RBC Theory Developments

Criticisms of early RBC models

Things true in model, not true in reality.
Recall that a business cycle in the model must be $\tilde{A}$ not $\tilde{G}$ (causes countercyclical C

1) In model, real wage strongly procyclical.
   In reality, $\frac{\text{Wage index}}{\text{Price index}}$ not.

2) In model, variation in employment comes from variation in hours/worker ("intensive margin")
   In reality, mostly variation in number employed ("extensive margin")

3) In model, individual's labor supply responds strongly to temporary fluctuations in real wage & real interest rate ("intertemporal substitution of labor supply").
   No evidence people do this.

4) In model, C is procyclical but only a little, not as strongly procyclical as in reality.

And the biggest criticism....

RBC Thy Developments

Technological regress?

In model, a recession in which $Y\downarrow$, $L\downarrow$ as in real recessions must be caused by an absolute decline of TFP (not just below-trend growth) that persists for several quarters.

Yes, estimates of TFP backed out from

$$\underbrace{Y}_{\text{real GDP}} = A \underbrace{K}_{\text{capital stock}}^{\alpha} \underbrace{L}_{\text{employment hours}}^{1-\alpha}$$

& data do show big, persistent $A\downarrow$

because capital stock doesn't fall in recession, & Y falls more than L

but there's a plausible alternative explanation for this: in recession,

— labor input per employment hours falls ("labor hoarding")

— lots of capital is unused or used less ("capital utilization" falls).

How can technology get worse?

Wouldn't we notice?

RBC Theory Developments
Technological regress? (cont.)
Why won't below-trend growth do the trick?

Why does model require technological regress?

Suppose $\tilde{A}_{t-1} = \tilde{K}_{t-1} = 0$ (variables were at LRSS values last period)

hence $\tilde{K}_t \approx 0$

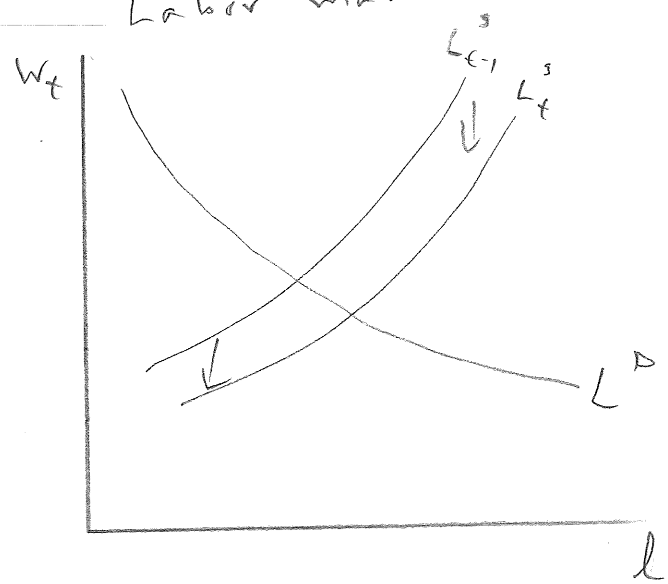
capital stock near LRSS this period,

$$\frac{K_t}{K_{t-1}} = e^{g+n}$$ (capital stock grew at rate $g+n$

Now, $\tilde{A}_t < 0$. A growth <u>slowdown</u> is $1 \leq \frac{A_t}{A_{t-1}} < e^g$

Technological <u>regress</u> is $\frac{A_t}{A_{t-1}} < 1$, $A_t < A_{t-1}$

Labor market:



$$w^s_t = b C_t \frac{1}{1 - l_t}$$

If $C_t < C_{t-1}$, L^s shifted <u>down</u>.

To make l fall, L^D must shift down enough to overcome this.

$$W^D_t = (1-\alpha) K_t^{\alpha} A_t^{1-\alpha} (N_t l_t)^{-\alpha}$$

Since $K_t > K_{t-1}$, to make l fall, A_t must be less than A_{t-1}.

Possible ways to make l fall as c falls: change assumptions so that

— L^S falls while c falls

— L^D falls even though $K_t > K_{t-1}$, $A_t = A_{t-1}$. A third factor?

RBC Thy Developments

Technological regress? (cont.)

Why must A↓ persist for many quarters?

In the model, a short-lived drop in A can't cause a multi-period recession.

