RBC THY

LRSS without shocks ("balanced-growth path")

Without ε's, model is RCK with endogenous L.

$$L_t = N_t \ell_t$$

In LRSS, ℓ must be fixed (ℓ can't grow or shrink forever)

so L_t grows at rate n (

$$\ln A_t = \ln \bar{A} + gt$$

As in RCK,

Y grows at rate $n+g$

G grows at rate $n+g$ (otherwise $\frac{G}{Y}$ grows or shrinks)

$k = \frac{K}{AL} = k^*$ fixed where $y = \frac{Y}{AL} = f(k) = k^\alpha$

$r = MPK - \delta = f'(k) - \delta = \alpha k^{*\alpha-1} - \delta$ fixed

$w_t = MPL = (1-\alpha) k^{*\alpha} A_t$ grows at rate g

$c = \frac{C}{L} = A_t \underbrace{\left(f(k^*) - (n+g+\delta) k^* \right)}_{\text{Fixed}}$ grows at rate g

Note $e^g = \frac{A_{t+1}}{A_t} = \frac{c_{t+1}}{c_t} = \frac{w_{t+1}}{w_t}$

Why does felicity function take natural-log form?

$$u_t = \ln c_t + b \ln(1 - l_t) \qquad 0 < b \leq 1$$

Note utility is "separable" in c vs. l

(i.e. $\frac{\partial u_t}{\partial c_t}$ unaffected by l_t)

Natural-log form is only thing that works if u is separable in c vs. l and U is separable across time

Recall $u_c'(c_t) = u_{1-l}'(1-l_t)\frac{1}{w_t}$

hence $$\frac{u_c'(c_{t+1})}{u_c'(c_t)} = \frac{u_{1-l}'(1-l_{t+1})}{u_{1-l}'(1-l_t)} \frac{w_t}{w_{t+1}}$$

In LRSS, l fixed, $c_{t+1} = e^g c_t$, $w_t / w_{t+1} = \frac{1}{e^g}$

So $u(\;)$ must be such that

$$\frac{u_c'(e^g x)}{u_c'(x)} = \frac{1}{e^g}$$

hence $u_c'(z) = \frac{1}{z}$ so $u_c(z) = \ln(z)$

RBC THY

LRSS without shocks

LRSS value of r^*, k^*

Euler equation: $u'_c(c_t) = e^{-\rho} E_t[u'(c_{t+1})(1+r_{t+1})]$

In nonstochastic LRSS with ln utility

$$\frac{1}{c_t} = e^{-\rho} \frac{1}{c_{t+1}}(1+r^*)$$

$$\Rightarrow r^* = \frac{c_{t+1}}{c_t} e^{\rho} = e^{g} e^{\rho} - 1 = e^{g+\rho} - 1$$

Note: if you have a number for r^* (trend value of r)
and g (trend TFP growth)
you can infer ρ

Also, since $r^* = f'(k^*) - \delta = \alpha k^{*\,\alpha-1} - \delta$

$$e^{g+\rho} - 1 = \alpha k^{*\,\alpha-1} - \delta$$

$$\Rightarrow k^* = \left(\frac{\alpha}{e^{g+\rho} + \delta - 1}\right)^{\frac{1}{1-\alpha}}$$

If you know δ (depreciation rate)
α (share of capital in income)

you can infer k^*.

RBC THY

LRSS w/o shocks (cont.)

LRSS l^*

Intratemporal f.o.c.: $u'_{1-l}(1-l_t) = w\, u'_c(c_t)$

with $u_t = \ln c_t + b \ln(1-l_t)$

$$\frac{b}{1-l^*} = w_t \frac{1}{c_t} = \frac{A_t(1-\alpha)k^{*\alpha}}{A_t\left(k^{*\alpha}-(n+g+\delta)k^*\right)}$$

$$\text{recall } k^* = \left(\frac{\alpha}{e^{g+\rho}+\delta-1}\right)^{\frac{1}{1-\alpha}}$$

$$\frac{b}{1-l^*} = \left.\frac{1-\alpha}{1-\alpha\frac{n+g+\delta}{e^{g+\rho}+\delta-1}}\right\} \text{call this } Z$$

so

$$l^* = 1 - \frac{b}{Z}$$

$$b = \frac{1-l^*}{Z}$$

Recall l^* is fraction of potential labor time devoted to work (not leisure)

so if you know values of other parameters, you can infer value of b from l^*.

