u_{t+i}]$$

Also recall $u_{t+1} = \rho u_t + \hat{u}_t$

so $E_t[u_{t+i}] = \rho^i u_t$

$$\Rightarrow \pi_t = (1-\lambda\omega) u_t \sum_{i=0}^{\infty} (\rho\beta)^i$$

apply infinite-sum thing

gives...

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Easy...

Optimal ω

$$x_t = -\omega u_t$$

$$\pi_t = \frac{\lambda}{1-\beta\rho} x_t + \frac{1}{1-\beta\rho} u_t = -\frac{\lambda\omega}{1-\beta\rho} u_t + \frac{1}{1-\beta\rho} u_t$$

$$= \frac{1}{1-\beta\rho}(1-\lambda\omega)u_t$$

Single-period loss $\frac{1}{2}(\alpha x^2 + \pi^2)$ is thus

call it

$$N_t = \frac{1}{2}\left[\alpha(-\omega u_t)^2 + \left(\frac{1}{1-\beta\rho}(1-\lambda\omega)u_t\right)^2\right]$$

$$= \underbrace{\frac{1}{2}\left[\left(\alpha(-\omega)^2 + \left(\frac{1}{1-\beta\rho}\right)^2(1-\lambda\omega)^2\right)\right]}_{Q} u_t^2$$

$$N_{t+i} = Q u_{t+i}^2 = Q u_t^2 \frac{u_{t+i}^2}{u_t^2}$$

hence

$$L_t = \frac{1}{2} E_t\left[\Sigma(\quad)\right] = E_t \frac{1}{2}\sum_{i=0}^{\infty} \beta^i Q u_t^2 \frac{u_{t+i}^2}{u_t^2}$$

$$= \underbrace{Q u_t^2}_{\frac{1}{2}\left[\alpha x_t^2 + \pi_t^2\right]} \underbrace{E_t \sum \beta^i \frac{u_{t+i}^2}{u_t^2}}_{\text{CGG call this } L_t} \qquad (4.11)$$

Just minimize Q!

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Easy (cont.)

Commitment gets you a better tradeoff:

$$\pi_t = \frac{\lambda}{1-\beta\rho} x_t + \frac{1}{1-\beta\rho} u_t \qquad (4.10)$$

greater than one

$$\text{so } \frac{\partial \pi}{\partial x_t} = \lambda \frac{1}{1-\beta\rho}$$

$$\pi_t = \lambda x_t + \beta \pi^e_{t+1} + u_t$$

$$\text{so } \frac{\partial \pi}{\partial x_t} = \lambda$$

A given loss of x buys you <u>more</u> in low inflation. Better!

Of course, under commitment CB isn't choosing x_t, it's choosing a <u>rule</u> ω; each ω implies a (π, x) pair for given u_t. (4.10) maps out these possibilities across different values of ω.

So what's best value of ω?

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Easy....

Optimal ω (cont.)

$$\frac{\partial Q}{\partial \omega} = 0 = \alpha(-\omega)(-1) + \left(\frac{1}{1-\beta\rho}\right)^2 (1-\lambda\omega)(-\lambda)$$

$$\Rightarrow \omega = \frac{\lambda}{\lambda^2 + \alpha(1-\beta\rho)^2} = \frac{\lambda}{\lambda^2 + \alpha(1-\beta\rho)(1-\beta\rho)}$$

hence following this rule means

$$x_t^c = -\frac{\lambda}{\lambda^2 + \underbrace{\alpha(1-\beta\rho)}(1-\beta\rho)} u_t \qquad (4.14)$$

vs. discretion

$$x_t = -\frac{\lambda}{\lambda^2 + \underbrace{\alpha}(1-\beta\rho)} u_t$$

difference is $\alpha(1-\beta\rho)$ vs. α

- Under commitment, have a <u>bigger</u> reduction in x in response to u_t

CGG notation

- CB could reproduce this commitment rule under discretion if it acts <u>as if</u> $\alpha = (1-\beta\rho)\alpha = \alpha^c$ which is less than "true" α.

"Responsible behavior" means pretending $\alpha = \alpha^c$ as well as pretending $k = 0$.

NK ISLM & ...

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Easy ....