Recall complete model

$$\tilde{C}_t = 0.59\,\tilde{K}_t + 0.38\,\tilde{A}_t - 0.13\,\tilde{G}_t$$

$$\tilde{L}_t = -0.31\,\tilde{K}_t + 0.35\,\tilde{A}_t + 0.15\,\tilde{G}_t$$

$$\tilde{K}_{t+1} = 0.95\,\tilde{K}_t + 0.08\,\tilde{A}_t - 0.004\,\tilde{G}_t$$

Say $\tilde{A} < 0$ in period t, then pops back up to zero.

See that $\tilde{K}_{t+1} < 0$ (because investment down to spread the pain over time)

In $t+1$, with $\tilde{A}_{t+1} = 0$ & $\tilde{K}_{t+1} < 0$, $\tilde{L}_{t+1} > 0$!

(because smaller capital stock lowers lifetime income looking forward from $t+1$)

So one quarter of recession will immediately be followed by <u>high</u> employment & output!

<u>Can an RBC-type model get recessions from something other than technological regress?</u>

RBC Theory Developments

Home Production (Benhabib, Rogerson & Wright, 1991)

Single-period utility is

$$u(c, 1-l) = \ln(c) + b\ln(1-l)$$

but $$c_t = \left[a\, c_{Mt}^{\phi} + (1-a)\, c_{Ht}^{\phi}\right]^{1/\phi}$$

$$l_t = h_{Mt} + h_{Ht}$$

c_M : "Market" (store-bought) consumption

c_H : "Home-produced" consumption

h_M : Hours work in "market," for a wage

h_H : Hours "worked" at home.

Home production function: $C_{Ht} = A_{Ht} K_t^{\eta} H_t^{1-\eta}$

(all home prodn is consumed; there's no home-produced capital)

When $\tilde{A}_{Ht} \uparrow$ (a positive home productivity shock),

$c_M \downarrow$, $h_M \downarrow$, investment in "market" capital falls.

Hence, downturns without technological regress

(also, h_M responds more to A_{Mt} ; market L^S more elastic)

RBC Theory Developments

King & Rebelo (1999), "Resuscitating Real ...." Section 8.

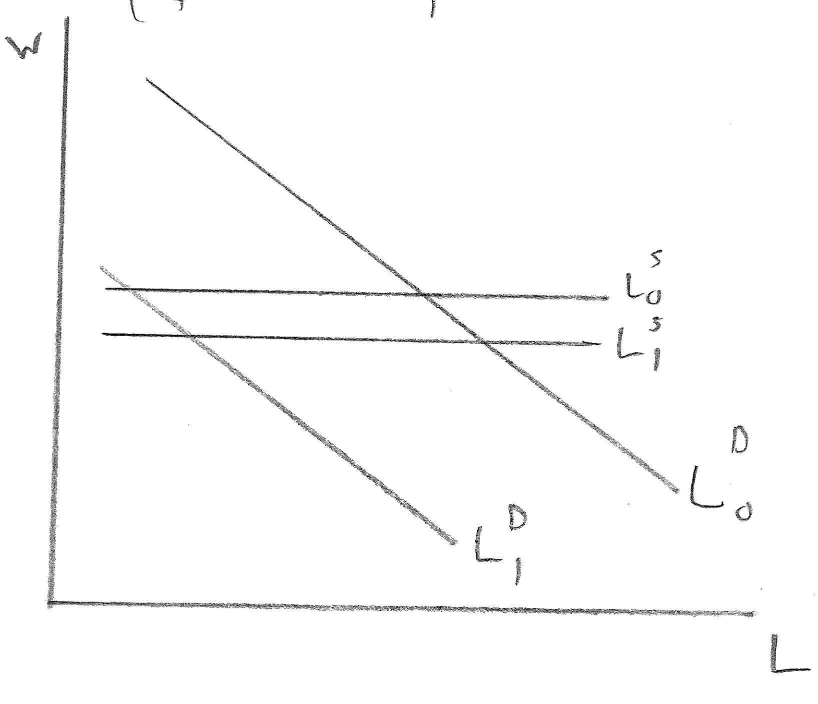
An RBC model in which slowdown in TFP growth can cause a recession. Don't need absolute decline in TFP.

"realistic business cycles from small, nonnegative changes in technology"

Introduce a new factor of production, "capital utilization," which falls when $r < r^*$ (LRSS) (endogenously.)

