REAL BUSINESS CYCLE THEORY

More about utility function & labor supply

We saw that if utility function is separable across time & felicity function is separable across consumption & leisure so that $u_t = u_c(c_t) + u_{1-l}(l_t)$,

$u_c(c_t)$ must be $\ln(c_t)$

in order to have LRSS "balanced growth."

Are there other possibilities for felicity? Yes.

What is this about? "Income effects" vs. "substitution effects" in labor supply.

Income & substn effects in LS with $u = \ln(c_t) + b\ln(1-l_t)$

Consider l_t

c_t

$$Z_t = \sum_{\tau=0}^{\infty}\left(\frac{1}{(1+r_t)}\right)^{\tau} Y_{t+\tau}$$ Lifetime income looking forward from t

In LRSS, l is fixed, c & lifetime income grow at rate g, so c is a fixed fraction of lifetime income $z = \frac{c}{Z}$

MRPL = real wage also grows at rate g

For simplicity, set $N=1$ $(n=0)$

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More about...

Income & subsdn effects...

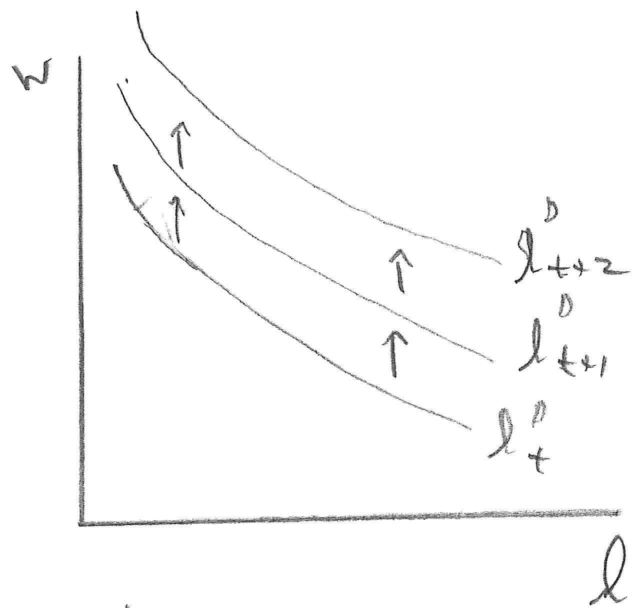
Labor demand curve

crawls up at

rate g

reflecting $A\uparrow, K\uparrow$

$$w_t^D = (1-\alpha)K^\alpha A^{1-\alpha} \ell^{D^{-\alpha}}$$



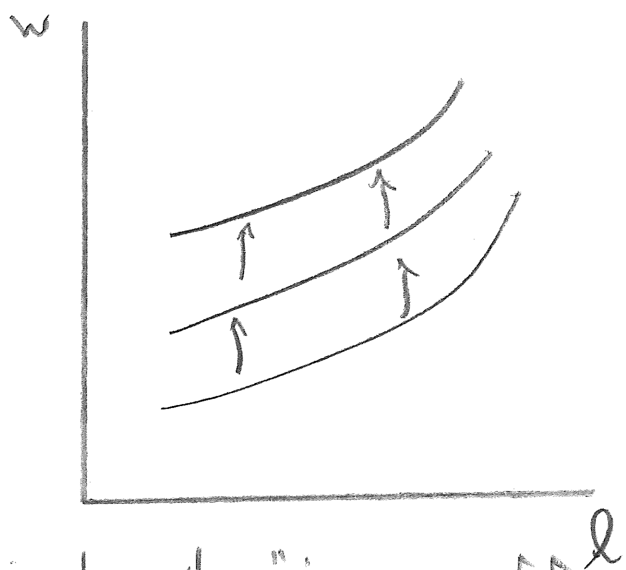
We need labor supply curve to crawl up at exactly the same rate to keep ℓ fixed which it does given our Felicity function

$$w_t^s = b\, c_t \frac{1}{1-\ell^s}$$

or

$$w_t^s = b\, x\, Z_t \frac{1}{1-\ell^s}$$

$\frac{c}{Z}$ in LRSS



Upward crawl in L^S is due to "income effect" on L^S; it exactly counteracts "substitution effect" of $w\uparrow$.