Interest rates & π under optimal ω

Given ω, from $\pi_t = \dfrac{1-\lambda\omega}{1-\beta\rho} u_t$ get

$$\pi_t = \frac{\alpha^c}{\lambda^2 + \alpha^c(1-\beta\rho)} u_t \qquad (4.15)$$

Note article contains error here, sign of coefficient

which means

$$_t\pi^e_{t+1} = \frac{\alpha^c}{\lambda^2+\alpha^c(1-\beta\rho)} \underbrace{\rho\, u_t}_{E_t[u_{t+1}]}$$

From

$$\omega u_t = x_t = -\varphi r_t + {}_t x^e_{t+1} + g_t$$

$-\omega\rho u_t$

get

$$r_t = \frac{1}{\varphi} g_t + (1-\rho) q^c u_t$$

$\dfrac{1}{\lambda^2 + \alpha(1-\beta\rho)^2}$

$$i_t = \frac{1}{\varphi} g_t + \underbrace{\left(1 + \frac{(1-\rho)\lambda}{\rho\varphi\alpha(1-\beta\rho)}\right)}_{\text{“}\gamma^c_\pi\text{”}} {}_t\pi^e_{t+1}$$

Response of r_t & i_t to g_t are same as under discretion,
but response to u_t or $_t\pi^e_{t+1}$ is <u>stronger</u> than under discretion.

Summary: optimal ω vs. discretion

Discretion

$$x_t = \frac{-\lambda}{\lambda^2 + \alpha(1-\beta\rho)} u_t$$

$$\pi_t = \frac{\alpha}{\lambda^2 + \alpha(1-\beta\rho)}$$

Optimal ω

$$x_t = \frac{-\lambda}{\lambda^2 \underbrace{\alpha(1-\beta\rho)}_{\alpha^c}(1-\beta\rho)} u_t$$

$$\pi_t = \frac{\alpha(1-\beta\rho)}{\lambda^2 + \alpha(1-\beta\rho)(1-\beta\rho)}$$

$$= \frac{\alpha}{\frac{\lambda^2}{1-\beta\rho} + \alpha(1-\beta\rho)} u_t$$

See:

1. Optimal ω is equivalent to discretion with "as if" value of α (what was α? Loss $\alpha(x_{t+i}+k)^2 + \pi_{t+i}^2$)

2. Under precommitment ω,
   - volatility in x is greater
   - volatility in π is smaller (because $\frac{\lambda^2}{1-\beta\rho} > \lambda^2$)

Why? Because of that better tradeoff of π vs x you can get with precommitment.

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Optimal commitment policy (harder to derive)

Writing dynamic optimization problem & allowing C. B. to set x_t as a function of u_{t-1}, u_{t-2}, etc. as well as u_t)

F.o.c's imply

$$x_{t+i} - x_{t+i-1} = -\frac{\lambda}{\alpha}\pi_{t+i} \qquad (4.18)$$

Change in x (not level) is related to π.

except in initial period in which $x_t = -\frac{\lambda}{\alpha}\pi_t$.

Nominal interest rate is then

$$i_t = \frac{1}{\varphi} g_t + \left(1 - \frac{\lambda}{\alpha\varphi}\right) {}_t\pi^e_{t+1}$$

Note coeff. on π^e is less than one.

So if for some reason π^e deviates from R.E.E., everything blows to hell.

NK IS-LM &...

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Easy (cont'd)

$$\pi_t = (1-\lambda\omega)\frac{1}{1-\beta\rho}\, u_t = \frac{1-\lambda\omega}{1-\beta\rho}\, u_t \qquad (4.9)$$

$$= \frac{1}{1-\beta\rho}\, u_t - \frac{\lambda\omega}{1-\beta\rho}\, u_t$$

Recall $x_t = -\omega u_t$ put in here

$$\pi_t = \frac{\lambda}{1-\beta\rho}\, x_t + \frac{1}{1-\beta\rho}\, u_t \qquad (4.10)$$

compare with AS or Phillips curve:

$$\pi_t = \lambda x_t + \beta \pi^e_{t+1} + u_t \qquad (2.2)$$

(2.2) shows relation between $\left.\begin{matrix} x_t \\ u_t \end{matrix}\right\}$ π_t taking π^e as given.

Under "discretion" this is relevant tradeoff between x_t & π_t, because C.B. does take π^e as given.

(4.10) shows relation between $\left.\begin{matrix} x_t \\ u_t \end{matrix}\right\}$ π_t coming out of pre-commited rule, where rule (and u_t) have also determined π^e. Shows available tradeoff between x & π under such a pre-commit rule.