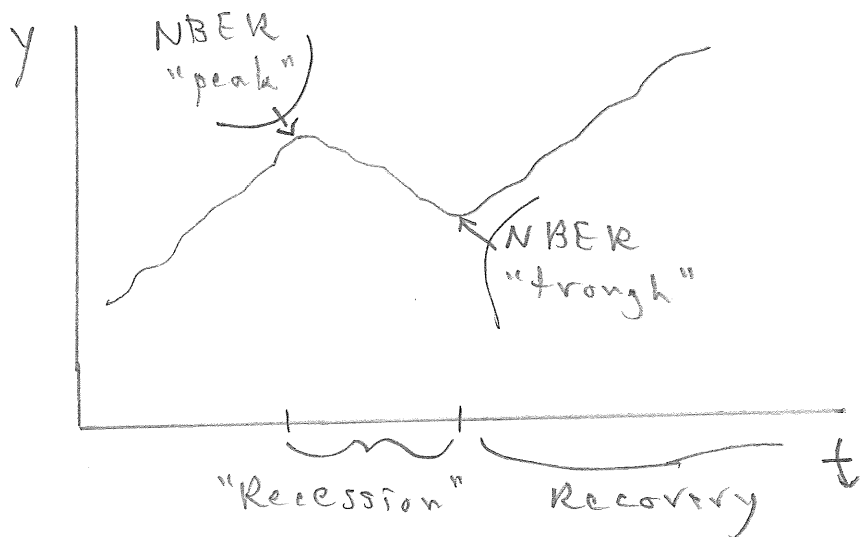


## SOME FACTS ABOUT MACROECONOMIC FLUCTUATIONS

Usually called "business cycles," but they do not occur at regular intervals (e.g. every 7 years) so we do not think they are actually cycles (where downturn eventually causes upturn & vice-versa). Rather, they seem to be result of randomly-occurring shocks + propagation mechanisms.

A "recession" is a fall in absolute level of  $Y$ , not just a slowdown in growth.



Recessions vary in length, from 3 months to  $1\frac{1}{2}$  years

When  $Y$  falls,  $L$  (labor hours) falls

Employment  $\downarrow$  most  $L \downarrow$  due to employment  $\downarrow$   
Hours/Worker  $\downarrow$

$C$  (consumption) falls some.

$I$  (investment) falls a lot.

Imports  $\downarrow$ ,  $NX \uparrow$

# SOME FACTS (cont.)

A variable is

"Procyclical": falls in recessions (+ corr w/Y)

"Countercyclical": rises in recessions (- corr w/Y)

"Acyclical"

Real wages w/P acyclical or procyclical

Inflation  $\pi$  in P or W procyclical

U unemployment rate countercyclical

Aggregate TFP or "Solow residual" (from growth accounting) is procyclical.

$$Y = A K^\alpha L^{1-\alpha}$$

$$y = a + \alpha k + (1-\alpha) z$$

in recession  
falls

falls a  
tiny bit

falls less than y

## REAL BUSINESS CYCLE THEORY

Apparent  $A \downarrow$  in recession is an exogenous negative "productivity shock" that causes a recession as households choose to work less, consume more leisure in response to temporarily-low productivity.

What causes decline in  $A$ ? Technology gets worse?

(Alternative explanation for  $A \downarrow$  in recessions, not RBC theory: input of  $L$  &  $K$  to production is mismeasured:  
"labor hoarding"  
"capital utilization")

# REAL BUSINESS CYCLE THEORY

(2)

## Big idea

BC cycles are Pareto-optimal response of a competitive economy, with perfectly rational agents, to "real" shocks, not to "monetary" shocks (shocks to  $\$M^D$  or  $\$M^S$ ).

## Structure of models

RCK + variable endogenous  $L$   
No  $M^D$ ,  $M^S$  or  $P$

Exogenous stochastic shocks to TFP,  
+ maybe other shocks

Shut down shocks, model converges to LKSS  
"balanced-growth" path that looks like RCK  
Response to a shock is temporary fluctuations  
of endogenous variables ( $Y$ ,  $L$ ,  $K$ ,  $\frac{w}{p}$  etc.)  
from balanced-growth path.

$$\ln X_t = \underbrace{\bar{X} + g t}_{\text{balanced-growth path}} + \underbrace{X_t}_{\substack{\uparrow \\ \text{deviation of } \ln \text{ of variable from} \\ \text{balanced-growth path}}}$$

RBC TAY (cond.)