RBC THEORY

How do you choose parameter values?

Take a "period" to be $\frac{1}{4}$, matching NIPA data

1) Make model match LR trends in U.S. data.

$\alpha = 1/3 \quad (1-\alpha) = 2/3 \sim$ labor's share of national income in NIPAs

$g = 0.5\%$ (Annual TFP growth of 2%)

$n = 0.25\%$ (Labor Force growth 1%)

$\delta = 2.5\%$ (Annual depreciation 10%)

2) Estimate ρ_G & $Var(\varepsilon_G)$

3) Infer quarterly values of A from data on Y, L, K & $\alpha = 1/3$.
From inferred quarterly A, estimate ρ_A & $Var(\varepsilon_A)$

4) A "reasonable" value for r^*: 1.5% (6% annually)
hence $f'(k^*) = r^* + \delta$ and k^*

5) ρ is determined by g & r^*, given log utility

Recall $c_t = c_{t+1} \frac{1}{e^{-\rho}(1+r)}$

in LRSS $c_{t+1} = e^g c_t$

hence $e^g = e^{-\rho}(1+r)$ take logs of both sides

$$\rho = \ln(1+r) - g \approx r - g$$

6) Given all these, infer b from data on l^*

RBC Thy

Log-linear approximation of model

$$\tilde{X}_t = \text{Log}(X_t) - \text{Log}(X_t \text{ on balanced-growth path})$$

Aggregate production function:

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha} = K_t^{\alpha}(A_t \ell_t N_t)^{1-\alpha}$$

$$\ln Y_t = \alpha \ln K_t + (1-\alpha)\ln A_t + (1-\alpha)\ln \ell_t + (1-\alpha)\ln N_t$$

doesn't deviate from LRSS path

$$\tilde{Y}_t = \alpha \tilde{K}_t + (1-\alpha)\tilde{A}_t + (1-\alpha)\tilde{\ell}_t$$

$$\rho_A \tilde{A}_{t-1} + \varepsilon_{A,t} \qquad \tilde{L}_t$$

together with

$$\tilde{C}_t \approx a_{CK}\tilde{K}_t + a_{CA}\tilde{A}_t + a_{CG}\tilde{G}_t$$

$$\tilde{L} \approx a_{LK}\tilde{K}_t + \ldots$$

$$\tilde{K}_{t+1} \approx b_{KK}\tilde{K}_t + b_{KA}\tilde{A}_t + b_{KG}\tilde{G}_t$$

and $\tilde{A}_t = \rho_A \tilde{A}_{t-1} + \varepsilon_{A,t}$ $\qquad \tilde{G}_t = \rho_G \tilde{G}_{t-1} + \varepsilon_{G,t}$

Feed in shocks for ε's, get paths over time for everything.

Note: ε_t affects $\tilde{X}_{t+1}$ through $\rho\,\varepsilon_t$ — serial correlation in shock

<u>and</u> b_{KA} — effect of $\tilde{A}_t$ on future capital stock

or

b_{KG}

How parameters enter loglinear approximation

Simplified model

Consider simplified version of model with fixed l and no G

(Campbell, 1994)

$$\tilde{C}_t = a_{ck}\tilde{k}_t + a_{cA}\tilde{A}_t$$

$$\tilde{K}_{t+1} = b_{kk}\tilde{k}_t + b_{kA}\tilde{A}_t$$

$$b_{kk} = \lambda_1 + (1-\lambda_1-\lambda_2)a_{ck} \qquad b_{kA} = \lambda_2(1-\lambda_1-\lambda_2)a_{CA}$$

$$\lambda_1 = \frac{1+r}{1+g} \qquad \lambda_2 = \frac{\alpha(r+\delta)}{(1-\alpha)(1+g)} \qquad \lambda_3 = \frac{\alpha(r+\delta)}{1+r}$$