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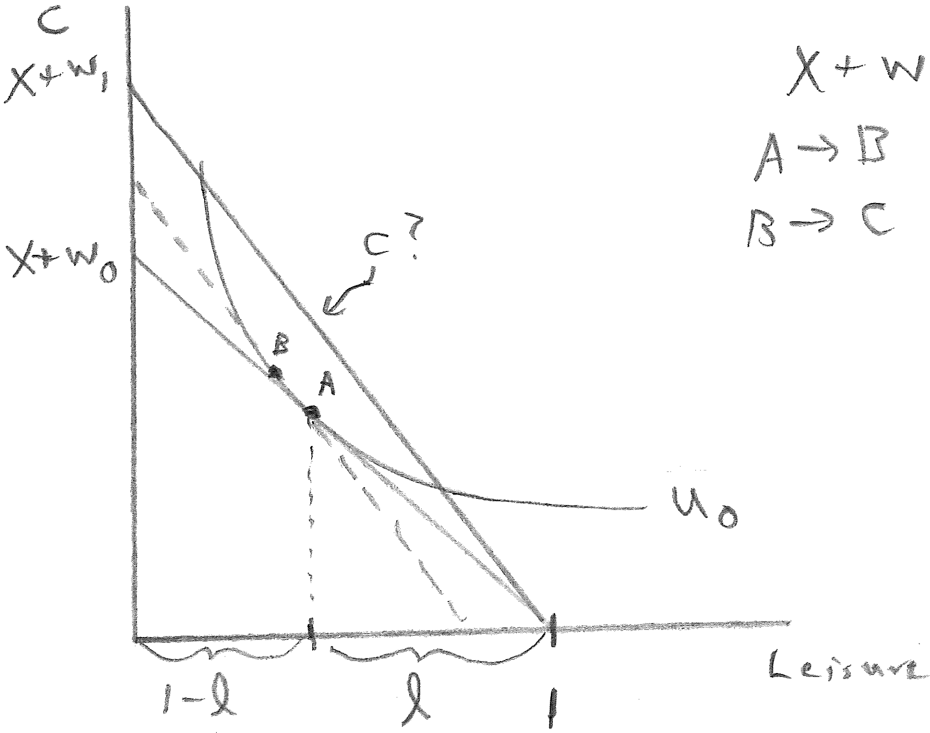
More about ...

Income & substn effects

~~Test Felicity function in a static (one-period) model~~

Income & substn. effects in static model

Recall L^s income & substn. effects in static model



X Nonlabor income

$X + wl$ Total income

$A \to B$ Substn effect

$B \to C$ Income effect

$$\frac{\partial l^s}{\partial w} = \underbrace{\left.\frac{\partial l^s}{\partial w}\right|_{u=u_0}}_{\text{Substn. effect, moving along indifference curve}} + \underbrace{\frac{\partial l^s}{\partial X} \cdot \frac{\partial X}{\partial w}}_{\text{Income effect on } l^s \text{ times change in income due to change in wage}}$$

$$\frac{\partial X}{\partial w} = l$$

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More about...

Income & substn effects...

Test felicity function

Put it in a static model, see if effects are equal so that $\frac{\partial L^s}{\partial w} = 0$. If yes, it will be ok for RBC model. (See K&R (1999) p. 945

Our felicity function in static model

Lagrangian

$$\mathcal{L} = \ln C + b\ln(1-l) + \lambda(X + wl - C)$$

(From $C = X + wl$)

$$\frac{\partial \mathcal{L}}{\partial C} = \frac{1}{C} - \frac{1}{\lambda} = 0$$

$$\frac{\partial \mathcal{L}}{\partial L} = b\frac{1}{1-l} + \lambda w = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = X - wL - C = 0$$

First two f.o.c.'s together give MU Leisure = MUC · W

$$b\left(\frac{1}{1-l}\right) = \frac{1}{C}W$$

hence $l^s = \frac{1}{1+b} - \frac{b}{1+b}\frac{X}{W}$. For $X = 0$, $l^s = \frac{1}{1+b}$

hence $\frac{\partial l^s}{\partial X} = -\frac{b}{1+b}\frac{1}{W}$

so in Slutsky

$$\frac{\partial l^s}{\partial w} = \left.\frac{\partial l^s}{\partial w}\right|_{u=u_0} \underbrace{-\frac{b}{1+b}\frac{1}{w}l}_{\text{income effect}}$$

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More about…

Our felicity function in static model

So what's substn effect? What's $\frac{\partial l}{\partial w}$ moving along indifference curve from A to B?

$$U_0 = \ln C + b \ln(1-l)$$

At both A & B, $b\left(\frac{1}{1-l}\right) = \frac{1}{C} w$

$$\text{so } C = (1-l)\frac{w}{b}$$

$$U_0 = \ln\left[(1-l)\frac{w}{b}\right] + b \ln(1-l)$$

Use "implicit function theorem": Fixing $\partial U = 0$,

$$\frac{\partial l}{\partial w} = \frac{\partial U}{\partial w} \Big/ \frac{\partial U}{\partial l}$$

gives $\partial l / \partial w = \frac{1}{w} \frac{1-l}{1+b}$

so what's

$$\frac{\partial l^s}{\partial w} = \frac{1}{w}\frac{1-l}{1+b} - \frac{b}{1+b}\frac{1}{w} l$$

at $l = \frac{1}{1+b}$ (all income is wages)

$$= \frac{1}{w}\frac{1-\frac{1}{1+b}}{1+b} - \frac{b}{1+b}\frac{1}{w}\frac{1}{1+b}$$

$$= \frac{1}{w}\frac{b}{(1+b)^2} - \frac{1}{w}\frac{b}{(1+b)^2} = 0$$

OK!

