RBC Theory Developments

Some criticisms of original RBC models & early literature

1) Some implications of model are inconsistent with behavior of macro time-series data.

— Model's C & L are too stable — they aren't as procyclical (positively correlated with Y) as true C & L

— Model's real wage W is too procyclical

— In model, L varies because of 2 (hours per worker) while number of employees grows with popn.
In reality, L varies mostly because number of employees varies.

2) Key assumptions (fundamental for model) are silly

— Model needs strong intertemporal substn. of labor supply (desired work hours respond strongly to temporary movements in real wage).
In reality, not so.

— Model needs technological regress (absolute decline in A) to generate cyclical downturn ($C\downarrow$, $L\downarrow$, $Y\downarrow$).
Slowdown in A growth is not enough.
Technological regress?

RBC Theory Developments

Some criticisms...

2) Key assumptions....

Why does model require technological regress?

Suppose $\tilde{A}_{t-1} = \tilde{K}_{t-1} = 0$ (variables were at LRSS values last period)

hence $\tilde{K}_t \doteq 0$

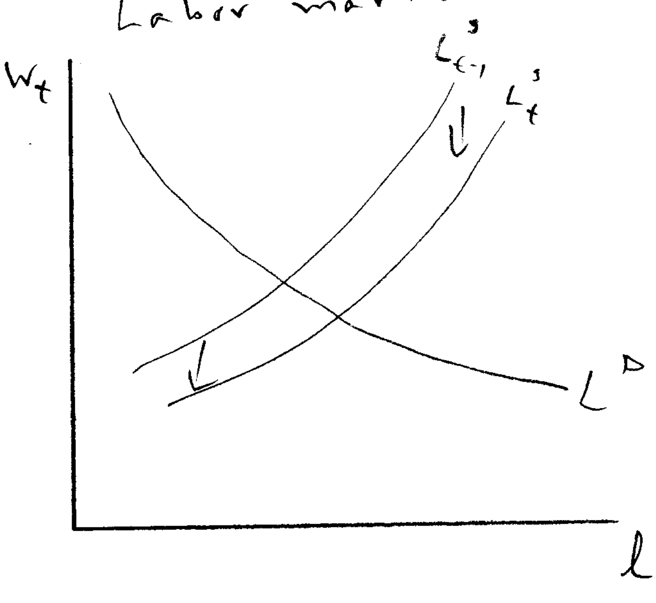
capital stock near LRSS this period,

$$\frac{K_t}{K_{t-1}} = e^{g+n}$$ (capital stock grew at rate $g+n$

Now, $\tilde{A}_t < 0$. A growth <u>slowdown</u> is $1 \leq \frac{A_t}{A_{t-1}} < e^{g}$

Technological <u>regress</u> is $\frac{A_t}{A_{t-1}} < 1$, $A_t < A_{t-1}$

Labor market:



$$w_t^s = b\, c_t \frac{1}{1 - l_t}$$

If $c_t < c_{t-1}$, L^s shifted <u>down</u>.

To make l fall, L^D must shift down enough to overcome this.

$$W_t^D = (1-\alpha)\, K_t^{\alpha} A_t^{1-\alpha} (N_t l_t)^{-\alpha}$$

Since $K_t > K_{t-1}$, to make l fall, A_t must be less than A_{t-1}.

Possible ways to make l fall as c falls: change assumptions so that

— L^s falls while c falls

— L^D falls even though $K_t > K_{t+1}$, $A_t = A_{t-1}$. A third factor?

RBC Theory Developments

Home Production (Benhabib, Rogerson & Wright, 1991)

Single-period utility is

$$u(c, 1-l) = \ln(c) + b\ln(1-l)$$

but $c_t = \left[a\, c_{Mt}^{\phi} + (1-a) c_{Ht}^{\phi}\right]^{1/\phi}$

$$l_t = h_{Mt} + h_{Ht}$$

c_M : "Market" (store-bought) consumption

c_H : "Home-produced" consumption

h_M : Hours work in "market," for a wage

h_H : Hours "worked" at home.