This makes L^D shift back, hence L & Y can fall.

To make effect on L&Y as big as possible, also add something to the model that makes L^S very elastic: "indivisible labor & lotteries" (+ "consumption insurance")



so when TFP slows down, $r < r^*$, $L^D \downarrow$ because capital utilization $\downarrow$, $L^S \downarrow$ (as in baseline model, because lifetime income $\downarrow$), but L^S extremely elastic so...

RBC thy Developments

King & Rebelo (1999)

Capital utilization

Z_t How hard you run capital

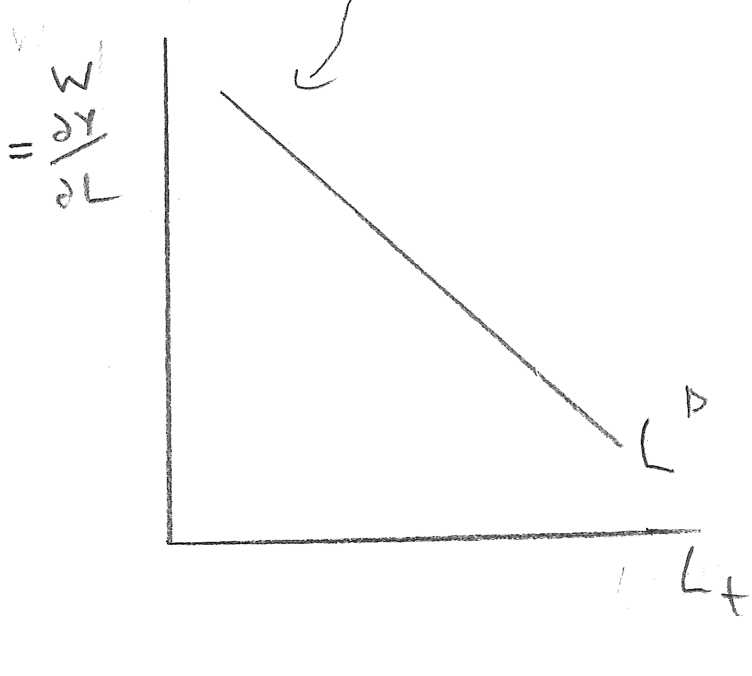
affects labor productivity, hence L^D curve;

$$Y_t = (Z_t K_t)^{\alpha} (A_t L_t)^{1-\alpha} \qquad \left[\text{in K\&R, } Y_t = A_t (ZK)_t^{1-\alpha} N_t^{\alpha}\right]$$

(labor input

What is labor demand curve?

$$MPL \ \frac{\partial Y}{\partial L} = (1-\alpha) Z_t^{\alpha} K_t^{\alpha} A_t^{1-\alpha} L_t^{-\alpha}$$



so if $Z \downarrow \rightarrow L^D \downarrow$

just as

if $A \downarrow \rightarrow L^D \downarrow$

Capital utilization also affects depreciation...

RBC Thy Developments

K & R (1999)

Capital utilization (cont.)

Depreciation rate δ increases with Z

$\delta_t(Z_t)$ A function $\delta'(Z_t) > 0$, $\delta''(Z_t) > 0$

Convex! This will matter!

$$K_{t+1} = K_t + Y_t - C_t - \delta(Z_t)K_t$$

In period t, taking as given K_t & A_t, household (social planner) chooses Z_t, L_t, C_t (hence investment $I = Y_t - C_t$)

to maximize

$$E \sum_{\tau=0}^{\infty} \beta^{\tau} u(C_{t+\tau}, L_{t+\tau})$$

Note: in choosing Z_t, tradeoff

$Z_t \uparrow \longrightarrow Y_t \uparrow$ GOOD

$\searrow \delta_t \uparrow$ BAD

RBC Thy Developments

K&R (1999)

Capital utilization (cont.)

