

NEW KEYNSIAN PHILLIPS CURVE

FISCHER MODEL PHILLIPS CURVE

Recall Fischer model with assumption that firms set price path based on information from $(t-1)$:

$$\frac{p_t^1}{p_{t-1}^2} \quad \frac{p_{t+1}^1}{p_t^2} \quad \text{based on info from } (t-1)$$
$$\frac{p_{t-1}^2}{p_t^2} \quad \text{based on info from } (t-2)$$

price level at time t is $\frac{1}{2}(p_t^1 + p_t^2)$

$$\text{and } y_t = m_t - p_t \quad \text{and } p_t^* = p_t + \phi y_t$$

$$\text{Result: } p_t = E_{t-2}[m_t] + \frac{\phi}{1+\phi} (E_{t-1}[m_t] - E_{t-2}[m_t])$$

$$y_t = \frac{1}{1+\phi} (E_{t-1}[m_t] - E_{t-2}[m_t]) + (m_t - E_{t-1}[m_t])$$

What kind of Phillips curve would this generate?

To make results more comparable with NKPC discussion, assume firms set price path with information from t :

$$\frac{p_t^1}{p_{t+1}^2} \quad \frac{p_{t+1}^1}{p_t^2} \quad \text{based on info from } t$$
$$\frac{p_{t+1}^2}{p_t^2} \quad \text{based on info from } t-1$$

$$\text{means } p_t^1 = p_t^* = p_t + \phi y_t$$

$$p_t^2 = {}_{t-1}P_t^* = {}_{t-1}P_t^e + \phi {}_{t-1}y_t^e$$

NEW KEYNSEIAN PHILLIPS CURVE

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FISHER MODEL PHILLIPS CURVE (cont.)

$$p_t = \frac{1}{2}(\phi y_t + p_t) + \frac{1}{2}(\phi_{t-1} y_t^e + p_{t-1}^e)$$

solve for p_t

$$p_t = \phi y_t + \phi_{t-1} y_t^e + p_{t-1}^e$$

subtract p_{t-1} from both sides

$$p_t - p_{t-1} = \dots \dots \dots p_t - p_{t-1}$$

$$\pi_t = \phi (y_t + y_t^e) + p_{t-1}^e$$

$$= \phi y_t + \underbrace{E_{t-1} [\phi y_t + \pi_t]}$$

past beliefs about current conditions

$$E_{t-1} [\pi_t] = \phi (E_{t-1} [y_t] + y_t^e) + E_{t-1} [p_{t-1}^e]$$

IF expectations rational,

$$E_{t-1} [\pi_t] = p_{t-1}^e$$

$$E_{t-1} [p_{t-1}^e] = p_{t-1}^e$$

$$E_{t-1} [y_t] = y_t^e$$

hence

$$p_{t-1}^e = \phi \underbrace{(y_t^e + y_t^e)}_{\text{must be zero}} + p_{t-1}^e$$

hence...

NEW KEYNESIAN PHILLIPS CURVE

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FISCHER MODEL PHILLIPS CURVE (cont.)

$$\pi_t = \phi Y_t + \pi_{t-1}$$

Same as LSF
Friedman-Phelps Phillips curve!

Means $y_t > 0$ does not forecast future π . Good.

But $E_t[y_{t+1}] = 0$ always. Bad.

? What if we combine Fischer model assumption that price setters predetermine path for p_i , not necessarily fixed i ,

with Calvo assumption that a price setter resets with fixed probability each period, not at predetermined intervals?

Result: Mankiw-Reis "Sticky Information,"

But why call it "Sticky Information"?

STICKY INFORMATION (MANKIW & REIS, 2002)

Like Fischer, price setter sets a path for future p 's instead of fixing p

Like Calvo, reset at random time

Motivation/interpretation of reset times: when price setter gets new info & updates forecast about economy, current & future w_i^* 's.

Two questions:

1) Phillips curve?

2) Can we get persistent recessions/booms from m shocks? Recall in LSF, answer was no.

Also no for "sticky info" with annual updating.

STICKY INFO (cont.)

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Assumptions using Romer notation

$y_t = m_t - p_t$ & monetary policy sets a path form.
 α ← reset probability

M&R's "λ"

M&R's "α"

$$p_{it}^* = (1-\phi)p_t + \phi m_t$$

hence expected future p_{it}^* 's depend on
expected future m 's & p 's ← (behavior of other price setters)

A firm that updates info at time t sets

$$p_{it+j} = E_t p_{it+j}^* = (1-\phi)E_t p_{it+j} + \phi E_t m_{t+j}$$

Rational expectations.