$$a_{ck} = \frac{1}{2Q_2}\left(-Q_1 - \sqrt{Q_1^2 - 4Q_0Q_2}\right)$$

$$a_{CA} = \frac{-a_{CA}\lambda_2 + \lambda_3(\rho_A - \lambda_2)}{\rho_A - 1 + (1-\lambda_1-\lambda_2)(a_{ck}+\lambda_3)}$$

$$Q_0 = \lambda_3\lambda_1$$

$$Q_1 = \lambda_1 - 1 + \lambda_3(1-\lambda_1-\lambda_2)$$

$$Q_2 = 1-\lambda_1-\lambda_2$$

Note: change any parameter $r, g, \alpha, \delta, \rho_A$

and all coeffs a's, b's change

RBC Thy

Complete model

Assuming "reasonable" values for $r, g, \alpha, \delta, \rho_A, \rho_G, b$ gives

$$\tilde{C}_t = 0.59\,\tilde{K}_t + 0.38\,\tilde{A}_t - 0.13\,\tilde{G}_t$$

$$\tilde{L}_t = -0.31\,\tilde{K}_t + 0.35\,\tilde{A}_t + 0.15\,\tilde{G}_t$$

$$\tilde{K}_{t+1} = 0.95\,\tilde{K}_t + 0.08\,\tilde{A}_t - 0.004\,\tilde{G}_t$$

Effect of $\tilde{K}_t$

$\tilde{K}_t \uparrow \rightarrow C_t \uparrow,\ L_t \downarrow,\ K_{t+1} \uparrow$

More K means we're richer, higher lifetime income

So C↑, leisure ↑, labor ↓

Because MU_C is diminishing, we want to transfer some consumption to future by hiking K_{t+1}

Effect of $\tilde{G}_t$

$\tilde{G}_t \uparrow \rightarrow C_t \downarrow,\ L_t \uparrow,\ K_{t+1} \downarrow$

More G means higher lifetime tax burden, lower lifetime after-tax income,

so C↓, leisure ↓, labor ↑

Transfer some loss to future by $K_{t+1} \downarrow$

RBC Thy

Complete model (cont.)

Effect of $\tilde{A}_t$

$\tilde{A}_t \uparrow \rightarrow C_t \uparrow, L_t \uparrow, K_{t+1} \uparrow$

Higher productivity means higher lifetime income,
higher current real wage,
higher r_{t+1} for any given K_{t+1} because of persistence in A shock.

C_t rises because lifetime income is higher.
More persistent $\tilde{A}$ (ρ_A higher) increases a_{CA}

L_t rises because current real wage higher:

$$\underbrace{U'_{1-l}(1-l_t)}_{\text{For } l_t\uparrow\text{, this must rise}} = w_t \underbrace{U'_c(C_t)}_{\text{but this falls because } C_t\uparrow}$$

so $w_t \uparrow$ allows $l_t \uparrow$.

More persistent $\tilde{A}$ decreases a_{LA} as it increases a_{CA}

K_{t+1} rises to spread some consumption to future periods, and because $r_{t+1}\uparrow$.

Note: apart from ρ_A, $\hat{A}_t$ has some effect on $t+1$, $t+2$...
through b_{KA}.

RBC THY

Complete model (cont.)

Look at Figures 5.2–5.4 (effects of ε_A)
5.5–5.7 (effects of ε_G)

See: effects of ε_A look like a business cycle.
" " ε_G don't.