How to choose Z_t

Bellman equation:

$$V(K_t, A_t, etc.) = \max_{C, L, Z} \{U(C_t, L_t) + \beta E_t[V(K_{t+1}, \ldots)]\}$$

$$\text{s.t. } Y_t = (Z_t K_t)^{\alpha} (A_t L_t)^{1-\alpha}$$

$$K_{t+1} = K_t + Y_t - C_t - \delta(Z_t) K_t$$

$$= K_t + (Z_t K_t)^{\alpha} (A_t L_t)^{1-\alpha} - C_t - \delta(Z_t) K_t$$

At optimal value of Z_t, it must be true that holding other choice variables fixed (C, L)

$$0 = \frac{\partial V_t}{\partial Z_t} = \beta E_t[V_K(K_{t+1}, \ldots)] \frac{\partial K_{t+1}}{\partial Z_t}$$

$$\& \quad \frac{\partial K_{t+1}}{\partial Z_t} = \alpha (Z_t K_t)^{\alpha-1} K_t (A_t L_t)^{1-\alpha} - \delta'(Z_t) K_t$$

so $0 = \nearrow$ so at optimal Z_t

$$\delta'(Z_t) K_t = \alpha K^{\alpha} (AL)^{1-\alpha} Z^{\alpha-1}$$

This implies Z is related to r!

How? Recall $r = MPK - \delta$

$$= \alpha (Z_t K_t)^{\alpha-1} Z_t (A_t L_t)^{1-\alpha} - \delta(Z_t)$$

$$= \alpha Z_t^{\alpha} K_t^{\alpha-1} (A_t L_t)^{1-\alpha} - \delta(Z_t) \ldots$$

RBC Thy Developments

K&R (1999)

Capital utilization

How to choose Z_t (cont.)

From $\delta'(Z_t)K_t = \alpha K^{\alpha}(AL)^{1-\alpha} Z^{\alpha-1}$ (previous page)

$$Z_t \delta'(Z_t) = \underbrace{\alpha Z^{\alpha}(AL)^{1-\alpha} K^{\alpha-1}}_{\text{this is MPK! (from previous page)}}$$

And $MPK = r + \delta$! So...

$$Z_t \delta'(Z_t) = r + \delta(Z_t)$$

$$r = Z_t \delta'(Z_t) - \delta(Z_t)$$

See that Z_{LRSS} is determined by r_{LRSS}.

$$\frac{\partial r}{\partial Z} = \delta'(Z_t) + Z_t \delta''(Z_t) - \delta'(Z_t) = Z_t \delta''(Z_t)$$

so $$\frac{\partial Z}{\partial r} = \frac{1}{Z_t \delta''(Z_t)}$$

Assumption that $\delta''(Z) > 0$ (convex) implies $\frac{\partial Z}{\partial r} > 0$.

So when $r < r_{LRSS}$, $Z < Z_{LRSS}$.

Recall from simple RBC model that $r_t < r_{LRSS}$ whenever $\tilde{A}_t < 0$ (A below trend growth)

so slow growth in A can cause $Z\downarrow$.

RBC Thy Developments

K&R (1999)

Indivisible labor & employment lotteries

(& consumption insurance)

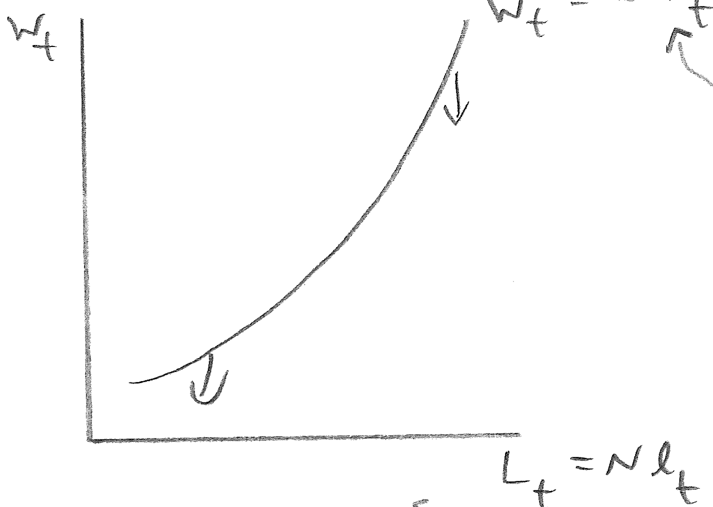
How can we make effect of $L^D \downarrow$ on L & Y as big as possible? Make L^s elastic!