Information intake changes expectations of future:

$$E_{t-1} m_t - E_{t-2} m_t \quad \text{Effect of info arriving in } (t-1) \\ \text{on forecast of } m_t$$

$$m_t - E_{t-1} m_t \quad \text{Error in time } t\text{'s forecast}$$

$$m_t = m_t - E_{t-1} m_t + \underbrace{(E_{t-1} m_t - E_{t-2} m_t)}_{\text{info arriving in } t-1} + \underbrace{(E_{t-2} m_t - E_{t-3} m_t)}_{\text{info arriving in } t-2} + \dots$$

STICKY INFO (cont.)

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Solution (Following Romer)

Conjecture form of soln:

- 1) y_t & p_t depend only on m_t & previous periods' forecasts of m_t (only exogenous variable)
- 2) If m_t is exactly what everyone forecast at all points in the past, $y_t = 0$ (no surprises)
- 3) Soln. is loglinear (y is loglinear fn. of m 's)

1) means

$$p_t = a_0 (m_t - E_{t-1} m_t) + a_1 (E_{t-1} m_t - E_{t-2} m_t) + a_2 (E_{t-2} m_t - \dots)$$

effect on p_t of info arriving in period $(t-1)$

"fraction... passed into the aggregate price level"

$$\begin{aligned} y_t &= m_t - p_t \\ &= m_t - E_{t-1} m_t + (E_{t-1} m_t - E_{t-2} m_t) + \dots \leftarrow (m_t) \\ &\quad - a_0 (m_t - E_{t-1} m_t) - a_1 (E_{t-1} m_t - E_{t-2} m_t) - \dots \leftarrow (p_t) \\ &= \sum_{i=0}^{\infty} (1 - a_i) (E_{t-i} m_t - E_{t-i-1} m_t) \end{aligned}$$

So, what are values for a_i 's?

STICKY INFO

Soln. (cont.)

What are a_i 's?

a_i is effect on p_t of $(E_{t-i} m_t - E_{t-i-1} m_t)$

Firms that have updated info in $(t-i)$ or later have adjusted p_t in response to that info. Firms that last updated prior to $(t-i)$ haven't adjusted p_t to that info.

Figure out:

- 1) For a firm that has updated since $(t-i)$, what's effect of that info on p_t ?
- 2) What fraction of firms has updated since $(t-i)$?

1) & 2) together give effect of $(E_{t-i} m_t - E_{t-i-1} m_t)$ on p_t , which is a_i .

STICKY INFO

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Soln

What is a_i ?

1) For a firm that has updated, effect of info

When a firm updates,

$$\begin{aligned}\Delta p_{it} &= \Delta E p_{it}^* = \Delta E((1-\phi)p_t + \phi m_t) \\ &= (1-\phi)\Delta E p_t + \phi \Delta E m_t\end{aligned}$$

Individual firm takes a_i as given, so response to $(E_{t-i} m_t - E_{t-i-1} m_t)$ is

$$\begin{aligned}\Delta p_{it} &= (1-\phi)a_i(E_{t-i} m_t - E_{t-i-1} m_t) + \phi(E_{t-i} m_t - E_{t-i-1} m_t) \\ &= [(1-\phi)a_i + \phi](E_{t-i} m_t - E_{t-i-1} m_t)\end{aligned}$$

Effect on price level is

$$\Delta p_t = \lambda_i [(1-\phi)a_i + \phi] (\dots)$$

↑
fraction of firms that have updated
since i

and $a_i = \lambda_i [(1-\phi)a_i + \phi]$

STICKY INFO

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Soln.

What is a_i ?

2) What is λ_i ?

$(1-\alpha)$ Prob. firm has not updated since $t-1$

$(1-\alpha)(1-\alpha)$ " " " " " " $t-2$
 $= (1-\alpha)^2$

$(1-\alpha)^{i+1}$ Prob. firm has not updated since $(i-1)$

$\lambda_i = 1 - (1-\alpha)^{i+1}$ Prob. firm has updated
in period i or after period i

So $a_i = \lambda_i [(1-\phi)a_i + \phi]$

solve for a_i :

$$a_i = \frac{\phi [1 - (1-\alpha)^{i+1}]}{1 - (1-\phi)[1 - (1-\alpha)^{i+1}]} \quad (7.81)$$

see: as $i \uparrow$, $(1-\alpha)^{i+1} \downarrow$, $a_i \uparrow$

Info about m_t that arrived long ago has big effect on p_t . Almost all firms have had a chance to update & change p_t plan.