Thus, the explanation of business cycles implied by RBC theory:
bad TFP shocks
"technology shocks"

RBC Thy

Labor Market

Recall in Romer

$$\frac{1}{c_t} = e^{-\rho} E_t\left[\frac{1}{c_{t+1}}(1+r_{t+1})\right]$$

hence

$$c_t = e^{\rho} \frac{1}{E_t\left[\frac{1}{c_{t+1}}(1+r_{t+1})\right]}$$

G and expected future A affect C thru this.
Anything that is expected to
raise C_{t+1} raises C_t
raise r_{t+1} Lowers C_t

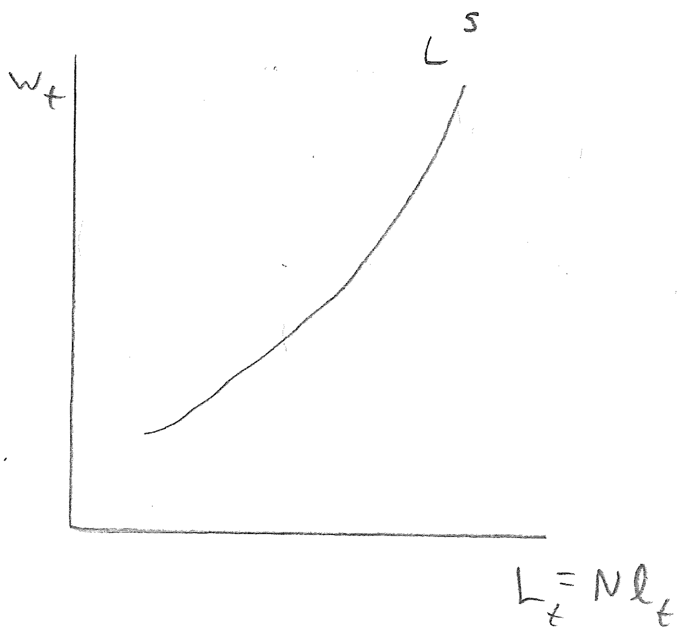
$$\frac{c}{1-l_t} = w_t / b$$

hence

$$w_t^s = b\, c_t \frac{1}{1-l^s}$$

Wage at which household is willing to supply l^s

("labor supply" shock comes along with c_t



L^s is shifted by anything that affects C_t, like G.

Anything that shifts L_t^s out (reduces leisure at a given real wage)

also causes C_t to fall

Anything that raises lifetime income raises C_t, shifts L_t^s back.

RBC thy

Labor Market (cont.)

Labor demand:

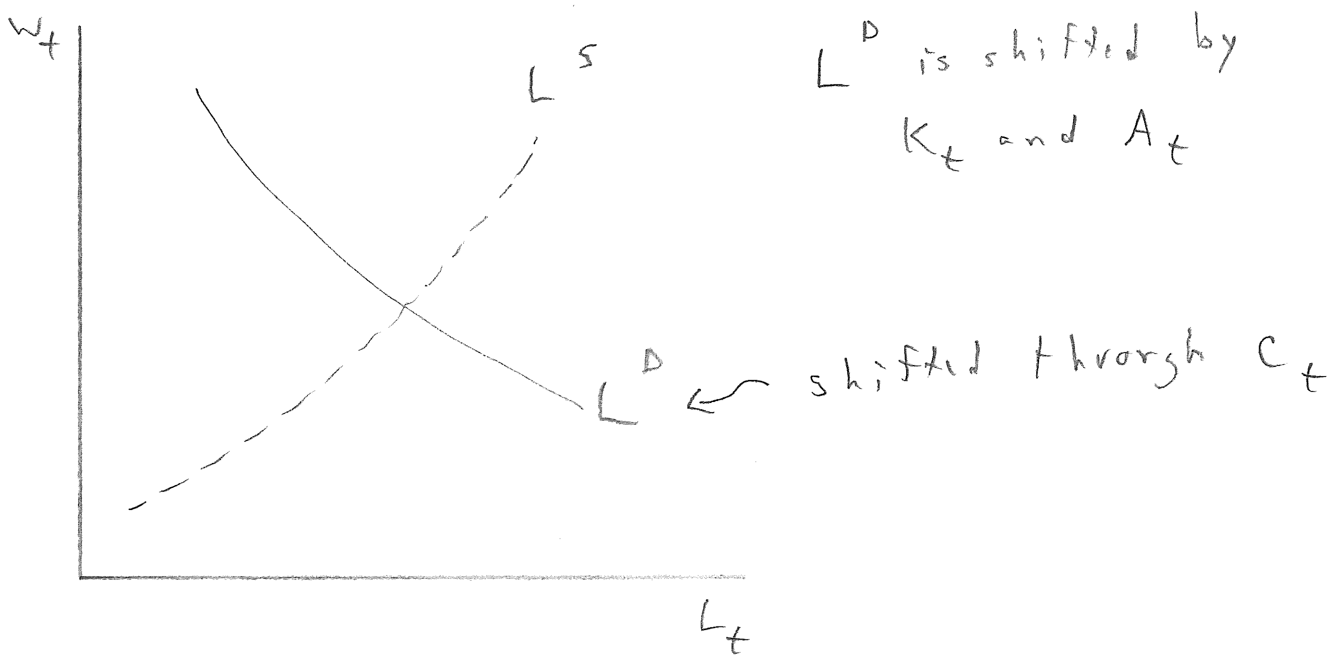
· Recall

$$W = MPL = (1-\alpha)\left(\frac{K}{AL}\right)^{\alpha} A_t$$

hence

$$W_t^D = (1-\alpha)\, K_t^{\alpha} A_t^{1-\alpha} (N \ell^D)^{-\alpha}$$

wage firms are willing to pay for ℓ^D



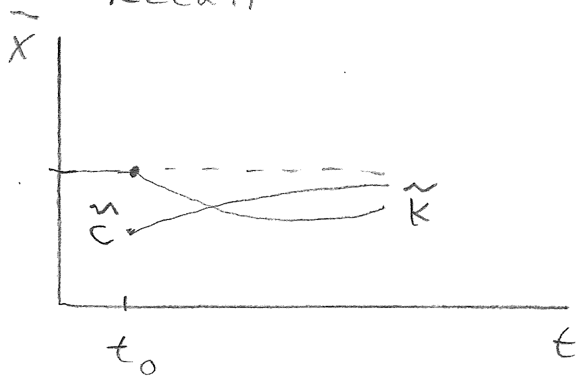
L^D is shifted by K_t and A_t

RBC Thy

Labor Market (cont.)

Response to $\tilde{G} \uparrow$

Recall

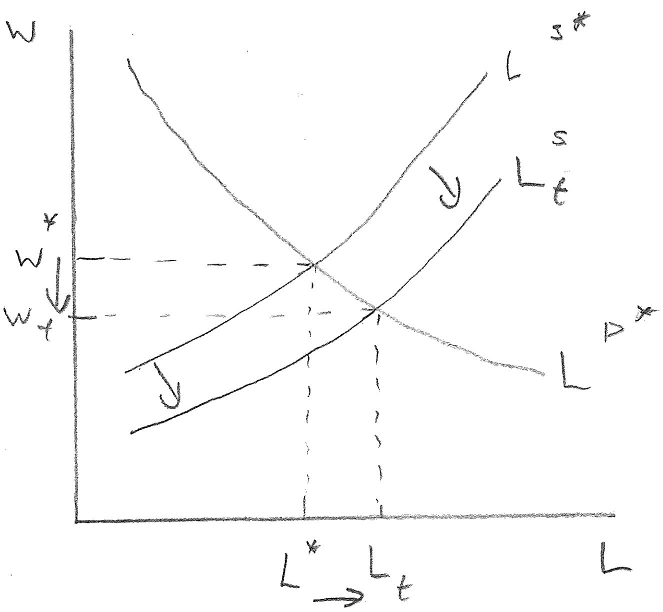


C jumps down, then gradually rises back to C^*.