Recall from simple RBC model that when $\tilde{A} \downarrow$ L^s curve shifts down/out, reducing effect on L:

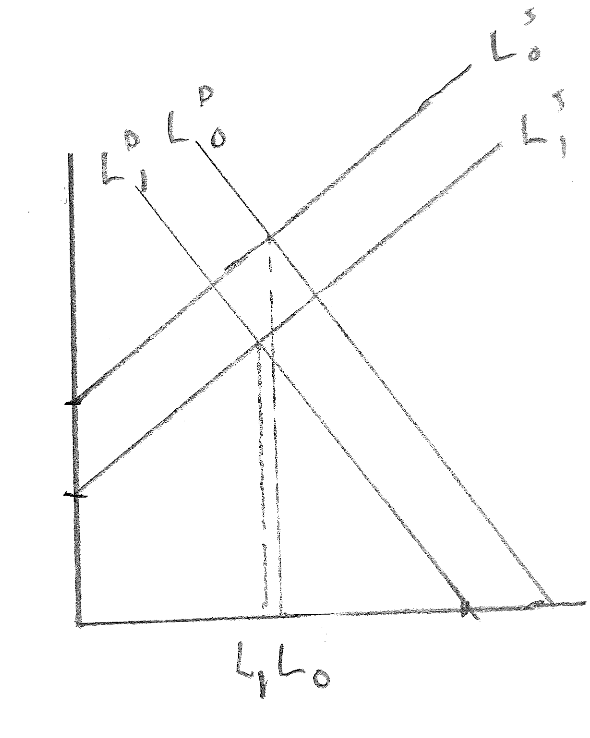
$$w_t^s = b\, c_t \frac{1}{1-\ell^s}$$

$$\left(c_t = e \frac{1}{e\, E_t\left[\frac{1}{c_{t+1}}(1+r_{t+1})\right]} \right.$$

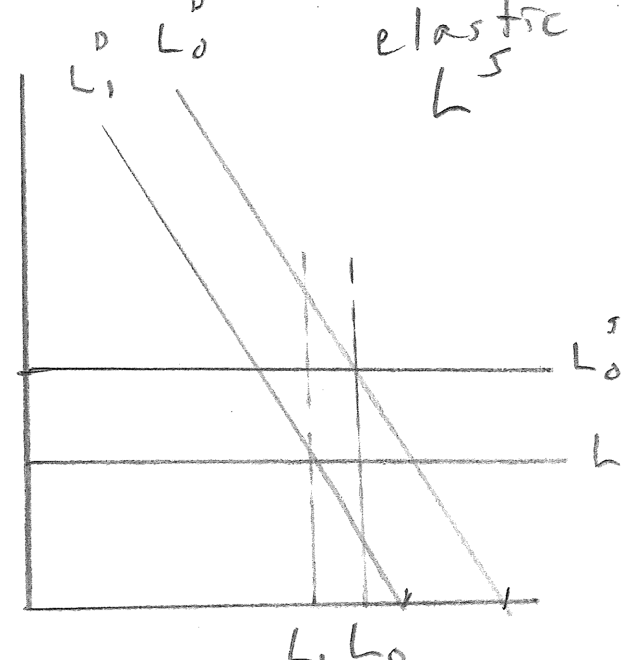
Lower lifetime income reduces c_t, so



Less elastic L^s



More elastic L^s



Bigger drop in L

RBC Thy Developments

Indivisible Labor

Makes L^S very elastic, without messing up LRSS

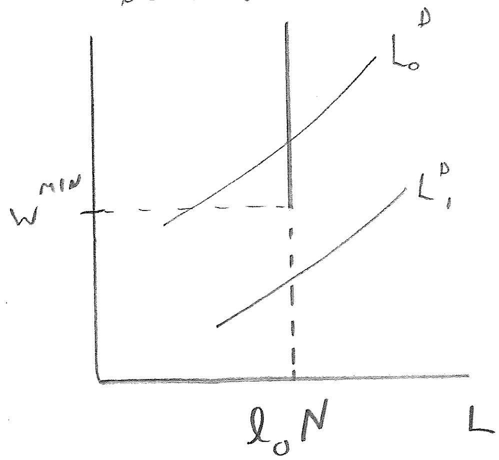
Extra assumption: you must work l_0 hours or not at all.

By itself, this would cause L^S to be inelastic: everyone works l_0 hours unless

$$\underbrace{u_c(w l_0) + u_{1-l}(1-l_0)}_{\text{utility I get if I work at wage } w} < \underbrace{u_{1-l}(1)}_{\text{utility from lots of leisure, no consumption}}$$

or $$u_c(w l_0) < u_{1-l}(1) - u_{1-l}(1-l_0)$$

Defines a minimum wage w^{MIN}



At L_1^D, no employment or output.