Romer's "baseline" RBC model

Two shocks

TFP  $\tilde{A}_t = \rho_A \tilde{A}_{t-1} + \epsilon_{At}$

↑  
i.i.d

Govt. purchases of output  $\tilde{G}_t = \rho_G \tilde{G}_{t-1} + \epsilon_{Gt}$

$\tilde{G}$  is output taken by "government" and thrown away (doesn't enter prodn. or util. functions)

Preview: what we get from the model

A set of log linear equations that give endogenous variables as functions of current or lagged endogenous variables,  $\tilde{A}$  &  $\tilde{G}$

$$\left. \begin{aligned} \tilde{C}_t &\cong a_{cK} \tilde{K}_t + a_{cA} \tilde{A}_t + a_{cG} \tilde{G}_t \\ \tilde{L}_t &\cong a_{lK} \tilde{K}_t + a_{lA} \tilde{A}_t + a_{lG} \tilde{G}_t \\ \tilde{K}_{t+1} &\cong b_{kK} \tilde{K}_t + b_{kA} \tilde{A}_t + b_{kG} \tilde{G}_t \end{aligned} \right\} \begin{array}{l} \text{linear approximations} \\ \text{around LKSS} \\ \text{path} \end{array}$$

$$\tilde{Y}_t = \alpha \tilde{K}_t + (1-\alpha) \tilde{L}_t + (1-\alpha) \tilde{A}_t$$

From  $Y = K^\alpha (AL)^{1-\alpha}$

What do these equations mean?

RBC TNY

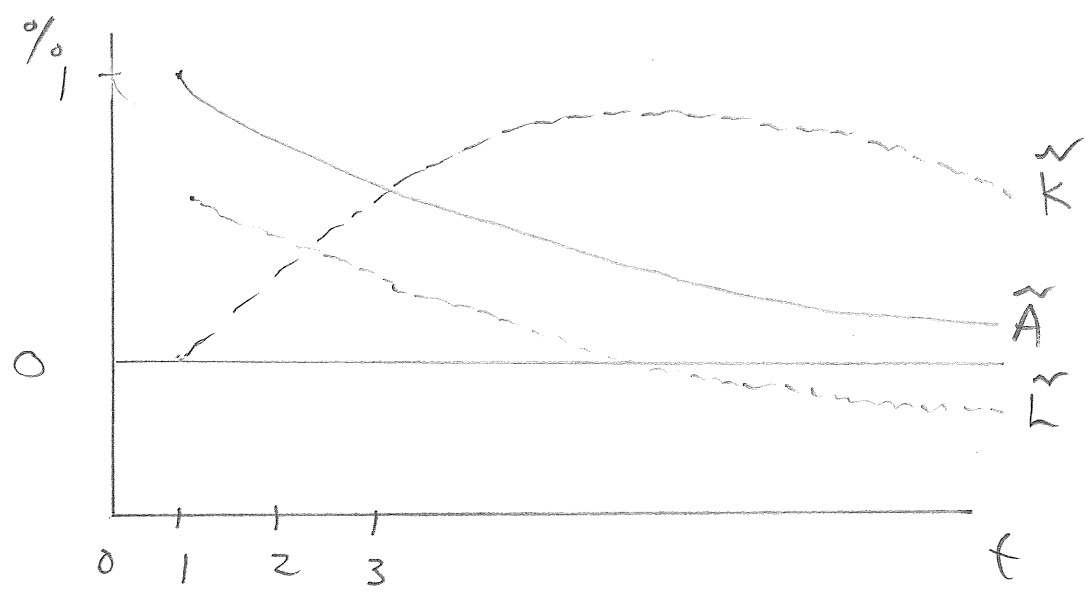
baseline model

Preview (cont.)

Imagine Excel spreadsheet

$t$	$\varepsilon_A$	$\varepsilon_G$	$\tilde{A}$	$\tilde{G}$	$\tilde{C}_t$	$\tilde{L}_t$	etc.
0	0	0	0				
1	1SD	0	1SD				equations give values for
2	0	0	$\rho_A 1SD$				these
3	0	0	$\rho_A^2 1SD$				
4	0	0	etc.				
5	0	0					

Make time-series plots of  $\tilde{X}$ 's



Compare with data from a "typical" business cycle. Do they look alike?

Do same for  $\varepsilon_G$ .