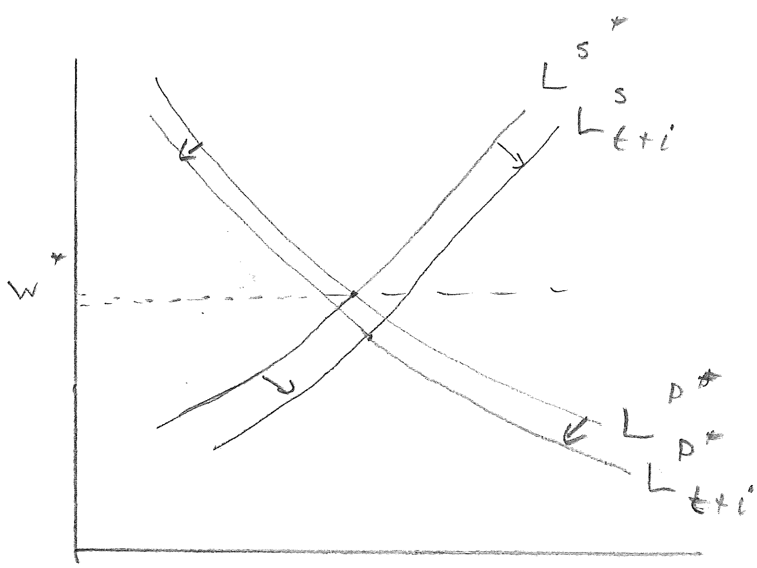
Hence LS curve jumps out, then gradually returns to original position.

Meanwhile, K gradually falls, then rises. Hence LD gradually shifts in, then gradually returns.

Just after $\tilde{G} \uparrow$



A bit later on



Note: real wage jumps down, then gradually rises back to W^* as L and Y jump up, then....

RW is <u>countercyclical</u>

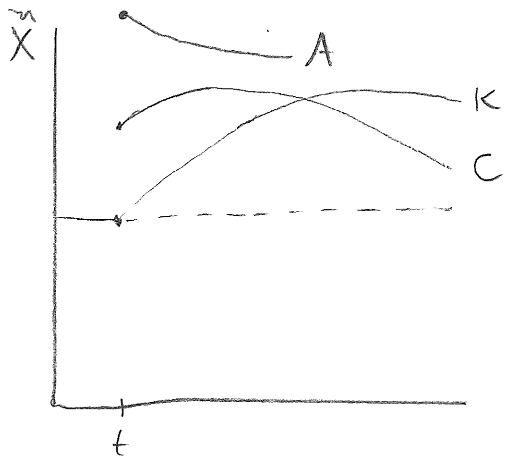
(so is consumption)

RBC Thy

Labor Market (cont.)

Response to $\tilde{A}\uparrow$

Recall

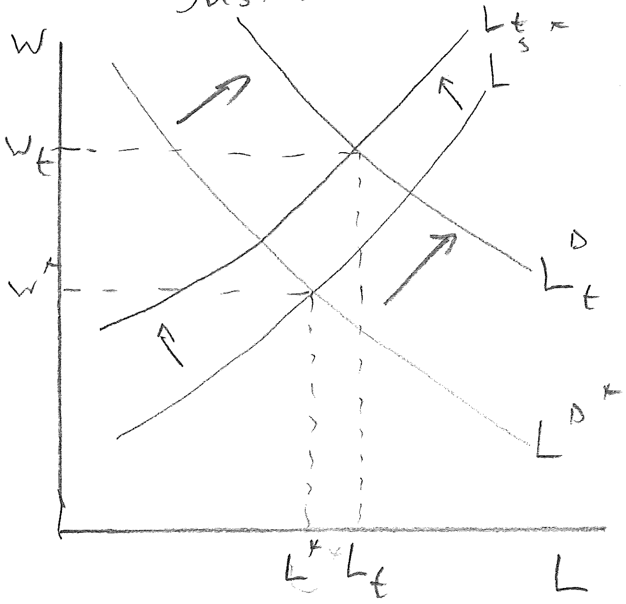


C jumps up, then rises a bit more before falling back.

Hence L^S jumps back, then slides back further before shifting out again.

Meanwhile L^D jumps out & remains out: A returns to LRSS, but K has been building up.

Just after $\tilde{A}\uparrow$,



How do we know L^S shift is small enough, relative to L^D shift, that L rises?

Recall we chose utility function so that L is unaffected by permanent change in A (trend growth):

for permanent change, "lifetime income" effect on labor supply (thru C) just balances labor demand/real wage effect.

For an expected-to-be temporary change in A, "lifetime income" effect is smaller, so we know L must rise in response to temporary change in A.

Note: RW is procyclical (so is consumption)