TIME-DEPENDENT PRICE ADJUSTMENT, A GENERAL RESULT

(ROMER 7.1, "Firms")

__Question:__ if a firm knows that the price it sets today will remain in effect for more than one period (more than one realization of aggregate demand, etc.), what price will it choose?

__Notation__

R_t Real profit $\left(\frac{\$\text{profit}}{P}\right)$ firm receives in period t

p_i Price of firm's product

p_t^* price that __would__ maximize R_t, if firm were able to adjust its price every period

t_0 Period in which firm __can__ set a new price

q_t Probability the price set at time t_0 is still in effect at time t

[Note: we assume q_t is exogenous, unaffected by p_i or by realization of AD, hence pricesetting is __not__ "state-dependent"]

$$\omega_t = q_t \Big/ \underbrace{\sum_{\tau=0}^{\infty} q_\tau}$$

(expected value of the number of periods price will remain stuck at p_i

Result we'll get:

$$p_i = \sum_{t=0}^{\infty} \omega_t \underbrace{E_0[p_t^*]}$$

expected value of p_t^* as of t_0

Example: if price will remain in effect for two periods, & I can adjust in the third period (in other words, firm can adjust its price every other period),

$$q_0 = 1 \qquad q_1 = 1 \qquad q_2 = q_3 = \dots = 0$$

$$\sum_{\tau=0}^{\infty} q_\tau = 1 + 1 + 0 + \dots = 2$$

$$\omega_0 = \frac{q_0}{2} = \frac{1}{2} \qquad \omega_1 = \frac{q_1}{2} = \frac{1}{2} \qquad \omega_2 = 0$$

$$p_i = \frac{1}{2} E_0[p_0^*] + \frac{1}{2} E_0[p_1^*]$$

if p_0^* known at t_0 (no uncertainty),

$$p_i = \frac{1}{2} p_0^* + \frac{1}{2} E_0[p_1^*]$$

TIME-DEPENDENT....

Why is $\sum_{T=0}^{\infty} q_T$ expected value of price duration?

Say T_F is last period a price could possibly still be in force.

Expected value of life is

= 1 · Prob. price dies in one year (not later)

+ 2 · Prob. price dies in two years (not earlier or later)

+ ....

+ T_F · Prob. price dies in T_F years.

So what is prob. price lives to year T, and dies in year T?

prob. price is still alive in $T+1$

$(1-q_{T+1})$ is prob. price is <u>not</u> still alive in $T+1$

= Prob. dies in 1 + Prob. dies in 2 + ... + Prob. dies in T

$(1-q_T)$ is prob. price not still alive in T

= Prob. price dies in 1 + ... + Prob. dies in $T-1$

so

$(1-q_{T+1})-(1-q_T)$ = Prob. dies in T

$= q_T - q_{T+1}$

so....

TIME-DEPENDENT ... (cont.)

Why is $\sum_{\tau=0}^{\infty} q_\tau$ ... ? (cont.)

Expected value of life is

$= 1 \cdot (q_0 - q_1)$

$+ 2 \cdot (q_2 - q_1)$

$+ \ldots$ zero

$+ \tau_F (q_{\tau_F} - q_{\tau_{F+1}})$

$= q_0 - q_1 + 2q_1 - 2q_2 + 3q_2 - 3q_3 + \ldots$

$= q_0 + q_1 + q_2 + \ldots + q_F$

$= \sum_{\tau=0}^{\infty} q_\tau$

Note this allows for possibility that there's no T_F.

In Taylor model price lives a certain number of periods (two), then dies

but you could assume price has a constant probability of dying in any given period, so that there is a small, but positive, prob. price will still be in effect in τ_∞.

We'll be working with a model like this: (Calvo model)

Derivation

Firm's prodn fn is $Y_{it} = L_{it}$

and cost of labor is W_t so $MC_t = W_t$

Demand for firm's product is

$$Y_{it} = Y_t \left(P_{it} / P_t \right)^{-\eta}$$

so profit-maxxing markup is $\frac{\eta}{\eta - 1}$

$$R_t = \frac{1}{P}\left(P_{it} Y_{it} - W_t L_{it}\right) = \left(\frac{P_{it}}{P_t}\right) Y_{it} - \left(\frac{W_t}{P_t}\right) Y_{it}$$

$$= Y_t \left[\left(\frac{P_{it}}{P_t}\right)^{1-\eta} - \left(\frac{W_t}{P_t}\right)\left(\frac{P_{it}}{P_t}\right)^{-\eta} \right] \qquad (7,8)$$

Firm acts to maximize

$$E\left[\sum_{t=0}^{\infty} \lambda_t R_t\right]$$

an odd discount factor. Firm is owned by h'holds with $E\left[\sum_{t=0}^{\infty} \beta^t U(C, L)_t\right]$

Utility value of a unit of real income at time t is $\beta^t \cdot MUC_t$

so $\lambda_t = \dfrac{\beta^t MUC_t}{\beta^0 MUC_0}$ time zero, i.e. now

TIME-DEPENDENT... (cont.)