RBC THYModel assumptionsProd. Fn, etc.

$$Y_t = K_t^\alpha (A_t L_t)^{1-\alpha} \quad (S.1)$$

$$Y_t = C_t + I_t + G_t \quad \text{closed economy}$$

$$\begin{aligned} K_{t+1} &= K_t + I_t - \delta K_t & (S.2) \\ &= K_t + Y_t - C_t - G_t - \delta K_t \end{aligned}$$

$G_t$  = lump-sum tax on household (see below)  
(Recall "Ricardian equivalence")

$Z_n N_t = \bar{N} + n t$  Exogenous rate of popn. growth

$L_t = N_t l_t$  where  $0 \leq l_t \leq 1$   
 ← fraction of potential labor hours (=1) supplied as labor by a person

$$\begin{aligned} \text{"Real wage" } w_t = \text{MPL} &= (1-\alpha) K_t^\alpha (A_t L_t)^{-\alpha} A_t & (S.3) \\ &= (1-\alpha) (K_t/A_t L_t)^\alpha A_t \end{aligned}$$

$$\text{Real interest rate } r_t = \text{MPK} - \delta = \alpha (K_t/A_t L_t)^{\alpha-1} - \delta$$

Note: if  $(K/AL)$  fixed, as it is in LKSS,  
 $r$  is fixed

real wage grows at same rate as  $A$

RBC

Model assumptions (cont.)

Utility Function

$$u_t = E \left[ \sum_{t=0}^{\infty} e^{-\rho t} u(c_t, 1-l_t) \frac{N_t}{H} \right] \quad (5.5)$$

$$\& u(c_t, 1-l_t) = \ln c_t + b \ln(1-l_t) \quad 0 < b < 1 \quad (5.7)$$

“felicity,” “instantaneous” or “flow” or “momentary” utility  
 (consumption per person)

Note:

— In this expression, Romer is sloppy: uses  $t$  to denote horizon looking forward from a point in time, as well as time periods.

Should be:

$$u_t = E_t \left[ \sum_{\tau=0}^{\infty} e^{-\rho \tau} u(c_{t+\tau}, 1-l_{t+\tau}) \frac{N_{t+\tau}}{H} \right]$$

$t$  “calendar time” (see King & Rebelo p. 943)

$\tau$  “Planning time”

- Agent is household not person
- Population grows by new people in fixed number of households

— Other ways to write subjective discounting:

$$e^{-\rho \tau} = \left( \frac{1}{1+\rho} \right)^\tau = \beta^\tau \quad \left( 0 < \beta < 1 \right)$$

How are they equivalent? Take logs!

$$-\rho \tau = \tau \ln \left( \frac{1}{1+\rho} \right) = -\tau \ln(1+\rho) \approx -\tau \rho$$



"Benthamite" utility function

$$U = \sum e^{-\rho t} (\ln c_t + b (\ln(1-l_t))) \frac{N_t}{H}$$

Means population growth increases felicity & utility

Requires  $n < \rho$  (otherwise  $U$  is infinite/can't deal with that),

Why assume this?

1) Need to allow for popn. growth

2) With popn. growth, Benthamite utility simplifies Euler equation etc.

Without Benthamite:

$$c_t = e^{\rho} c_{t+1} (1+r_t)^{-1} (1+n)$$

With:

$$c_t = e^{\rho} c_{t+1} (1+r_t)$$

simpler

Derive Euler equation

Generally, utility maximization requires

$$\frac{\partial U}{\partial c_t} = \frac{\partial U}{\partial c_{t+1}} \cdot \frac{\partial c_{t+1}}{\partial c_t}$$

where  $\frac{\partial c_{t+1}}{\partial c_t} = (1+r)(1+n)^{-1}$

Why?  $c_{t+1} = (1+r)(Y - C)_t + Y_{t+1}$

$$\frac{\partial c_{t+1}}{\partial c_t} = -(1+r) \frac{\partial c_t}{\partial c_t}$$

Little  $c$ :  $\frac{\partial c_{t+1}}{\partial c_t} = -(1+r) \frac{\partial c_t}{\partial c_t} \frac{1}{1+n}$

From this we get Euler equation.

this is big C

RBC

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"Benthamite" utility (cont.)

Derive Euler equation (cont.)

$$\partial U / \partial c_t = \partial U / \partial c_{t+1} (1+r)(1+n)^{-1}$$

Without Benthamite

$$U = \ln c_t + b(1-l_t) + e^{-\rho} [\ln c_{t+1} + b(1-l_{t+1})] + \dots$$

$$\frac{\partial U}{\partial c_t} = \frac{1}{c_t} \quad \frac{\partial U}{\partial c_{t+1}} = e^{-\rho} \frac{1}{c_{t+1}}$$

Put that into above, get previous page.