Home production function: $C_{Ht} = A_{Ht} K_t^{\eta} H_t^{1-\eta}$

(all home prodn is consumed; there's no home-produced capital)

When $\tilde{A}_{Ht} \uparrow$ (a positive home productivity shock),

$c_M \downarrow$, $h_M \downarrow$, investment in "market" capital falls.

Hence, downturns without technological regress

(also, h_M responds more to A_{Mt}; market L^s more elastic)

RBC Theory Developments

Capital utilization (King & Rebelo, 1999)

Z_t Capital utilization, how hard you run your capital

$$Y_t = (Z_t K_t)^{\alpha} (A_t L_t)^{1-\alpha}$$

$N l_t$

[Note: King & Rebelo have $Y_t = A_t (Z_t K_t)^{1-\alpha} (N_t l_t)^{\alpha}$]

$$K_{t+1} = K_t + Y_t - C_t - \delta(Z_t) K_t \quad \text{where } \delta'(Z_t) > 0, \; \delta''(Z_t) > 0.$$

Depreciation rate increases with Z.

Note: labor demand at given A, K increases with Z:

$$\frac{\partial Y}{\partial L} = w = (1-\alpha) Z_t^{\alpha} K_t^{\alpha} A_t^{1-\alpha} L_t^{-\alpha}$$

also

$$\frac{\partial Y}{\partial K} = \alpha Z^{\alpha} (A L)^{1-\alpha} K^{\alpha-1}$$

$$\frac{\partial Y}{\partial Z} = \alpha K^{\alpha} (A L)^{1-\alpha} Z^{\alpha-1} =$$

Z is another choice variable.

LRSS value of Z must be fixed.

What determines value of Z chosen by social planner or profit-maximizing firms?

RBC Thy Developments

Capital utilization (cont.)

Social planner's Bellman equation:

$$V(k_t, \text{etc.}) = \max_{c,l,z} \left\{ U_c(C_t) + U_{1-l}(1-l) + E_t\left[V(k_{t+1}, \text{etc.}) \right] \right\}$$

F.O.C. for Z: $0 = E_t\left[V_k(k_{t+1}),\right] \dfrac{\partial K_{t+1}}{\partial z_t}$

$$\frac{\partial K_{t+1}}{\partial z_t} = \frac{\partial Y}{\partial z_t} - \delta'(z_t) K_t$$

$$\delta'(z_t) K_t = \alpha K^{\alpha} (AL)^{1-\alpha} z^{\alpha-1}$$

$$z_t \delta'(z_t) = \alpha z^{\alpha} (AL)^{1-\alpha} K^{\alpha-1} = \frac{\partial Y}{\partial K} = r + \delta(z_t)$$

Hence optimal z is value for which

$$z_t \delta'(z_t) - \delta(z_t) = r$$

LRSS value

Note: LRSS value of z is determined by r^*.

Also: $\dfrac{\partial r}{\partial z} = \delta'(z_t) + z_t \delta''(z_t) - \delta'(z_t) = z_t \delta''(z_t)$

$$\frac{\partial z}{\partial r} = \frac{1}{z_t \delta''(z_t)} > 0$$

If $r < r^*$, $z < z^*$

Recall: in baseline model, $r_t < r^*$ if $\tilde{A} < 0$ – not only if technology regresses, but also if technology growth is positive but slower than usual (slowdown).

RBC Thy Developments
Capital utilization (cont.)

What happens to Z in a technology slow down, holding ℓ fixed?

Suppose $\tilde{A}_{t-1} = \tilde{k}_{t-1} = 0$,

$\tilde{k}_t = 0$, so $\frac{k_t}{k_{t-1}} = e^{g+n}$

and now $A_t = A_{t-1}$ (slowdown not regress)