Otherwise, variations in L^D affect w but not l

But add another extra assumption:

"employment lotteries" Hansen (1985)

"individuals will be randomly assigned to employment or unemployment each period, with consumption insurance against the possibility of unemployment"

RBC Thy Developments

Indivisible Labor (cont.)

Indivisible labor + employment lotteries + consumption insurance

K & R show that in RBC model, this whole bundle can make L^s very elastic.

For an added bonus, they use a nonseparable Felicity Function which makes consumption more procyclical.

But the effect on L^s elasticity can be shown with our old Romer Felicity Function so that's what I'll use here.

RBC Thy Developments
Indivisible Labor (cont.)

$0 < p_t < 1$ Probability a household member is assigned "work"

hence $L_t = p_t \, l_0 \, N_t$, $p_t = \dfrac{L_t}{l_0 N_t}$

· Expected single-period utility before work/leisure assignment:

$$p_t\left[u_c(c_1) + u_{1-l}(1-l_0)\right] + (1-p_t)\left[u_c(c_2) + u_{1-l}(1)\right]$$

where c_1 is consumption given to a worker,

c_2 " " " " " person at leisure

and $p_t c_1 + (1-p) c_2 = C_t$ ← (average consumption per person)

Note: $c_2 = \dfrac{C_t}{1-p} - \dfrac{p}{1-p} c_1$

Bellman equation (for social planner)

$$V_t = \text{Max}\left\{ \text{above single-period utility} + \beta E_t\left[V(k_{t+1})\right]\right\}$$

Choice variables:

C_t Average consumption per person

c_{1t} Consumption given to "worker" (which determines c_2)

p_t Fraction of household members assigned work

RBC Thy Developments

Indivisible Labor

Choice of c_1 (& c_2) given c_t

We'll begin solving the problem by taking c_t & p_t as given and choosing c_{1t} (c_{2t}).

$$V_t = \max\left\{ p_t\left[u_c(c_1) + u_{1-l}(1-l_0)\right] + (1-p_t)\left[u_c(c_2) + u_{1-l}(1)\right] + \beta E_t[V_{..}] \right\}$$

$$\text{s.t. } c_2 = \frac{c_t}{1-p} - \frac{p}{1-p}c_1$$

$\partial c_2 / \partial c_1$

At optimum c_{1t},

$$0 = \frac{\partial V_t}{\partial c_{1t}} = p\,u_c'(c_1) + (1-p)\,u_c'(c_2)\left(-\frac{p}{1-p}\right)$$

gives optimality condition $u_c'(c_1) = u_c'(c_2)$

Equalize MU of consumption across workers & nonworkers

Given that "marginal felicity" of c_t depends only on quantity of c_t, that means $c_1 = c_2$.

Everybody gets same consumption.

An individual's consumption is independent of his own employment or labor income.

This is called "consumption insurance."

(Note that an "unemployed" household member is happier – same c, more leisure.)

RBC Thy Developments

Indivisible Labor

Choice of C_1.. (cont.)

Note that "separability" of felicity function across l & c has mattered here.

Because of separability, equalizing MU_c means equalizing C_1 & C_2.

If felicity were not separable, social planner would still want to equalize MU_c, but that might not mean equalizing C_1

Example:

$$u(c, 1-l) = \frac{1}{1-\sigma}\left\{[c\, v(1-l)]^{1-\sigma} - 1\right\}$$

$$\frac{\partial u}{\partial c} = \frac{1}{[c_t v(1-l_t)]^{\sigma}} \cdot v(1-l_t)$$

(different for $l = 0$ (nonworker)
$l = l_0$ (worker))

RBC ...

Indivisible Labor :

Choice of p_t

Now choose p_t given that $c_{1t} = c_{2t} = c_t$.