With Benthamite:

$$U = N_t (\ln c_t + b(1-l_t)) + e^{-\rho} N_{t+1} [\ln c_{t+1} + b(1-l_{t+1})] + \dots$$

$$\frac{\partial U}{\partial c_t} = N_t \frac{1}{c_t} \quad \frac{\partial U}{\partial c_{t+1}} = N_{t+1} e^{-\rho} \frac{1}{c_{t+1}}$$

Put that into above, get...

# RBC THT

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## Model assumptions

### Utility Function (cont.)

$$U = \sum e^{-\rho t} (\ln c_t + b \ln(1-l_t)) \frac{N_t}{H}$$

Utility is "additively separable" across periods

$$U = u(c_t, 1-l_t) + e^{-\rho} u(c_{t+1}, 1-l_{t+1}) + e^{-2\rho} (\dots)$$

so  $\frac{\partial U}{\partial c_t}, \frac{\partial U}{\partial l_t}$  unaffected by  $c_{t+1}, l_{t+1}$ , i.e.  $\frac{\partial U}{\partial c_t \partial c_{t+1}} = 0$

How much I ate, worked last period does not directly affect what I want to do today.

Example of not separable across periods:

$$U = \sum e^{-\rho t} (\ln(c_t - j c_{t-1})) + \dots$$

so  $\frac{\partial U}{\partial c_{t+1}} = \frac{1}{c_{t+1} - j c_t}$

See:  $c_t \uparrow \rightarrow \frac{\partial U}{\partial c_{t+1}} \uparrow \quad \frac{\partial U}{\partial c_t \partial c_{t-1}} > 0$

The more I ate yesterday, the more I want to eat today.

A function of this form is called "habit formation" in consumption.

Utility Function (cont.)

$$u(c_t, 1-l_t) = \ln c_t + b \ln(1-l_t)$$

Felicity is "additively separable" in  $c, l$

so  $\frac{\partial u}{\partial c_t}$  unaffected by  $l$  and vice-versa.

Example of not separable:

$$u(c_t, 1-l_t) = \frac{1}{1-\sigma} \left\{ [c_t v(l_t)]^{1-\sigma} - 1 \right\}$$

where  $0 < \sigma < 1$

$$\frac{\partial u}{\partial c_t} = \frac{1}{1-\sigma} (1-\sigma) (c_t v(l_t))^{-\sigma} v(l_t) = \frac{v(l_t)^{1-\sigma}}{c_t^{-\sigma}}$$

etc. (see King & Rebelo p. 945)

RBC TNY

Household's problem

Representative household owns capital, gets paid  $r_t K_t$  and  $w_t l_t \frac{N_t}{H}$  in period  $t$ .

Pays lump-sum tax equal to  $\frac{G_t}{H}$ .

Can borrow at  $r_t$  if he wants (but in equilibrium he won't).

Ricardian equivalence: it makes no difference to him if govt. doesn't tax this period but instead borrows to pay for  $G_t$  now and taxes him  $(1+r_t)G_t$  to repay the "govt bonds" in  $t+1$ .

Household's budget constraint:

$$\underbrace{\sum_{t=0}^{\infty} \frac{1}{(1+r)^{t-t}} c_{t-t} \frac{N_{t-t}}{H}}_{\text{present discounted value of consumption}} \leq \underbrace{\frac{K_t}{H}}_{\text{wealth today}} + \underbrace{\sum_{t=0}^{\infty} \frac{1}{(1+r)^{t-t}} \left( \underbrace{l_{t-t} \frac{N_{t-t}}{H} w_{t-t}}_{\substack{\text{p.d.v.} \\ \text{wage} \\ \text{income}}} - \underbrace{\frac{G_{t-t}}{H}}_{\substack{\text{p.d.v.} \\ \text{lump-sum} \\ \text{tax}}} \right)}_{}$$

Household takes as given current & expected  $w, r, G$  and existing wealth  $(K/N)_t$

and chooses  $c_t, l_t$ .

Note:  $N_t$  is exogenous, so when  $L$  falls, that's just  $l \downarrow$

RBC TNY

Solving model as a "Social planner"

"First Fundamental Theorem of welfare economics" says outcome of perfect competition, etc. is Pareto optimal.

