"NEW KEYNESIAN" PHILLIPS CURVE

Taylor Model and the Phillips Curve

For Romer discussion of Taylor model, we assumed
rational expectations,
$y = m - p$ ← AD curve
beliefs about path for m.

(1) If we don't assume those things, what does Taylor model imply about relation between π and y?

(2) Is it consistent with expectations-augmented Phil Curve

$$\pi_t = {}_{t-1}\pi^e_t + \alpha y_t \quad ?$$

Recall Taylor model:

$$x_t = \tfrac{1}{2}(p_t + \phi y_t + E_t p_{t+1} + \phi E_t y_{t+1})$$

$$p_t = \frac{x_t + x_{t-1}}{2} \qquad p_{t+1} = \frac{x_{t+1} + x_t}{2}$$

$$\Rightarrow p_t = \tfrac{1}{2}\left(\tfrac{1}{2}(p_t + E_t p_{t+1} + \phi y_t + \phi E_t y_{t+1}) + \tfrac{1}{2}(p_{t-1} + E_{t-1} p_t + \phi y_{t-1} + \phi E_{t-1} y_t)\right)$$

Multiply both sides by 4

$$4p_t = p_t + E_t p_{t+1} + p_{t-1} + E_{t-1} p_t + \phi(y_t + y_{t-1} + E_t y_{t+1} + E_{t-1} y_t)$$

Subtract $3p_t$ from both sides

$$p_t = -2p_t + E_t p_{t+1} + p_{t-1} + E_{t-1} p_t + \ldots$$

Subtract p_{t-1} from both sides
(next page)

"New Keynesian" Phillips Curve

Taylor Model (cont.)

$$\underbrace{p_t - p_{t-1}}_{\pi_t} = \underbrace{E_t p_{t+1} - p_t}_{E_t \pi_{t+1}} - p_t + E_{t-1} p_t + \ldots$$

$$\pi_t = E_t \pi_{t+1} + \underbrace{(E_{t-1} p_t - p_t)}_{\text{error in last period's expectation of this period's } p} + \phi(y_t + y_{t-1} + E_t y_{t+1} + E_{t-1} y_t)$$

This expression matches Roberts (8)

Another way to write it: add $+p_{t-1} - p_{t-1}$ to RHS

$$\pi_t = E_t \pi_{t+1} + \underbrace{E_{t-1} p_t - p_{t-1}}_{E_{t-1}\pi_t} \underbrace{- p_t + p_{t-1}}_{-\pi_t} + \ldots$$

Add π_t to both sides

$$2\pi_t = E_t \pi_{t+1} + E_{t-1} \pi_t + \ldots$$

$$\pi_t = \frac{1}{2}(E_t \pi_{t+1} + E_{t-1} \pi_t) + \frac{\phi}{2}(y_t + y_{t-1} + E_t y_{t+1} + E_{t-1} y_t)$$

Compare with expectations-augmented Phil curve:

$$\pi_t = E_{t-1} \pi_t + \alpha y_t$$

Taylor model gives something similar to e-a Phil curve, but not the same:

$$E_{t-1} \pi_t \quad \text{vs.} \quad \frac{1}{2}(E_t \pi_{t+1} + E_{t-1} \pi_t)$$

"forward-looking"

"NEW KEYNESIAN" PHILLIPS CURVE

Taylor Model (cont.)

How is experienced output related to future inflation?

Recall $\pi_t = E_t \pi_{t+1} + (E_{t-1} p_t - p_t) + \phi(\ldots\ldots)$

$$\Rightarrow E_t \pi_{t+1} = -E_{t-1} p_t + p_t + \pi_t - \phi(\ldots\ldots)$$

Add $-p_{t-1} + p_{t-1}$ to RHS:

$$E_t \pi_{t+1} = -\underbrace{(E_{t-1} p_t - p_{t-1})}_{E_{t-1}\pi_t} + \underbrace{(p_t + p_{t-1})}_{\pi_t} + \pi_t - \phi(\ldots\ldots)$$

$$= \pi_t - \phi(y_t + y_{t-1} + E_{t-1} y_t + E_t y_{t+1}) + (\pi_t - E_{t-1}\pi_t)$$

$$\pi_{t+1} = E_t \pi_{t+1} + \underbrace{(\pi_{t+1} - E_t \pi_{t+1})}_{\text{error in period } t\text{'s expectation of } \pi_{t+1}}$$

