Economics 613
Problem set on rational expectations

1) Suppose aggregate demand in a economy follows

$$y_t = a\ (m_t - p_t)$$

and the economy has a Friedman-Phelps Phillips curve:

$$\pi_t = {}_{t-1}\pi_t^e + fy_t$$

where *y* denotes the log of the output gap (*y* is zero if output is equal to the natural rate of output). Expectations are model-consistent rational expectations (what you get in rational expectations equilibrium). Everyone knows the money supply evolves according to a random walk:

$$m_t = m_{t-1} + \varepsilon_t$$

where ε is a mean-zero, i.i.d. random variable with variance σ_ε^2. As of time *(t-1)*, no one knows what the *realized* value of ε_t will be.

a) Derive ${}_{t-1}p_t^e$ in terms of m_{t-1}.

b) Derive p_t in terms of $m_{t-1}$and ε_t.

c) Derive π_t in terms of m_{t-1} , p_{t-1} and ε_t .

d) Derive y_t in terms of ε_t.

e) What is the variance of *y*?

2) Suppose aggregate demand in a economy follows

$$y_t = a\ (m_t - p_t) + \varepsilon_t$$

where ε is a mean-zero, i.i.d. random variable with variance σ_ε^2. As of time *(t-1)*, no one knows what the *realized* value of ε_t will be. The economy has a Friedman-Phelps Phillips curve:

$$\pi_t = {}_{t-1}\pi_t^e + fy_t$$

where *y* denotes the log of the output gap (*y* is zero if output is equal to the natural rate of output). Expectations are model-consistent rational expectations (what you get in rational expectations equilibrium). Everyone knows the money supply is always equal to a fixed value m.

a) Derive ${}_{t-1}p_t^e$ in terms of m.

b) Derive p_t in terms of m and ε_t.

c) Derive π_t in terms of m, p_{t-1} and ε_t .

d) Derive y_t in terms of ε_t.

e) What is the variance of *y*?

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3) Suppose an economy has an IS curve corresponding to:

$$y_t = -ar_t$$

and a Friedman-Phelps Phillips curve:

$$\pi_t = {}_{t-1}\pi_t^e + fy_t$$

where *y* denotes the log of the output gap (*y* is zero if output is equal to the natural rate of output) and *r* denotes the difference between the real interest rate and the natural rate of interest.

The central bank sets interest rates, not the money supply, and follows an interest-rate rule:

$$r_t = by_t + c(\pi_t - \pi)$$

where π is a fixed value. Note that this interest-rate rule is consistent with the general form $r(\pi, y)$ assumed for the IS/MP model.

Expectations are model-consistent rational expectations (what you get in rational expectations equilibrium).

Solve for π_t.

4) Suppose an economy has an IS curve corresponding to:

$$y_t = -ar_t + \varepsilon_t$$

and a Friedman-Phelps Phillips curve:

$$\pi_t = {}_{t-1}\pi_t^e + fy_t$$

where *y* denotes the log of the output gap (*y* is zero if output is equal to the natural rate of output) , *r* denotes the difference between the real interest rate and the natural rate of interest, and ε is a mean-zero, i.i.d. random variable with variance σ_ε^2.

The central bank sets interest rates, not the money supply, and follows an interest-rate rule:

$$r_t = by_t + c(\pi_t - \pi)$$

where π is a fixed value. Expectations are model-consistent rational expectations (what you get in rational expectations equilibrium).

a) Solve for ${}_{t-1}\pi_t^e$.

b) Derive π_t in terms of ε_t and π.

c) Derive y_t in terms of ε_t .

d) What is the variance of *y*?