Thus we can assume outcome of this economy maximizes utility of rep. agent given constraints on economy as a whole, which we get by maximizing  $E[U]$  s.t.  $K_t, G_t$  and production  $F_t$ .

You can solve model either way.

Dynamic Programming

1) Define "value function"  $V_t$   
(value of maximand (here,  $E[U]$ ), looking forward from time  $t$ , as function of variables taken as given at time  $t$  (here,  $K_t, \tilde{A}_t, \tilde{B}_t$ ) assuming control variables ( $C, Z$ ) are set at optimum)

2) Write down Bellman equation

3) Write down f.o.c.'s

— "Intratemporal" relationships between time  $t$  values

— "Intertemporal" relationships between values at  $t$  and at  $t+1$

# RBC Thy

## Bellman Equation

Describes intertemporal optimization problem as a "problem to be solved each period, looking just one step ahead, taking as given variables determined by past history that matter for now & future. Here,  $K_t$  and  $A_t$  (and  $G_t$ , if we include govt. in model).

fixed population  
↓

For simplicity, & to avoid Benthamite issue, set  $N=1$  ( $n=0$ )

$$V(K_t, A_t) = \text{Max}_{c, l} \left\{ U_c(c_t) + U_{1-l}(1-l_t) + e^{-\rho} E_t [V(K_{t+1}, A_{t+1})] \right\}$$

### F.O.C.'s

Take derivatives w.r.t. choice variables, set  $\frac{\partial V}{\partial m} = 0$

Note: choice of  $c$  &  $l$  will affect  $K_{t+1}$ , but not  $A_{t+1}$

$$K_{t+1} = K_t + Y_t - C_t - \delta K_t = K_t + F(K_t, A_t, l_t) - C_t - \delta K_t$$

hence  $\frac{\partial K_{t+1}}{\partial c_t} = -1$        $\frac{\partial K_{t+1}}{\partial l_t} = \frac{\partial Y}{\partial l} = w \leftarrow \text{MPL}$

w.r.t. consumption:

$$\frac{\partial V}{\partial c_t} = 0 = U'_c(c_t) + e^{-\rho} E_t \left[ V_K(K_{t+1}, A_{t+1}) \frac{\partial K_{t+1}}{\partial c_t} \right]$$

$$= U'_c(c_t) + e^{-\rho} E_t [V_K(\quad) (-1)]$$

w.r.t.  $l$ :

$$\frac{\partial V}{\partial l_t} = 0 = U'_{1-l}(1-l) + e^{-\rho} E_t [V_K(\quad) \frac{\partial K_{t+1}}{\partial l_t}]$$

$$= U'_{1-l}(1-l) + e^{-\rho} E_t [V_K(\quad) w]$$

LOOK!  $E_t [V_K(\quad)]$  is in both expressions!

# RBC Thy

## F.O.C.s (cont.)

From  $\frac{\partial V}{\partial l_t} = 0$ , we have

$$\frac{1}{w_t} U'_{1-l} = e^{-\rho} E_t [V_k(\dots)]$$

From  $\frac{\partial V}{\partial c_t} = 0$ , we have

$$U'_c(c_t) = e^{-\rho} E_t [V_k(\dots)]$$

hence  $\frac{1}{w_t} U'_{1-l} = U'_c(c_t)$

$$U'_{1-l} (1-l_t) = w_t U'_c(c_t)$$

Romer calls this

"Intra-temporal F.O.C."

What it means:

I have an endowment of time, which I can "eat" directly as leisure, or convert into consumption & "eat" more c. Rate of exchange between l and c is real wage w.

When I work one more hour, I lose  $MU_{LEISURE} = U'_{1-l}$   
I gain  $MU_c w = U'_c w$

At optimum, can't be  $MU_{LEISURE} > MU_c w$   
work less!

can't be  $MU_{LEISURE} < MU_c w$   
work more!

so it must be...

But how do I determine  $U'_c(c_t)$ ?



# RBC Thy

(13)

## Benveniste-Scheinkman equation

Recall

$$V(K_t, A_t) = \max_{c, l} \left\{ U_c(c) + U_{1-l}(1-l) + e^{-\rho} E_t[V(K_{t+1}, A_{t+1})] \right\}$$

What is  $V_K$ ? (which will tell us  $E_t[V_K(\cdot)]$ )

Recall the "envelope theorem":

if we are maximizing a function subject to constraints, the effect of a marginal change in constraint on the maximized value of the function

is equal to the marginal effect of choice variable on optimand,

times marginal effect on the choice variable of the change in constraint,

holding fixed other choice variables.