This part of social planner's problem:

$$V_t(K_t) = \underset{c,p}{\text{Max}} \left\{ u_c(c_t) + p_t u_{1-l}(1-l_0) + (1-p_t) u_{1-l}(1) + \beta E_t V(K_{t+1}) \right\}$$

s.t. $K_{t+1} = (1-\delta_t(z_t))K_t + Y(K_t, z_t, L_t) - C_t$

Note $\frac{\partial K_{t+1}}{\partial C_t} = -1$, $\frac{\partial K_{t+1}}{\partial L_t} = \frac{\partial Y}{\partial L_t} = MPL = W$

Choice of p_t will give L_t^s since $L_t = p_t l_0 N_t$

To skip a step, substitute into above $p_t = \frac{L_t}{l_0 N_t}$

$$V_t(K_t) = \underset{c,L}{\text{Max}} \left\{ u_c(c_t) + u_{1-l}(1) - \frac{L_t}{l_0 N_t}\left(u_{1-l}(1) - u_{1-l}(1-l_0)\right) + \beta E_t[\ ] \right\}$$

Take f.o.c.'s for c & L

$$\frac{\partial V_t}{\partial C_t} = 0, \quad \frac{\partial V_t}{\partial L_t} = 0$$

per-capita consumption in felicity fn.

and keep in mind that $\frac{\partial c_t}{\partial C_t} = \frac{1}{N_t}$

aggregate consumption

RBC Theory...

Indivisible Labor

Choice of p_t (cont.)

$$0 = \frac{\partial V}{\partial c_t} = \frac{1}{N_t} u'_c(c_t) + \beta E_t V_{K_{t+1}} (-1)$$

$$0 = \frac{\partial V}{\partial L_t} = \frac{1}{l_0 N_t} \left(u_{1-l}(1) - u_{1-l}(1-l_0) \right) + \beta E_t V_{K_{t+1}} \cdot W_t$$

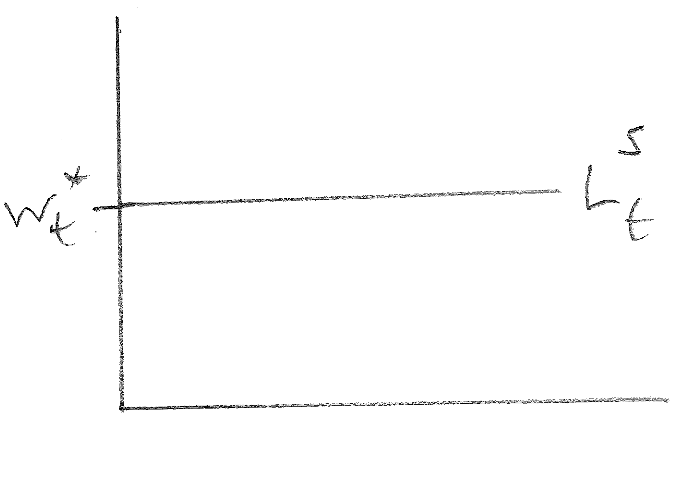
gives $w_t u'_c(c_t) = \frac{1}{l_0} \left(u_{1-l}(1) - u_{1-l}(1-l_0) \right)$

Wait a minute! This was supposed to define L^s.

I don't see L in there.

Right! It means L^S is perfectly elastic.

$$W_t^* = \frac{\frac{1}{l_0} \left(u_{1-l}(1) - u_{1-l}(1-l_0) \right)}{u'_c(c_t)}$$



Social planner supplies any quantity of labor demanded as long as wage is W_t.

RBC Thy

Indivisible Labor

Compare with Romer

Here, with $u_t = \ln c(_t) + b\ln(1-l_t)$:

$$w_t^s = c_t \, b(0 - \ln(1-l_0))$$

Romer:

$$w_t^s = c_t \, b\frac{1}{1-l^s}$$

Both move up & down with c_t, but Romer's is also upward-sloping.

Extra bonus: nonseparable felicity

K & R use a nonseparable felicity function so that $l \uparrow \rightarrow MUC \uparrow$ $\left(\frac{\partial^2 u}{\partial c \, \partial(1-l)} < 0 \right)$

Hence, when social planner equalizes MUC across workers & nonworkers, workers get more C.

Result: RBC's without technological regress

K & R calibrate their model & see effect of $\tilde{A}$.