Derivation (cont.)

Firm chooses p_i to max this.
In doing that, it only has to worry about periods
in which p_i might still be in effect
(firm's price still stuck at p_i).

Define $A \equiv E\left[\sum_{t=0}^{\infty} q_t \lambda_t R_t\right]$

By choosing p_i to max A, I'm making $E\left[\sum \lambda_t R_t\right]$.

$$A = E\left[\sum_{t=0}^{\infty} q_t \lambda_t Y_t \left(\left(\frac{p_i}{P}\right)_t^{1-\eta} - \left(\frac{W}{P}\right)_t \left(\frac{p_i}{P}\right)_t^{-\eta}\right)\right.$$

pull P out front

$$A = E\left[\sum_{t=0}^{\infty} q_t \lambda_t Y_t P_t^{\eta-1}\left(p_i^{1-\eta} - W_t p_i^{-\eta}\right)\right] \quad (7.9)$$

Now our goal is to put this in terms of p_i^*.
Notation change: $p_{it}^* = p_t^*$. (This is actually OK, because
all firms identical, so p_{it}^* same for all.)

Recall $p_t^* = \frac{\eta}{\eta-1} W_t$. So $W_t = \frac{\eta-1}{\eta} p_t^*$. So:

$$A = E\left[\sum_{t=0}^{\infty} q_t \lambda_t Y_t P_t^{\eta-1}\left(p_t^{1-\eta} - \underbrace{\frac{\eta-1}{\eta} p_t^*}_{W_t} p_i^{-\eta}\right)\right.$$

Rearrange to get...

TIME-DEPENDENT.... (cont.)

$$A = E\left[\sum_{t=0}^{\infty} q_t \lambda_t Y_t P_t^{\eta-1} \underbrace{P_i^{-\eta}\left(P_i - \frac{\eta-1}{\eta} P_t^*\right)}_{\text{call this } Z}\right]$$

Note Z is a function of P_i and P_t^* only.
It can be expressed in terms of logs of P_i & P^*:

$$Z_t = F(p_i, p_t^*)$$ This gives (7.10)

Taking a second-order Taylor approximation

$$F(p_i, p_t^*) \approx F(p_t^*, p_t^*) + F'(\ )(p_i - p^*) + \tfrac{1}{2}F''(\ )(p_i - p^*)^2$$

What do we know about F' and F''?

$R_t = Y_t P_t^{\eta-1} Z_t$ and p_t^* maximizes R_t, hence:

$$\frac{\partial R_t}{\partial p_i} = 0 = Y_t P_t^{-\eta-1} F'(p_i - p^*) \text{ at } p_i = p^*$$

(first-order condition)

$$\frac{\partial^2 R_t}{\partial p_i^2} = Y_t P_t^{-\eta-1} F''(p_i - p^*) < 0 \text{ at } p_i = p^*$$

(second-order condition)

hence at $p_i = p^*$, $F'(\ ) = 0$, $\underbrace{F''(\ )}_{\text{call this } -K} < 0$

TIME-DEPENDENT....

Taking a second-order.... (cont.)

Hence $F(p_i, p_t^*) \approx F(p_t^*, p_t^*) - K(p_i - p_t^*)^2$ (7.11)

Substitute approximation into A:

$$A \approx E\left[\sum_{t=0}^{\infty} q_t \lambda_t Y_t P_t^{\eta-1} \left(F(p_t^*, p_t^*) - K(p_i - p_t^*)^2\right)\right]$$

Another approximation: $\lambda_t Y_t P_t^{\eta-1}$ is $\approx$ constant

"Inflation is low" $\rightarrow P_t$ constant

"Economy is always close to flexible-price equilibrium"

$\rightarrow Y_t$ always close to $\overline{Y}$

$\rightarrow C_t$ always close to $\overline{C}$, so MUC constant

"Households' discount factor is close to one"
(households don't discount future utility)
Together with constant MUC,

$\rightarrow \lambda_t$ constant

$$A \approx \text{Constant} \cdot E\left[\sum_{t=0}^{\infty} q_t \left(F(p_t^*, p_t^*) - K(p_i - p_t^*)^2\right)\right]$$

rearrange to get...