Optimal Z satisfies

$$z_t \delta(z_t) = \alpha Z^{\alpha} (A \ell N_t)^{1-\alpha} k^{\alpha-1}$$

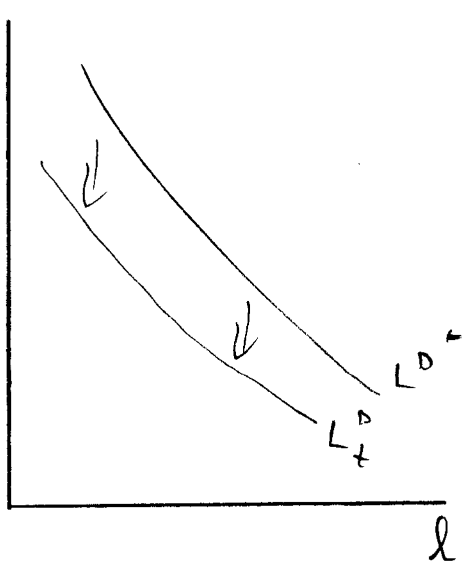
$$z_t^{1-\alpha} \delta(z_t) = \alpha (A \ell N_t)^{1-\alpha} k^{\alpha-1}$$

$$\frac{z_t^{1-\alpha}\delta(z_t)}{z_{t-1}^{1-\alpha}\delta(z_{t-1})} = \frac{A_t}{A_{t-1}}^{1-\alpha} \frac{L_t}{L_{t-1}}^{1-\alpha} \frac{k_t}{k_{t-1}}^{\alpha-1} = (e^n)^{1-\alpha} (e^{g+n})^{\alpha-1} =$$

$$= e^{-g(1-\alpha)} < 1$$

Hence, holding ℓ fixed, $Z_t < Z_{t-1}$

Labor demand:



$$w_t^D = \frac{\partial y}{\partial L} = (1-\alpha) Z_t^{\alpha} k_t^{\alpha} A_t^{1-\alpha} (N \ell_t)^{-\alpha}$$

Since $Z_t < Z_{t-1}$, labor demand shifts <u>down</u> even though $A_t = A_{t-1}$.

Can this make ℓ fall?

Depends on what happens to L^S.

RBC Thy Developments

Indivisible Labor

Makes L^S very elastic, without messing up LRSS

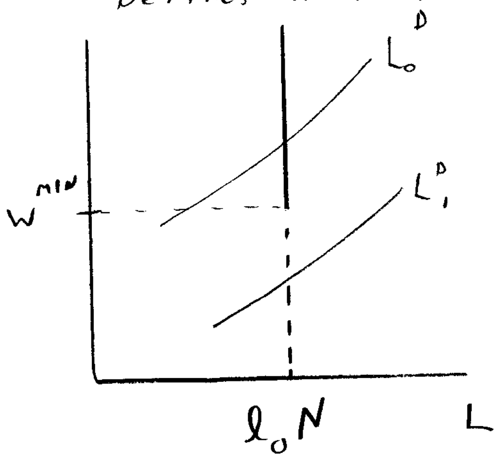
Extra assumption: you must work l_0 hours or not at all.

By itself, this would cause L^S to be inelastic: everyone works l_0 hours unless

$$\underbrace{U_C(wl_0) + U_{1-l}(1-l_0)}_{\text{utility I get if I work at wage } w} < \underbrace{U_{1-l}(1)}_{\text{utility from lots of leisure, no consumption}}$$

or $U_C(wl_0) < U_{1-l}(1) - U_{1-l}(1-l_0)$

Defines a minimum wage w^{MIN}



At L_1^D, no employment or output.

Otherwise, variations in L^D affect w but not l

But add another extra assumption:

"employment lotteries" Hansen (1985)

"individuals will be randomly assigned to employment or unemployment each period, with consumption insurance against the possibility of unemployment"

RBC Thy Developments
Indivisible Labor (cont)

$0 < p_t < 1$ Probability a household member is assigned "work"

hence $L_t = p_t \, l_0 \, N_t$, $p_t = \dfrac{L_t}{l_0 N_t}$

Expected single-period utility before work/leisure assignment:

$$p_t\left[u_c(c_1) + u_{1-l}(1-l_0)\right] + (1-p_t)\left[u_c(c_2) + u_{1-l}(1)\right]$$

where c_1 is consumption given to a worker,

c_2 " " " " " person at leisure

and $p_t c_1 + (1-p) c_2 = C_t$ ← consumption per person

Note: $c_2 = \dfrac{C_t}{1-p} - \dfrac{p}{1-p} c_1$

Bellman equation (for social planner)

$$V_t = \underset{C_t, c_1, p_t}{\text{Max}} \left\{ \text{above single-period utility} + E_t\left[V(k_{t+1})\right] \right\}$$

How is consumption distributed between c_1 & c_2?