If expectations are rational, this error must be uncorrelated with variables from $t, t-1, \ldots$

$$\pi_{t+1} = \pi_t - \phi(y_t + y_{t-1} + E_{t-1} y_t + E_t y_{t+1}) + (\pi_t - E_{t-1}\pi_t) + (\pi_{t+1} - E_t \pi_{t+1})$$

What would happen if you regress $\Delta \pi_{t+1} = \pi_{t+1} - \pi_t$ on y_t, y_{t-1}? Assuming $E_{t-1} y_t$ and $E_t y_{t+1}$ have positive or zero correlation with y_t, y_{t-1}, coefficients on y_t, y_{t-1} will be <u>negative</u>.

Romer p. 338: "anticipated [expected] disinflation is associated with a [current] output <u>boom</u>"

"NEW KEYNESIAN" PHILLIPS CURVE (cont.)

Other models of price setting

Rotemberg (1982): Quadratic Price Adjustment Cost

Assume "menu cost" of adjusting p is not fixed, but increases quadratically with size of price adjustment:

$$\text{cost} = c\,(p_{it} - p_{it-1})^2 \leftarrow \text{quadratic}$$

Also assume: cost of $(p_{it} - p_{it}^*)$ ← letting p deviate from p^*

$$= (p_{it} - p_{it}^*)^2$$

Firm's problem:

$$\text{minimize} \quad E_t \sum_{\tau=t}^{\infty} \underbrace{\Theta}_{\text{discount factor.}}{}^{\tau-1}\left[(p_{i\tau} - p_{i\tau}^*)^2 + c\,(p_{i\tau} - p_{i\tau-1})^2\right]$$

$$= E_t\left[(p_{it} - p_{it}^*)^2 + c\,(p_{it} - p_{it-1})^2 + \Theta\left((p_{it+1} - p_{it+1}^*)^2 + c\,(p_{it+1} - p_{it})^2\right) + \ldots\right]$$

First-order condition:

$$\frac{\partial E_t \cdots}{\partial p_t} = 0 = E_t\left[2\,(p_{it} - p_{it}^*) + 2c\,(p_{it} - p_{it-1}) - \Theta\, c2\,(p_{it+1} - p_{it})\right]$$

Assume $\Theta = 1$ (no discounting)

then above gives:

$$p_{it} - p_{it-1} = E\left[p_{it+1} - p_{it}\right] - \frac{1}{c}\,(p_{it} - p_{it}^*)$$

Let $p_{it}^* = p_t + \phi\, y_t$

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Rotemberg (1982) model (cont.)

$$\Rightarrow p_{it} - p_{it-1} = E[p_{it+1} - p_{it}] - \tfrac{1}{c}(p_{it} - p_t - \phi y_t)$$

Firm i knows all other firms are like Firm i, so:

$$E[p_{it+1} - p_{it}] = E[p_{t+1} - p_t] = E_t \pi_{t+1}$$

$$p_{it} - p_{it-1} = E_t \pi_{t+1} - \tfrac{1}{c}(p_{it} - p_t - \phi y_t)$$

As all the identical firms set their prices at p_{it},

$$p_{it} = p_t \quad \text{so:}$$

$$p_t - p_{t-1} = \pi_t = E_t \pi_{t+1} + \frac{\phi}{c} y_t$$

Calvo (1983): Staggered Adjustment with Random Time of Adjustment

Firm i must hold p_i fixed until it is time to adjust,
Adjustment time not on calendar schedule (like Taylor),
but arrives randomly:

α Probability firm i can adjust in any given period
= Fraction of firms adjusting in any given period

$(1-\alpha)$ Probability I won't be able to adjust again next period

$(1-\alpha)(1-\alpha)$ Probability I won't be able to adjust again next period or following period

etc.

[Note: Roberts (1995) calls α "π"]

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Calvo (1983) model (cont.)

