Problem set on maximization of expected utility and Jensen's inequality
(adapted from Romer problem 5.6)

Consider a model similar to the Diamond OLG mode. A person lives two periods, period 1 and period 2. He acts to maximize the expected value of his lifetime utility.
Lifetime utility is $U = \ln C_1 + \ln C_2$.

Hint: to get your answers to the questions below, do not set up and solve Lagrangians. Just use the budget constraint to get C_2 as a function of C_1, substitute that into the expected utility function and take one first order condition.

(1) Suppose a person receives labor income equal to W_1 in the first period and no labor income in the second period. Second-period consumption is thus $C_2 = (1+r)(W_1 - C_1)$ where r is the real return to holding a unit of capital in period 2.

a) Suppose that in period 1 people know with certainty that r will be equal to a value $\overline{r}$. What is C_1?

b) Now suppose that in period 1 r is uncertain. $r = \overline{r} + \epsilon$ where ϵ is mean-zero "white noise." Note that as of period 1 the expected value of r is equal to $\overline{r}$ from part a), and $E[\epsilon] = 0$. Will C_1 be greater than, less than or equal to the value of C_1 you found in part a)?

(2) Now suppose a person receives no labor income in the first period. Instead he receives labor income W_2 in the second period. To consume in the first period, he borrows at interest rate r. That is, in period 2 he must pay $(1+r)$ for each unit of consumption he received in period 1. Thus second-period consumption is $C_2 = W_2 - (1+r)C_1$.

a) Suppose that in period 1 people know with certainty that r will be equal to a value $\overline{r}$ and also know that W_2 will be equal to a value $\overline{W}$. What is C_1?

b) Now suppose that in period 1 r is certain, but W_2 is not. $W_2 = \overline{W} + \epsilon$ where ϵ is mean-zero "white noise." Note that as of period 1 the expected value of W_2 is equal to $\overline{W}$ from part a), and $E[\epsilon] = 0$. Will C_1 be greater than, less than or equal to the value of C_1 you found in part a)? Hint: apply Jensen's inequality.