Static GE Model with Imperfect Competition

Assumptions

Each Firm i is monopoly producer of a good

"Continuum" of goods along a line 0–1

like $\frac{1}{N}\sum_{i=1}^{N} C_i$ where $N \to \infty$

Production function: $Y_i = L_i$ (6.39)

Monopoly profits go to households.

Labor market is competitive (simple, but we'll find this is too simple)

One representative household (same as infinite number of tiny households).

$$U = C - \frac{1}{\gamma} L^{\gamma} \qquad \gamma > 1 \qquad (6.40)$$

$$\left[\int_{i=0}^{1} C_i^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \qquad \eta > 1 \qquad (6.41)$$

"Dixit-Stiglitz" setup

η will be elasticity of product demand.

If $\eta < 1$, monopoly would raise price infinitely high.

Static GE Model....

Assumptions (cont.)

No G, I or NX so $Y = C$ (6.42)

$$Y = \frac{M}{P} \qquad (6.43)$$

"M" is nominal AD.

Or, from money in utility function & static (no interest rate, no Euler eqn.)

M can be money supply times a constant.

example: $U = C^{\alpha}(M/P)^{1-\alpha} - \frac{1}{\gamma}L^{\gamma}$ where $C + \frac{M}{P} \leq Z$

$$= (\overbrace{Z - \tfrac{M}{P}}^{C})^{\alpha}(M/P)^{1-\alpha} - \frac{1}{\gamma}L^{\gamma}$$

$$\frac{\partial U}{\partial(M/P)} = 0 = C^{\alpha}(1-\alpha)(M/P)^{-\alpha} + \alpha C^{\alpha-1}(-1)(M/P)^{1-\alpha}$$

$$\Rightarrow C^{\alpha}(1-\alpha)(M/P)^{-\alpha} = \alpha C^{\alpha-1}(M/P)^{1-\alpha}$$

$$\Rightarrow C^{\alpha}C^{1-\alpha} = \frac{\alpha}{1-\alpha}(M/P)^{1-\alpha}(M/P)^{\alpha}$$

$$\Rightarrow C = Y = \frac{\alpha}{1-\alpha}\frac{M}{P}$$

Static GE Model (cont.)

What we want to derive

We want to think about costs vs. benefits to a firm of adjusting P_i when there is a change in M.

To do this, we need:

— demand for one firm's good

— costs, which here means labor costs, which here means wage W

Note model is "symmetric": all firms & households the same, so whatever we derive for one goes for all.

An equilibrium is:
"if all other firms behave like X, it's best for <u>me</u> to behave like X."

Static GE Model

Demand for one firm's good

Allocate total spending S across goods i to maximize C

$$\mathcal{L} = \left[\int_{i=0}^{1} C_i^{\frac{\eta-1}{\eta}}\, di\right]^{\eta/\eta-1} + \lambda\left[S - \int_{i=0}^{1} P_i C_i\, di\right] \qquad (6.44)$$

F.O.C.:

$$0 = \frac{\partial \mathcal{L}}{\partial C_i} = \frac{\eta}{\eta-1}\left[\int_{j=0}^{1} C_j^{\frac{\eta-1}{\eta}}\, dj\right]^{\frac{\eta}{\eta-1}-1} \frac{\eta-1}{\eta} C_i^{\frac{\eta-1}{\eta}-1} - \lambda P_i$$

(holding all other C's fixed)

$$\boxed{\frac{\eta}{\eta-1} - 1 = \frac{1}{\eta-1}} \quad \boxed{\frac{\eta-1}{\eta} - 1 = -\frac{1}{\eta}}$$

so

$$\frac{\eta}{\eta-1}\left[\int_{j=1}^{1} C_j^{\frac{\eta-1}{\eta}}\, dj\right]^{\frac{1}{\eta-1}} \frac{\eta-1}{\eta} C_i^{-\frac{1}{\eta}} = \lambda P_i \qquad (6.45)$$

(cancel out)

Solve for C_i, gives

$$C_i = \underbrace{\lambda^{-\eta}\left[\int_{j=1}^{1} C_j^{\frac{\eta-1}{\eta}}\, dj\right]^{\frac{\eta}{\eta-1}}}\, P_i^{-\eta} \qquad (6.46)$$

Call this "A." Has no subscript because it has same value for every i.

But what is A? A shortcut to find it:

Static GE Model
Demand for ... (cont.)