Recall in section 7.1 Romer defined

q_t Probability price firm sets in period zero is still in effect at future time t

so here $q_2 = (1-\alpha)(1-\alpha) = (1-\alpha)^2$

$$q_t = (1-\alpha)^t$$

Looking forward from period t to period $t+j$,

$$q_j = (1-\alpha)^j$$

$$\sum_{j=0}^{\infty} q_j = \sum_{j=0}^{\infty} (1-\alpha)^j = \frac{1}{\alpha}$$

Recall in section 7.1 Romer also derived

$$p_i = \sum_{t=0}^{\infty} \omega_t E_0[p_t^*] \qquad (7.13)$$

price set by a firm that is able to adjust this period

where $\omega_t = q_t \Big/ \sum_{\tau=0}^{\infty} q_\tau$

Here, $\omega_t = \dfrac{(1-\alpha)^j}{1/\alpha} = \alpha(1-\alpha)^j$

So ...

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Calvo (1983) model (cont.)

$$p_i = \sum_{t=0}^{\infty} \alpha(1-\alpha)^j E_0[p_j^*] = \alpha \sum_{t=0}^{\infty} (1-\alpha)^j E_0[p_j^*]$$

New notation:

x_t Price chosen by a firm that can adjust at time t [Roberts calls this z_t]

hence

$$x_t = \alpha \sum_{j=0}^{\infty} (1-\alpha)^j E_t\, p_{t+j}^*$$

This is Romer (7.56) with $\beta = 1$

Note: this makes sense. If $p_{t+j}^* = p^*$ for all j,

$$x_t = \alpha \sum_{j=0}^{\infty} (1-\alpha)^j p^* = \alpha \frac{1}{\alpha} p^* = p^*$$

Note also that expanding above gives

$$x_t = \alpha(1-\alpha)^0 p_{it}^* + \alpha(1-\alpha) E_t\, p_{t+1}^* + \alpha(1-\alpha)^2 E_t\, p_{t+2}^* + \dots$$

$$= \alpha\, p_{it}^* + (1-\alpha)\underbrace{\left(\alpha E_t\, p_{t+1}^* + \alpha(1-\alpha) E_t\, p_{t+2}^* + \dots\right)}_{\alpha \sum_{j=1}^{\infty}(1-\alpha)^j E_t\, p_{t+1+j}^*}$$

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Calvo (1983) model (cont'd)

and

$$\alpha \sum_{j=1}^{\infty} (1-\alpha)^j E_t p^*_{t+1+j} = E_t \, \alpha \sum_{j=1}^{\infty} (1-\alpha)^j p^*_{t+1+j} = E_t x_{t+1}$$

so

$$x_t = \alpha p^*_{it} + (1-\alpha) E_t x_{t+1} \qquad (7.57 \text{ with } \beta = 0)$$

price level (average p_i across all firms) is

$$p_t = \alpha x_t + (1-\alpha) p_{t-1} \qquad (7.53)$$

which can also be derived like this:

$$p_t = \alpha x_t + \underbrace{(1-\alpha)\alpha}_{} x_{t-1} + (1-\alpha)(1-\alpha)\alpha x_{t-2} + \cdots$$

probability a firm couldn't adjust this period, but was able to adjust last period

$$= \alpha \sum_{j=0}^{\infty} (1-\alpha)^j x_{t-j}$$

$$= \alpha x_t + (1-\alpha) \underbrace{\sum_{j=0}^{\infty} (1-\alpha)^j x_{t-1-j}}_{p_{t-1}}$$

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Calvo (1983) model (cont.)

Finally, as always

$p_{it}^* = p_t + \phi y_t$ [Romer calls ϕ "β" and adds a disturbance ε]

What does this imply for inflation π?

We'll find

$$\pi_t = E_t \pi_{t+1} + \frac{\alpha^2 \phi}{1-\alpha} y_t$$ (7.60 with $\beta = 1$)

similar to Rotemberg

How do we get (7.60)?

