MALTHUS' MODEL ("MALTHUSIAN GROWTH")

Production

$$Y_t = A_t F(L_t) \qquad F'(L_t) > 0 \qquad F''(L_t) < 0$$

Story behind production function

Constant returns to scale but fixed factor: land

Example: Cobb-Douglas

$$Y = B\,Land^{\alpha} L^{1-\alpha} \leftarrow B(c\,Land)^{\alpha}(cL)^{1-\alpha} = c\,B\,Land^{\alpha} L^{1-\alpha}$$

Fix land: $Y = \underbrace{(B\,Land^{\alpha})}_{A} L^{1-\alpha}$

$$\frac{\partial Y}{\partial L} = A\,F'(L_t) = (B\,Land^{\alpha})(1-\alpha)L^{-\alpha} > 0$$

$$\frac{\partial^2 Y}{\partial L^2} = A\,F''(L_t) = (B\,Land^{\alpha})(1-\alpha)(-\alpha)L^{-(\alpha+1)} < 0$$

Population Growth

n_t Population growth rate $\quad \dot{L}_t = n L_t$

σ "Subsistence" Consumption Level

$$n_t = G(\underbrace{A_t F'(L_t)}_{\substack{MPL\\ \text{or}\\ \text{wage}}} - \sigma) \qquad G'(\cdot) > 0 \quad G(0) = 0$$

$$A_t F'(L_t) - \sigma = 0$$
$$\sigma = A_t F'(L_t)$$

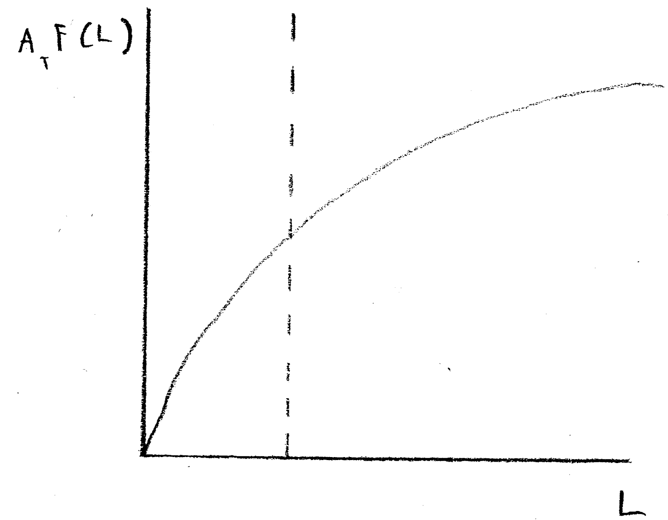
hence

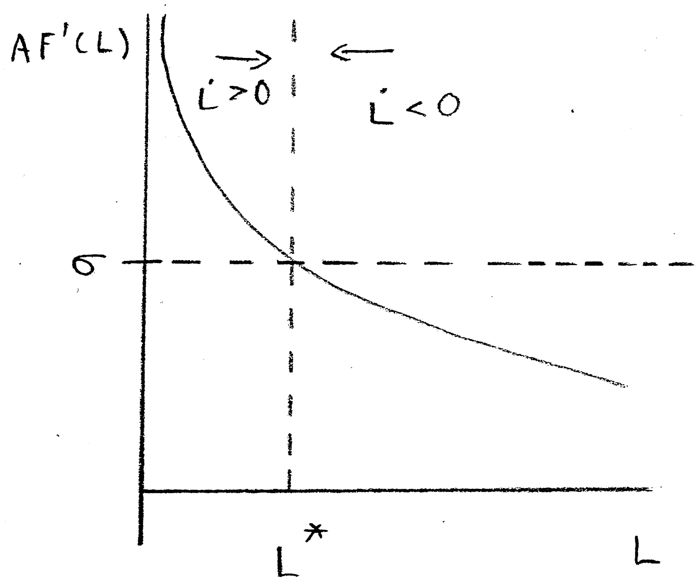
$$\dot{L}_t = L_t\,G(A_t F'(L_t) - \sigma)$$

$$\dot{L}_t \gtreqless 0 \quad \text{as} \quad A_t F'(L_t) \gtreqless \sigma$$

Malthus' Model

Graphical Representation of equilibrium





$$AF'(L^*) = \sigma$$

Example:

$$Y = A L^{1-\alpha}$$

Equilibrium

$$A(1-\alpha)L^{*-\alpha} = \sigma$$

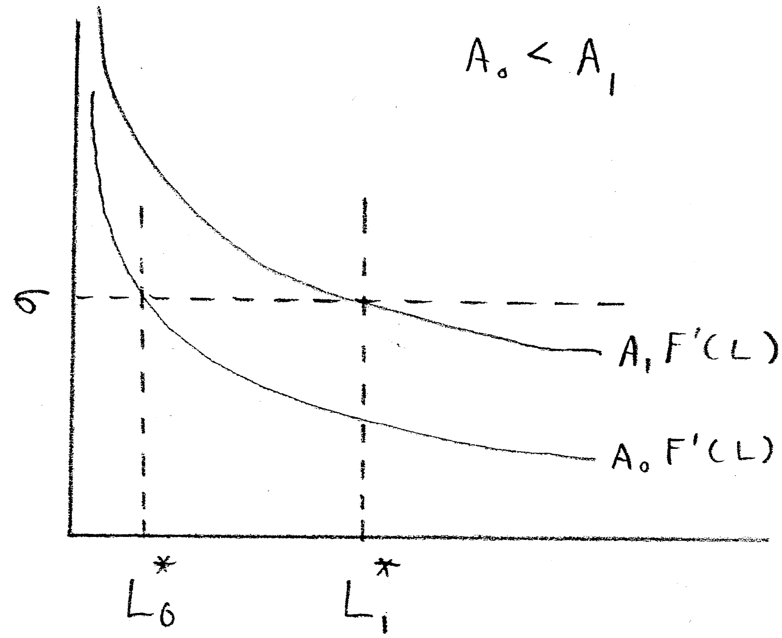
$$\Rightarrow L^* = (1-\alpha)^{1/\alpha} \sigma^{-1/\alpha} A^{1/\alpha}$$

or, in logs,

$$l^* = \text{Constant} + \frac{1}{\alpha} a$$

Malthus' Model

Effect of ΔA:



Implications:

— $A\uparrow$ does not improve welfare, except in short run ("Dismal Science")

— In the long run, population growth is proportional to technological improvement.

Taking above example, & assuming $L_t = L_t^*$,

$$\dot{l} = \tfrac{1}{\alpha}\dot{a}$$

Malthus' Model

Birth rates & death rates

Birth rate $\quad b = k_1 + \gamma W$

Death rate $\quad d = k_2 - \eta W$

$b = d$

What is σ? When $W = \sigma$, $u = 0$

$$\underbrace{k_1 + \gamma W}_{b} = \underbrace{k_2 - \eta W}_{d}$$

$$\sigma = \frac{k_2 - k_1}{\gamma + \eta}$$

Medical improvement: $k_2 \downarrow$, $\sigma \downarrow$