We know $\int_{i=0}^{1} P_i C_i \, di = S$

so $\int_{i=0}^{1} P_i A P_i^{-\eta} \, di = S$

so $$A = \frac{S}{\int_{j=0}^{1} P_j^{1-\eta} \, dj} \qquad (6.47)$$

so $$C = \Big[\int_{i=0}^{1} \underbrace{C_i}_{A P_i^{-\eta}}{}^{\frac{\eta-1}{\eta}} \Big]^{\frac{\eta}{\eta-1}} = \frac{S \left(\int_{i=0}^{1} P_i^{1-\eta} \, di \right)^{\eta/\eta-1}}{\int_{j=0}^{1} P_j^{1-\eta} \, dj} \qquad (6.48)$$

Now $\int_{j=0}^{1} P_j^{1-\eta} \, dj = \int_{i=0}^{1} P_i^{1-\eta} \, di$

so $$C = \frac{S}{\left(\int_{i=0}^{1} P_i^{1-\eta} \, di \right)^{1/1-\eta}}$$ ⟸ (a kind of price index

Call it "P"

so $C = S/P \qquad A = \frac{S}{P^{1-\eta}} = S P^{\eta-1}$

Static GE Model

Demand for... (cont.)

Understanding price index

Think

$$\frac{1}{N}\left(\sum_{i=0}^{N} P_i^{1-\eta}\right)^{\frac{1}{1-\eta}} \qquad \text{A geometric average}$$

Note: if all P_i's $= \bar{P}$, $P = \bar{P}$

Back to demand

Recall $C_i = A P_i^{-\eta} = S P^{\eta-1} P_i^{-\eta} = S P^{-1} \left(\frac{1}{P}\right)^{-\eta} P_i^{-\eta}$

$$= \left(\frac{P_i}{P}\right)^{-\eta} \frac{S}{P} = \left(\frac{P_i}{P}\right)^{-\eta} C \qquad (6.50)$$

in logs, $c_i = c - \eta(p_i - p)$ (elasticity

Static GE Model (cont'd)

Labor supply

Recall $U = C - \frac{1}{\gamma} L^{\gamma}$ s.t. $S \leq WL + R$ ← (profit income

$$C = S/P = (WL + R)/P$$

substitute in for C in utility function:

$$U = \frac{WL + R}{P} - \frac{1}{\gamma} L^{\gamma} \qquad (6.51)$$

Choose L taking W, P, R as given

$$\frac{\partial U}{\partial L} = 0 = \frac{W}{P} - L^{\gamma - 1}$$

$$\Rightarrow L = \left(\frac{W}{P}\right)^{1/\gamma - 1} \qquad (6.53)$$

in logs $l = \frac{1}{\gamma - 1}(w - p)$ ← (elasticity

Recall there is "one" household, so this is aggregate LS

Firm behavior

Choose P_i to max profit taking W, P, Y as given.

Real profit:

$$\frac{R_i}{P} = \frac{P_i \overbrace{Y_i}^{(P_i/P)^{-\eta} Y}}{P} - \frac{W \overbrace{L_i}^{Y_i}}{P} = \left(\frac{P_i}{P}\right)^{1-\eta} Y - \frac{W}{P}\left(\frac{P_i}{P}\right)^{-\eta} Y \quad (6.55)$$

$$\frac{\partial R_i/P}{\partial P_i} = 0 = (1-\eta)(P_i/P)^{-\eta} Y + \eta \frac{W}{P}\left(\frac{P_i}{P}\right)^{-\eta-1} Y \quad (6.56)$$

$$\Rightarrow P_i/P = \frac{\eta}{\eta-1}\frac{W}{P}$$

$$\text{or } P_i = \frac{\eta}{\eta-1} W$$

$W \leftarrow$ marginal cost

(markup

$$= \frac{1}{1-1/\eta} W$$

Bigger elasticity η, smaller mark-up. Won't work if $\eta < 1$.