We'll follow Roberts (1995), but use Romer's notation

Recall

$$x_t = \alpha p_{it}^* + (1-\alpha) E_t x_{t+1}$$

$$= \alpha p_{it}^* + E_t x_{t+1} - \alpha E_t x_{t+1}$$

Rearrange this (subtract x_t from both sides etc.)

$$\Rightarrow E_t(x_{t+1} - x_t) = \alpha E_t x_{t+1} - \alpha p_{it}^*$$ (Roberts 4)

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Calvo (1983) model (cont'd)

Recall $p_t = \alpha x_t + (1-\alpha) p_{t-1}$

$$= \alpha x_t + p_{t-1} - \alpha p_{t-1}$$

$\Rightarrow p_t - p_{t-1} = \alpha x_t + \alpha p_{t-1}$ (Roberts 5)

so $E_t(p_{t+1} - p_t) = \alpha(E_t x_{t+1} - p_t)$

$$\Rightarrow E_t x_{t+1} = p_t + \frac{1}{\alpha} E_t(p_{t+1} - p_t)$$

substitute this into Roberts (4) to get:

$$E_t(x_{t+1} - x_t) = \alpha p_t + E_t(p_{t+1} - p_t) - \alpha p_{it}^*$$

$$= E_t(p_{t+1} - p_t) - \alpha(\underbrace{p_{it}^* - p_t}_{\phi y_t})$$

$$E_t \Delta x_{t+1} = E_t(\overbrace{p_{t+1} - p_t}^{\pi_{t+1}}) - \alpha\phi y_t \text{ (Roberts 6)}$$

setting (Roberts 6) = (Roberts 4) gives..

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Calvo (1983) model (cont.)

$$\alpha E_t x_{t+1} - \alpha p_{it}^* = E_t \pi_{t+1} - \alpha \phi y_t$$

$$\Rightarrow E_t x_{t+1} = p_{it}^* + \tfrac{1}{\alpha} E_t \pi_{t+1} - \phi y_t$$

$$= p_t + \phi y_t + \tfrac{1}{\alpha} E_t \pi_{t+1} - \phi y_t$$

$$= p_t + \tfrac{1}{\alpha} E_t \pi_{t+1}$$

Subtract x_t from both sides

$$E_t x_{t+1} - x_t = \tfrac{1}{\alpha} E_t \pi_{t+1} + p_t - x_t$$

[From $p_t = \alpha x_t + (1-\alpha) p_{t-1}$, we know

$$x_t = \tfrac{1}{\alpha} p_t + p_{t-1} - \tfrac{1}{\alpha} p_{t-1}$$]

so

$$E_t \Delta x_{t+1} = \tfrac{1}{\alpha} E_t \pi_{t+1} + p_t - \tfrac{1}{\alpha} p_t - p_{t-1} + \tfrac{1}{\alpha} p_{t-1}$$

$$= \tfrac{1}{\alpha} \left(E_t \pi_{t+1} + (\alpha - 1) \pi_t \right)$$ (Roberts between 6 & 7)

Multiply both sides by α ....

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Calvo (1983) model (cont'd.)

$$\alpha E_t \Delta x_{t+1} = E_t \pi_{t+1} + (\alpha - 1)\pi_t$$

$$(1-\alpha)\pi_t = E_t \pi_{t+1} - \alpha E_t \Delta x_{t+1}$$

Recall $E_t \Delta x_{t+1} = E_t \pi_{t+1} - \alpha \phi y_t$ from Roberts 6

so

$$(1-\alpha)\pi_t = E_t \pi_{t+1} - \alpha E_t \pi_{t+1} + \alpha^2 \phi y_t$$

$$\Rightarrow \pi_t = E_t \pi_{t+1} + \frac{\alpha}{1-\alpha}\phi y_t$$

compare with

Rotemberg

$$\pi_t = E_t \pi_{t+1} + \frac{1}{c}\phi y_t$$

from adjustment cost $c(p_t - p_{t-1})^2$

Taylor

$$\pi_t = \tfrac{1}{2}\left(E_t \pi_{t+1} + E_{t-1}\pi_t\right) + \phi\left(y_t + y_{t-1} + E_t y_{t+1} + E_{t-1} y_t\right)$$