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More about utility function--

Income & substn. effects with $u = \ln(c_t) + \ldots$

Why does l increase in response to $\tilde{A}\uparrow$?

Why does a 1% ↑ in $\tilde{A}$ cause $l\uparrow$, while $g = 1\%$ (trend $A\uparrow$) does not?

The $\tilde{A}$ increases lifetime income & consumption, but not as much as the trend growth.

Also, $\tilde{A}\uparrow \rightarrow r > r^*$

From intratemporal F.O.C., $l_t = 1 - \frac{bc_t}{w_t}$

Combining with interdemporal,

$$l_t = 1 - \frac{b}{w_t} e^{\rho} c_{t+1} \frac{1}{1+r_{t+1}}$$

In both $\tilde{A}$ & trend growth, $w_t\uparrow$,

but in $\tilde{A}$, c_t rises less because c_{t+1} rises less and $r_{t+1}\uparrow$.

So l_t can increase.

K&R (1999) p. 973: "When the shock is temporary, there is a small wealth [e.g., lifetime-income] effect that depresses labor supply but temporarily high wages & real interest rates induce individuals to work hard. When the shock is permanent, there are much larger wealth effects..."

In our L^S/L^D graph, $\tilde{A}$ makes L^S curve ↑ a little;

trend growth makes L^S curve ↑ more.

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More about utility Function...

Can other Felicity Functions work?

Yes. Here's one:

$$u(c_t, 1-l_t) = \ln c_t + b\frac{1}{1-\eta}\left((1-l)^{1-\eta} - 1\right)$$ (K & R eqn. (4.2))

Intratemporal F.O.C. here is:

$$w_t \frac{1}{c_t} = b(1-l)^{-\eta}$$ (Note that as $\eta \to 1$ this becomes $\ln(1-l)$)

Across two periods in LRSS where $\frac{w_{t+1}}{w_t} = \frac{c_{t+1}}{c_t} = e^g$,

$$\frac{w_{t+1}\frac{1}{c_{t+1}}}{w_t\frac{1}{c_t}} = \frac{b(1-l_{t+1})^{-\eta}}{b(1-l_t)^{-\eta}} = \frac{b(1-\bar{l})^{-\eta}}{b(1-\bar{l})^{-\eta}} = 1$$

$$\frac{w_{t+1}}{w_t}\frac{c_t}{c_{t+1}} = 1$$

$$e^g \frac{1}{e^g} = 1$$ No problem!

Again, this is saying that income effect of growth in lifetime income (consumption) exactly counteracts substn. effect of $w\uparrow$.

If felicity separable, you can do what you want to $u_{1-l}(l)$ but you do need $u_c(c)$ to be $\ln(c_t)$.

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More about

What if Felicity Function is not separable?

In general, Felicity Function must be of form:

$$u(c, 1-l) = \frac{1}{1-\sigma}\left\{[c\, v(1-l)]^{1-\sigma} - 1\right\} \qquad \left(\begin{matrix} K \& K \\ (3.8) \end{matrix}\right)$$

a Function of $1-l$

Note that $\dfrac{\partial u}{\partial(c\,v(1-l))} = \dfrac{1}{[c\,v(1-l)]^{-\sigma}}$

As $\sigma \rightarrow 1$ this becomes

$$u(c, 1-l) = \ln[c\,v(1-l)] = \ln c + \ln[v(1-l)]$$

(Note that if $v(1-l) = 1-l$ this is Romer's log-log Felicity, & with the right $v(1-l)$ it can also give one on previous page.)

MUC & MU Leisure:

$$\frac{\partial u}{\partial c_t} = \frac{1}{[c_t\, v(1-l_t)]^{\sigma}} \cdot v(1-l_t)$$

$$\frac{\partial u}{\partial(1-l_t)} = \frac{1}{[c_t\, v(1-l_t))^{\sigma}} \cdot c_t \cdot v'(1-l_t)$$

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Move about time

What if Felicity is not separable? (cont.)

Intratemporal F.o.c.:

$$w_t \cdot \frac{1}{[\quad]^{\sigma}} \cdot v(1-l_t) = \frac{1}{[\ ]^{\sigma}} \cdot c_t \cdot v'(1-l_t)$$

$$w_t\, v(1-l_t) = c_t\, v'(1-l_t)$$

Across two periods in LRSS,

$$\frac{w_{t+1}\, v(1-\bar{l})}{w_t\, v(1-\bar{l})} = \frac{c_{t+1}\, v'(1-\bar{l})}{c_t\, v'(1-\bar{l})}$$

$$\frac{w_{t+1}}{w_t} = \frac{c_{t+1}}{c_t}$$

$e^g = e^g$ No problem!