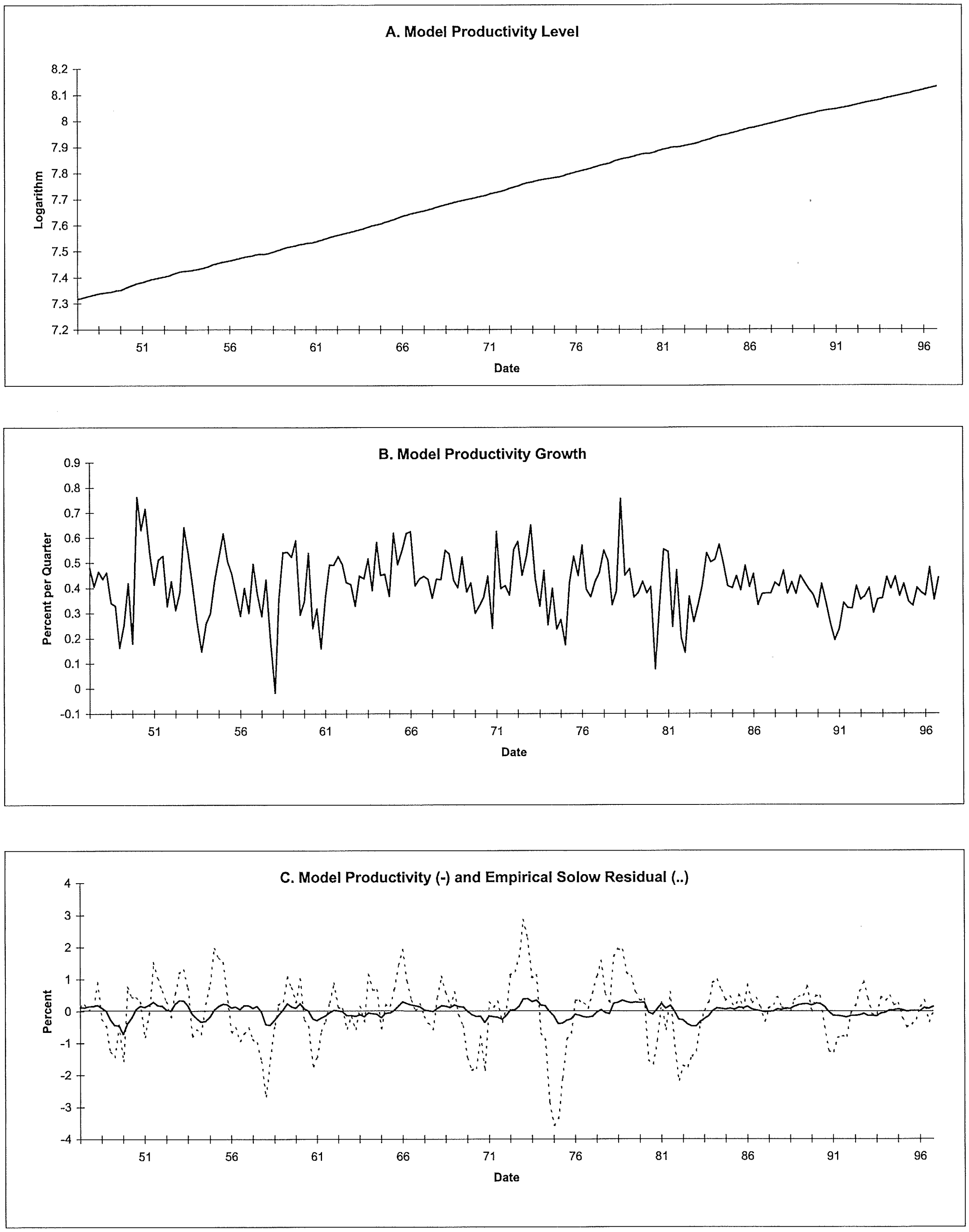
They see that small $\tilde{A} \downarrow$, which push A below trend growth (so that $r < r^{\alpha}$) but do not cause A to actually fall, can cause $Y \downarrow$, $L \downarrow$.

Then they take time series of actual real GDP 1948–1996.

They back out what path of A would have to be, in their model, to produce that actual path of Y (Figure 14).

No regress needed! See next page.

Figure 14


A. Model Productivity Level
Logarithm
8.2
8.1
8
7.9
7.8
7.7
7.6
7.5
7.4
7.3
7.2
51
56
61
66
71
76
81
86
91
96
Date
B. Model Productivity Growth
Percent per Quarter
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
51
56
61
66
71
76
81
86
91
96
Date
C. Model Productivity (-) and Empirical Solow Residual (..)
Percent
4
3
2
1
0
-1
-2
-3
-4
51
56
61
66
71
76
81
86
91
96
Date