Another approximation (cont.)

$$A \approx \text{Constant} \cdot E\left[\sum_{t=0}^{\infty} q_t F(p_t^*, p_t^*)\right]$$ (p_i isn't in here)

$$- \text{Constant} \cdot K \cdot \sum_{t=0}^{\infty} q_t E\left[(p_i - p_t^*)^2\right]$$ (this is only part we control)

(q_t:) this is certain, so we can bring it out front of E[ ]

To maximize A, choose p_t to minimize

$$J = \sum_{t=0}^{\infty} q_t E\left[(p_i - p_t^*)^2\right] \qquad (7.12)$$

How do we minimize this?

Use:

$$E[X^2] = (E[X])^2 + \text{Var}(X) \quad \text{(from defn. of variance)}$$

so here $E[(p_i - p_t^*)^2] = (E[p_i - p_t^*])^2 + \text{Var}(p_i - p_t^*)$

p_i is known (we're choosing it)

so $E[p_i - p_t^*] = p_i - E[p_t^*]$

$\text{Var}(p_i - p_t^*) = \text{Var}(p_t^*)$

so above

$$J = \sum_{t=0}^{\infty} q_t \left((p_i - E[p_t^*])^2 + \text{Var}(p_t^*)\right) \qquad (7.12)$$

Another approximation (cont.)

Find optimal p_i

Now take f.o.c. and solve for optimal value of p_i.

$$\frac{\partial J}{\partial p_i} = q_0 \cdot 2 \cdot (p_i - E[p_0^*]) + q_1 \cdot 2 \cdot (p_i - E[p_1^*])$$
$$+ q_2 \cdot 2 \cdot (p_i - E[p_2^*]) + \ldots$$

$$0 = q_0 \cdot 2 \cdot p_i + q_1 \cdot 2 \cdot p_i + q_2 \cdot 2 \cdot p_i + \ldots$$
$$- q_0 \cdot 2 \cdot E[p_0^*] - q_1 \cdot 2 \cdot E[p_1^*] - q_2 \cdot 2 \cdot E[p_2^*] - \ldots$$

$$0 = 2 p_i \sum_{\tau=0}^{\infty} q_\tau - 2 \sum_{t=0}^{\infty} q_t E[p_t^*]$$

switch to τ because these summations are done independently

$$\Rightarrow p_i = \frac{1}{\sum_{\tau=0}^{\infty} q_\tau} \sum_{t=0}^{\infty} q_t E[p_t^*]$$

Recall $\omega_t = \dfrac{q_t}{\sum_{\tau=0}^{\infty} q_\tau}$ so...

$$p_i = \sum_{t=0}^{\infty} \omega_t E[p_t^*] \qquad (7.13)$$

Done!

TIME-DEPENDENT... (cont.)

If you don't want to say $\lambda_t \approx 1$

A slightly better approximation is to say $\lambda_t = \beta^t$

Then (7.12) becomes:

$$J = \sum_{t=0}^{\infty} q_t \beta^t \left(\left(p_i - E[p_t^*] \right)^2 + Var(p_t^*) \right) \quad (7.14)$$

so

$$\frac{\partial J}{\partial p_i} = q_0 \cdot \beta^0 \cdot 2 \cdot (p_i - E[p_0^*]) + q_1 \cdot \beta^1 \cdot 2 \cdot (p_i - E[p_1^*]) + \ldots$$

$$0 = 2 p_i \sum_{t=0}^{\infty} \beta^t q_t - 2 \sum_{t=0}^{\infty} \beta^t q_t E[p_t^*]$$

$$\Rightarrow p_i = \frac{1}{\sum_{t=0}^{\infty} \beta^t q_t} \sum_{t=0}^{\infty} \beta^t q_t E[p_t^*]$$

Define $\tilde{\omega}_t = \dfrac{\beta^t q_t}{\sum_{t=0}^{\infty} \beta^T q_T}$

$$p_i = \sum_{t=0}^{\infty} \tilde{\omega}_t E[p_t^*] \quad (7.15)$$