Example: Max  $U(X_1, X_2)$  s.t.  $P_1 X_1 + P_2 X_2 = Y$

$U^*$  Maximized value of  $U$

$$\frac{\partial U^*}{\partial Y} = U_{X_1} \frac{1}{P_1} = U_{X_2} \frac{1}{P_2}$$

←  $\frac{\partial X_1}{\partial Y}$  holding fixed  $X_2$

Here,

$$V_K(K_t, A_t) = U'_c(c) \frac{\partial c}{\partial K_t} \text{ holding fixed } l_t, K_{t+1}, \text{ etc.}$$

# RBC Thy

## Benveniste-Scheinkman equation (cont.)

Recall  $K_{t+1} = K_t + F(K_t, A_t) - C_t - G_t - \delta K_t$

hence  $C_t = -K_{t+1} + F(K_t, A_t) - G_t + (1 - \delta)K_t$

$$\frac{\partial C_t}{\partial K_t} = \underbrace{F_K(\quad)}_{\text{or } \frac{\partial Y}{\partial K}} + (1 - \delta)$$

Recall  $r \equiv \frac{\partial Y}{\partial K} - \delta$

hence  $\frac{\partial C_t}{\partial K_t} = 1 + r_t$

hence  $V_K(K_t, A_t) = U'_C(C_t) (1 + r_t)$  ↖ (BS equation)

what this means: value of a marginal unit of capital is MV of eating another unit of output times marginal effect of capital on this period's output after covering extra depreciation, to keep  $K_{t+1}$  unchanged.

BS equation will hold next period too, so:

$$E_t[V_K(K_{t+1}, A_{t+1})] = E_t[U'_C(C_{t+1}) (1 + r_{t+1})]$$

or

$$= E_t\left[\frac{1}{w_{t+1}} U'_{1-l}(1-l_t) (1 + r_{t+1})\right]$$

RBC Thy"Intertemporal F.O.C."

Recall

$$\frac{1}{w_t} U'_{1-l}(c_t) = e^{-\rho} E_t [V_K (K_{t+1}, A_{t+1})]$$

$$U'_c(c_t) = e^{-\rho} E_t [V_K (c_t)]$$

hence from BS eqn

$$U'_c(c_t) = e^{-\rho} E_t [U'_c(c_{t+1}) (1+r_{t+1})]$$

$$U'_{1-l}(1-l_t) = w_t e^{-\rho} E_t [U'_c(c_{t+1}) (1+r_{t+1})]$$

We could also write these in terms of  $w_{t+1}, l_{t+1}$

Assuming Rower's specific utility function:

$$\text{Recall } U(c, 1-l_t) = \ln(c_t) + b \ln(1-l_t)$$

$$\text{hence } U'_c(\ ) = \frac{1}{c_t} \quad U'_{1-l}(\ ) = \frac{b}{1-l_t}$$

Intratemporal F.O.C.

$$U'_{1-l}(\ ) = w_t U'_c(c_t)$$

$$\text{becomes } \frac{b}{1-l_t} = w_t \frac{1}{c_t} \quad \text{or} \quad \frac{c_t}{1-l_t} = \frac{w_t}{b_t} \quad (5.26)$$

Intertemporal F.O.C.

$$\frac{1}{c_t} = e^{-\rho} E_t \left[ \frac{1}{c_{t+1}} (1+r_{t+1}) \right] \quad (5.23)$$

## RBC TNY

(2)

Now what?

If we know  $E_t \left[ \frac{1}{c_{t+1}} (1+r_{t+1}) \right]$ , we know  $c_t$

If we know  $c_t$ , we know  $z_t$ .

With  $c_t$  and  $z_t$  we know everything else.

But to know  $E_t \left[ \frac{1}{c_{t+1}} (1+r_{t+1}) \right]$  we need to know

$E_t \left[ E_{t+1} \left[ \frac{1}{c_{t+2}} (1+r_{t+2}) \right] \right]$  because intertemporal

F.O.C. will hold at time  $t+1$  too!

So we have to define expected values for all variables going into the infinite future!

How?

1) Assume  $E_t [X_{t+\infty}] = X_{LKSS}$  (if no shocks)

make this the "Final period"

2) Solve back from there.

To begin, figure out what LKSS looks like.

But note there's an approximation here.

$E_t [X_{t+\infty}] \neq X_{LKSS}$  (if no shocks).

because there will be shocks in the future!