How do p and L_t react to shocks?

RBC Thy Developments

Indivisible Labor (cont.)

Distribution of consumption

F.O.C.:

$$0 = p\, u_c'(c_1) + (1-p)\, u_c'(c_2) \underbrace{\frac{\partial c_2}{\partial c_1}}_{-\frac{p}{1-p}}$$

hence

$$u_c'(c_1) = u_c'(c_2)$$

hence $c_1 = c_2 = \dfrac{C}{N}$ ← aggregate consumption

Everybody gets the same consumption,

This means it's better to be unemployed:

$$u_c(c) + u_{1-\ell}(1) > u_c(c) + u_{1-\ell}(1-\ell_0)$$

Note this followed from separable utility across c & $(1-\ell)$.

More generally, you allocate consumption so that

$\dfrac{\partial u}{\partial c}$ for worker $= \dfrac{\partial u}{\partial c}$ for guy at leisure

If $\dfrac{\partial^2 u}{\partial c\, \partial(1-\ell)} < 0$ more leisure <u>reduces</u> MU of C,

you'll allocate more consumption to workers.

RBC Thy Developments

Indivisible Labor (cont.)

Labor supply

Setting $c_1 = c_2 = c$,

$$V_t = \underset{c,p}{\mathrm{Max}} \left\{ u_c(c_t) + p\, u_{1-l}(1-l_0) + (1-p)\, u_{1-l}(1) + E_t[\ldots\ldots \right\}$$

Re-write in terms of L_t, where $p_t = \dfrac{L_t}{l_0 N_t}$

$$= \underset{c,L}{\mathrm{Max}} \left\{ u_c(c_t) + u_{1-l}(1) - \frac{L_t}{l_0 N_t}\left(u_{1-l}(1) - u_{1-l}(1-l_0)\right) + \ldots\ldots \right\}$$

For Romer, $\ln(c_t) + \dfrac{L_t/l_0}{N_t} b \ln(1-l_0) + \dfrac{N_t - (L_t/l_0)}{N_t} b \ln 1$ (4.59)

F.O.C.:

$$0 = \frac{1}{N_t} u_c'(c_t) + e^{-\rho} E_t[\qquad]$$

$$0 = \frac{1}{l_0 N_t}\left(u_{1-l}(1) - u_{1-l}(1-l_0)\right) + e^{-\rho} E_t[\quad] \underbrace{\frac{\partial Y}{\partial L}}_{w}$$

hence

$$w_t u_c'(c_t) = \frac{1}{l_0}\left(u_{1-l}(1) - u_{1-l}(1-l_0)\right)$$

or

$$w_t = \frac{\frac{1}{l_0}\left(u_{1-l}(1) - u_{1-l}(1-l_0)\right)}{u_c'(c_t)}$$

RBC Thy Developments

Indivisible Labor

Labor supply (cont.)

Recall

$$W_t^s = \frac{\frac{1}{l^0}\left(U_{1-l}(1) - U_{1-l}(1-l_0)\right)}{U'_c(C_t)}$$

with log utility of consumption

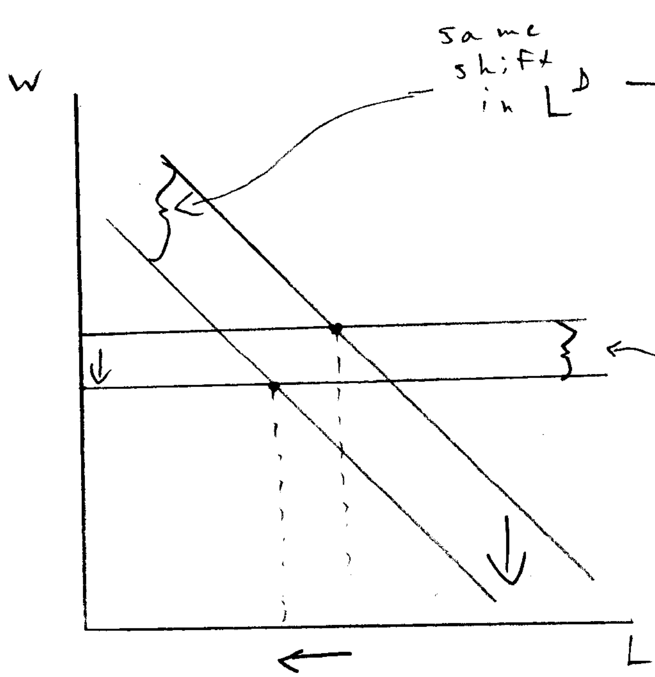
$$W_t^s = \frac{C_t}{l_0}\left(U_{1-l}(1) - U_{1-l}(1-l_0)\right)$$