Effect of m on output

$$\text{Recall } y_t = \sum_{i=0}^{\infty} (1-\alpha_i) (E_{t-i} m_t - E_{t-i-1} m_t)$$

$$\text{where } \alpha_i = \frac{\phi [1 - (1-\alpha)^{i+1}]}{1 - (1-\phi)[1 - (1-\alpha)^{i+1}]}$$

Example: m is random walk

$$m_t = m_{t-1} + \varepsilon_t \text{ means } (E_{t-i} m_t - E_{t-i-1} m_t) = \varepsilon_i$$

1) As $i \uparrow$, so that shock ε_i is further back in time, bigger effect on p_t , smaller effect on y_t .

But old ε 's are still affecting y_t

($1-\alpha_i$) is close to zero, but positive)

so variations in y are persistent.

AD shocks have persistent effects on y .

2) Real rigidity. Small ϕ means small α_i , ε 's have bigger effect on y .

STICKY INFO

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Effect of m on output

Possible lag in effect of m on π

For small ϕ (lots of real rigidity)

& "plausible" value of α_j

as time passes following an ϵ_j

α_i rises fast at first, then more slowly

& effect on π is greatest many periods after

shock. Hence a downward m shock

"first produces a recession, then a fall

in inflation."

STICKY INFORMATION

Sticky-Information Phillips Curve (following M&R)

$P_t^* = P_t + \alpha y_t$ (Romer's ϕ)

λ Probability firm can adjust it's path for p (Romer's α)

x_t^j Price for period t set by firm that last updated j periods ago

$x_t^j = E_{t-j} P_t^*$

Firms that adjusted period before last

$P_t = \lambda P_t^* + (1-\lambda)\lambda E_{t-1} P_t^* + (1-\lambda)(1-\lambda)\lambda E_{t-2} P_t^* + \dots$
firms that adjusted last period M&R's

$P_t = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-j} P_t^*$ (A3)

$P_t = \lambda (P_t + \alpha y_t) + \lambda \sum_{j=0}^{\infty} (1-\lambda)^{j+1} E_{t-1-j} (P_t + \alpha y_t)$ (A4)

$= \lambda (P_t + \alpha y_t) + \lambda \sum (1-\lambda)^j (1-\lambda) E_{t-1-j} (P_t + \alpha y_t)$

$= \lambda (P_t + \alpha y_t) + \lambda \sum (1-\lambda)^j E_{t-1-j} () - \lambda^2 \sum (1-\lambda)^j E_{t-1-j} ()$

From (A3), $P_{t-1} = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-1-j} (P_{t-1} + \alpha y_{t-1})$ (A5)

hence

$\pi_t = P_t - P_{t-1} = \lambda (P_t + \alpha y_t) + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-1-j} (\overbrace{P_t - P_{t-1}}^{\pi_t} + \alpha (\overbrace{y_t - y_{t-1}}^{\Delta y_t}))$

$- \lambda^2 \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-1-j} (P_t + \alpha y_t)$

what's this?

NKPC

STICKY INFORMATION

Sticky-information Phillips Curve (cont.)

$$\text{Recall } p_t = \lambda(p_t + \alpha y_t) + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-1-j}^{j+1} (p_t + \alpha y_t) \quad (A4)$$

$$\text{hence } p_t - \lambda p_t - \lambda \alpha y_t = \lambda \sum_{j=0}^{\infty} \dots$$

$$(1-\lambda) p_t - \lambda \alpha y_t =$$

Divide both sides by $(1-\lambda)$

$$p_t - \frac{\lambda \alpha}{(1-\lambda)} y_t = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-1-j}^{j+1} (p_t + \alpha y_t)$$

(note difference)
funny thing at bottom of previous page,
divided by λ . Substitute

$$\pi_t = \lambda(p_t + \alpha y_t) + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-1-j}^{j+1} (\pi_t + \alpha \Delta y_t) - \lambda \left(p_t - \frac{\lambda \alpha}{1-\lambda} y_t \right)$$

$$= \frac{\alpha \lambda}{1-\lambda} y_t + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-1-j}^{j+1} (\pi_t + \alpha \Delta y_t)$$

past expectations of current economic conditions, as in Fischer or Friedman-Phelps.