All firms identical so $P_i = P$

$$P = \frac{1}{1-1/\eta} W$$

$$\frac{W}{P} = 1 - \frac{1}{\eta}$$

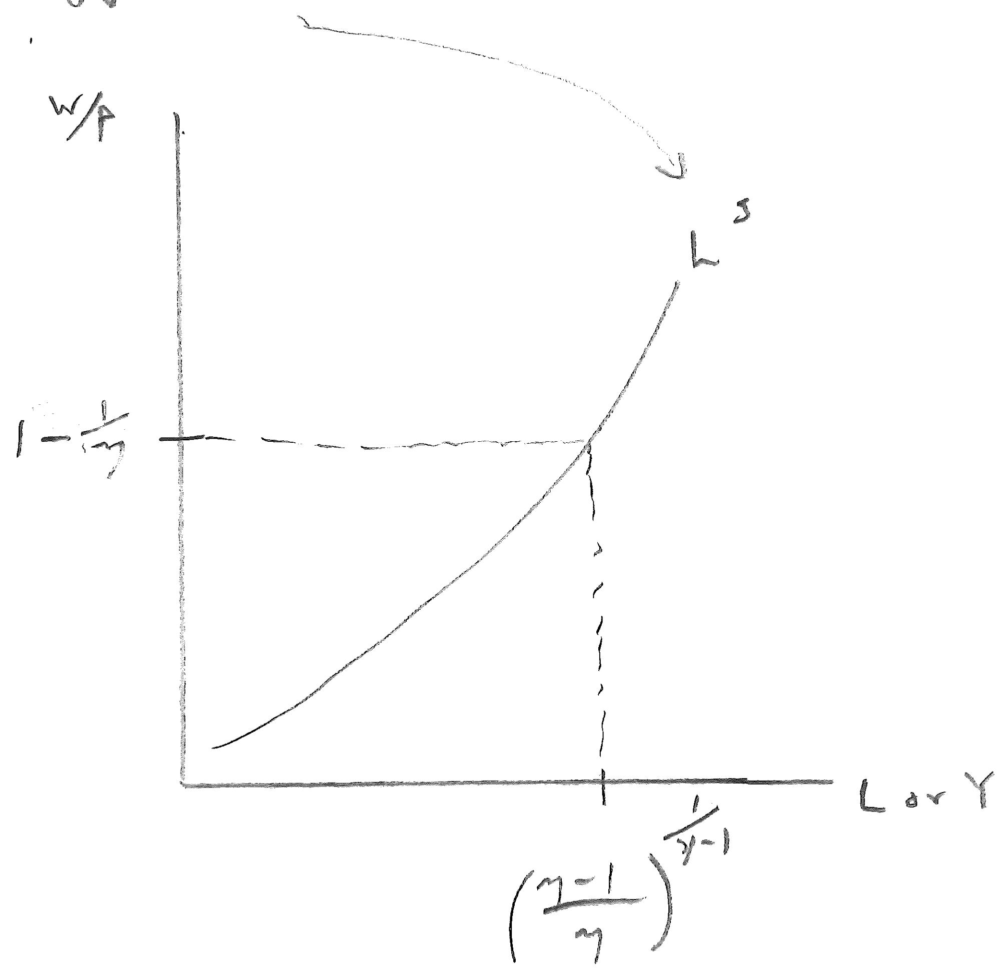
Real wage is reciprocal of markup.

Static GE Model (cont.)

Output & employment

Recall $L = Y = (W/P)^{\frac{1}{\gamma-1}}$

or $W/P = Y^{\gamma-1} = L^{\gamma-1}$



$$1 - 1/\eta = L^{\gamma-1}$$

$$\Rightarrow \bar{L} = \bar{Y} = \left(\frac{\eta-1}{\eta}\right)^{1/\gamma-1} \qquad (6.61)$$

From $Y = M/P$, $P = \frac{M}{Y} = M \Big/ \left(\frac{\eta-1}{\eta}\right)^{1/\gamma-1}$ (6.62)

Static GE Model (cont.)

Equilibrium output is "too low"

because economy is monopolized.

$$\bar{Y} = \left(\frac{\eta - 1}{\eta}\right)^{\frac{1}{\gamma - 1}} < 1 \quad \text{because } \eta > 1, \gamma > 1$$

Compare with "social optimum" output (max utility)

$$\max_{Y} U_i = C_i - \frac{1}{\gamma} L_i^{\gamma} \quad \text{given } C = Y, \; Y = L$$

$$= Y - \frac{1}{\gamma} Y^{\gamma}$$

$$\frac{\partial U}{\partial Y} = 0 = 1 - \gamma \frac{1}{\gamma} Y^{*\,\gamma - 1}$$ optimal Y

$$Y^* = 1 > \bar{Y}$$

Note this implies $\left.\frac{\partial U_i}{\partial Y}\right|_{Y = \bar{Y}} > 0$

increasing output above natural rate would be a good thing!

Static GE Model (cont.)

In terms of p_i^*, p, y, m

Recall $P_i^* = \frac{\eta}{\eta-1} P Y^{\gamma-1}$ (From 6.59)

profit-maxxing price

in logs $p_i^* = \ln\left(\frac{\eta}{\eta-1}\right) + p + (\gamma-1)y$

or $p_i^* - p = \underbrace{\ln\left(\frac{\eta}{\eta-1}\right)}_{c} + \underbrace{(\gamma-1)}_{\phi} y$ (6.61)

$= c + \phi\, y$

$0 < \gamma - 1 < 1$

Recall $y = m - p$

$\Rightarrow p_i^* = c + (1-\phi)p + \phi m$

1) Holding p fixed, why does $m\uparrow$ raise p_i^*?

Recall p^* is markup over mc, and $mc = W = PY^{\gamma-1}$

For given p, $M\uparrow \rightarrow Y\uparrow, L^D\uparrow$

With upward-sloping L^S, real wage $\uparrow \rightarrow W\uparrow \rightarrow mc\uparrow$

2) How do we know $\gamma > 1$?

$U = C - \frac{1}{\gamma} L^\gamma$

It's increasing marginal disutility of labor.