Look: labor supply is infinitely elastic at a reservation wage determined by expectations of future (through c).

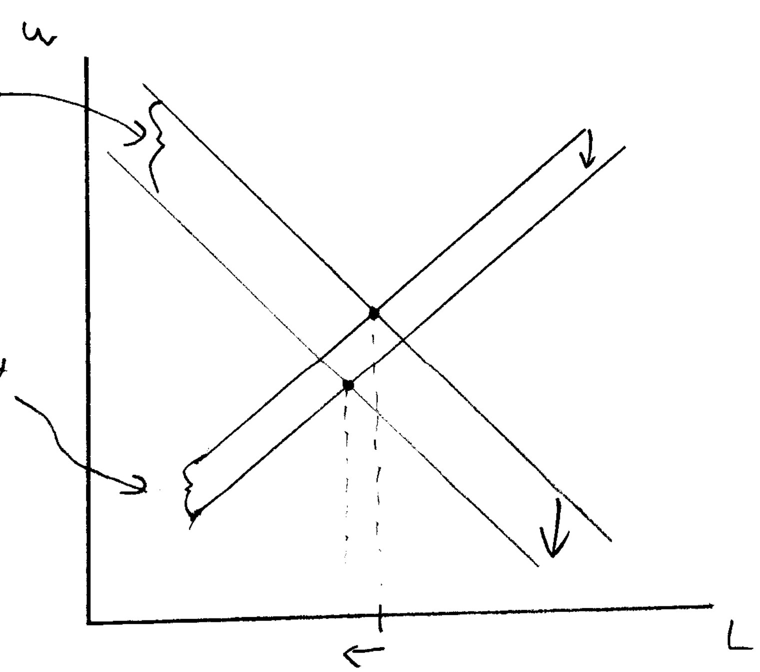
This makes L respond <u>more</u> to A_t, or $\underbrace{A_t \text{ and } Z_t}_{\text{capacity utilization stuff}}$

Say $\tilde{A}\downarrow$

<u>Indivisible Labor</u>



<u>Baseline Model</u>



RBC Theory Developments

Consumption or Leisure Nonseparable Across Time

Single-period utility is not a function of c_t (or $(1-l_t)$)

but rather a function of weighted average of

current & past c_t (or $(1-l_t)$)

Example: c_t

$$u(c_t, 1-l_t) = \ln(\hat{c}_t) + b \ln(1-l_t)$$

$$\text{where } \hat{c}_t = \sum_{i=0}^{\infty} a_i c_{t-i} \quad \text{and} \quad \sum_{i=0}^{\infty} a_i = 1$$

In LRSS (balanced-growth), this makes little difference:

- if c_t grows at constant rate, so does $\hat{c}_t$
- if l fixed, so is $\hat{l}_t$

But it strengthens intertemporal substn

(c_t or l_t respond <u>more</u> to $\tilde{A}_t$, $\hat{G}_t$)

because $\frac{\partial^2 u}{\partial c_t^2}$ and $\frac{\partial^2 u}{\partial (1-l_t)^2}$ are less negative,

marginal utility doesn't diminish as quickly

$$\frac{\partial u}{\partial c_t} = \frac{\partial u}{\partial \hat{c}_t}\frac{\partial \hat{c}_t}{\partial c_t} = \frac{1}{\hat{c}_t} a_i \quad \text{not} \quad \frac{1}{c_t}$$

$$\frac{\partial^2 u}{\partial c_t^2} = (-1)\frac{1}{\hat{c}_t^2} a_i^2 \quad \text{not} \quad (-1)\frac{1}{c_t^2}$$