Solow Model

Differences across countries

$\left(\frac{Y}{L}\right)_{it}$ Output per worker in country i at time t

caused by

— $y_{it} \neq y_i^*$ A country can be away from LRE

— $y_i^* \neq y_j^*$ Countries can have different LRE's because of differences in s vs. n

— $A_i \neq A_j$ Countries have different "technology"

recall $Y = K^\alpha (AL)^{1-\alpha}$

$$\frac{Y}{L} = A^{1-\alpha}\left(\frac{K}{L}\right)^\alpha = A^{1-\alpha} y$$

recall $A_t = A_0 e^{gt}$

so A can vary because of

— differences in A_0 $\Leftarrow$ initial level

— differences in g $\Leftarrow$ growth rate

— $\delta_i \neq \delta_j$ Countries have different depreciation rates

Solow Model

Relations between Y^*/L and s, n in LRE

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha} \Rightarrow \frac{Y}{L} = K^{\alpha} A^{1-\alpha} L^{-\alpha} \Rightarrow \frac{Y}{AL} = K^{\alpha} A^{-\alpha} L^{-\alpha}$$

divide both sides by L

divide both sides by A

$$= \left(\frac{K}{AL}\right)^{\alpha}$$

or

$$f(k) = k^{\alpha}$$

In LRE,

$$s f(k^*) = (n+g+\delta) k^*$$

$$s\left(\frac{K^*}{AL}\right)^{\alpha} = (n+g+\delta)\left(\frac{K^*}{AL}\right)$$

Solve for K^*

$$\Rightarrow K^* = \left[\frac{s}{n+g+\delta}\right]^{\frac{1}{1-\alpha}} AL$$

Substitute this into expression for $\frac{Y}{L}$

$$\left(\frac{Y}{L}\right)^* = \left[\frac{s}{n+g+\delta}\right]^{\frac{\alpha}{1-\alpha}} (AL)^{\alpha} A^{1-\alpha} L^{-\alpha} = A\left[\frac{s}{n+g+\delta}\right]^{\frac{\alpha}{1-\alpha}}$$

Take logs:

$A_0 e^{gt}$

$$\ln\left(\frac{Y}{L}\right) = \underbrace{\ln A}_{\ln A_0 + gt} + \frac{\alpha}{1-\alpha}\ln s - \frac{\alpha}{1-\alpha}(n+g+\delta)$$

If $\alpha = \frac{1}{3}$, $\frac{\alpha}{1-\alpha} = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{1}{2}$

If $\alpha = \frac{1}{3}$, this would mean

$$\ln(Y/L) = \ln A_0 + gt + \frac{1}{2}\ln s - \frac{1}{2}(n+g+\delta)$$

MANKIW, ROMER & WEIL (1992): SOLOW + HUMAN CAPITAL

Background

Convergence

After Solow, models with s derived from household utility functions

imply LRE s & n same across countries

hence, assuming Solow-like aggregate prodn fns (diminishing MPK)

$\Rightarrow y^*, k^*$ same across countries

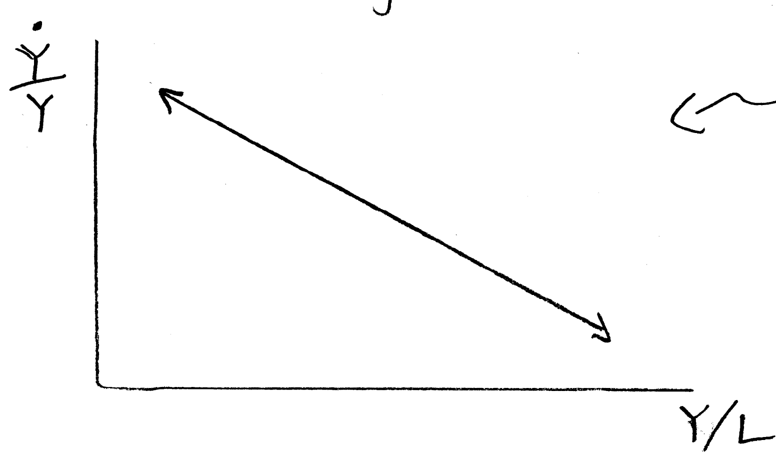
Implication:

- Observed variation in y reflects $(y_t - y^*)$ deviations from LRE

Poor countries are below LRE

- If $(y_t - y^*) < 0$, $\dot{y} > g$

Poor countries grow especially fast "Convergence"



Prediction for data

MRM (1992)

Background (cont.)

Convergence (cont.)

Facts about convergence

Hasn't happened.
Many poor countries (mostly in Africa, S. America) stay poor.

Response by economic theorists

- Models with no LRE levels of y, rather LRE $\dot{y}$ determined by s etc.

  To get this, non-diminishing MPK
  or non-diminishing MP(K + something)

  "Non-diminishing returns to reproducible factors"

- If MPK nondiminishing, why do firms choose finite capital stocks?

  "Externalities to capital accumulation"

$$MPK > MPK_{PRIVATE} = r$$

For country as a whole, in agg prodn fn

For individual firm buying capital

cost of K / income to K owner

SOLOW MODEL

ADDING HUMAN CAPITAL (MANKIW, ROMER & WEIL, 1992)

Production function: $Y = K^{\alpha} H^{\beta} (AL)^{1-\alpha-\beta}$ $\alpha + \beta < 1$

Notation: divide by AL to get

$$\frac{Y}{AL} = K^{\alpha} H^{\beta} AL^{-\alpha-\beta} = \left(\frac{K}{AL}\right)^{\alpha} \left(\frac{H}{AL}\right)^{\beta}$$

$$y = \quad = k^{\alpha} \quad h^{\beta}$$

Note: α is still equal to share of capital in national income

CRS, so diminishing returns to (K & H) holding L fixed.

(means LRSS like Solow model)

Savings rates

s_k for capital
s_h for human capital } same rate of depreciation

Evolution of k and h:

$$\dot{k}_t = s_k y_t - (n+g+\delta) k_t$$

$$\dot{h}_t = s_h y_t - (n+g+\delta) h_t$$

In LRSS,

$$\dot{k} = 0 = s_k k^{*\alpha} h^{*\beta} - (n+g+\delta) k^*$$

$$\dot{h} = 0 = s_h k^{*\alpha} h^{*\beta} - (n+g+\delta) h^*$$

rearrange to get

$$k^* = \left(\frac{s_k h^{*\beta}}{n+g+\delta}\right)^{\frac{1}{1-\alpha}} \qquad h^* = \left(\frac{s_h k^{*\alpha}}{n+g+\delta}\right)^{\frac{1}{1-\beta}}$$

} Two equations, two unknowns

Solow Model

Adding Human Capital (cont.)

Solving for k^* and h^* gives

$$k^* = \left(\frac{s_k^{1-\beta} s_h^{\beta}}{n+g+\delta}\right)^{\frac{1}{1-\alpha-\beta}} \qquad h^* = \left(\frac{s_k^{\alpha} s_h^{1-\alpha}}{n+g+\delta}\right)^{\frac{1}{1-\alpha-\beta}}$$

$$y^* = k^{*\alpha} h^{*\beta}$$

$$= (n+g+\delta)^{-\frac{\alpha+\beta}{1-\alpha-\beta}} \left(s_k^{1-\beta} s_h^{\beta}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(s_k^{\alpha} s_h^{1-\alpha}\right)^{\frac{\beta}{1-\alpha-\beta}}$$

$$= (n+g+\delta)^{\frac{-(\alpha+\beta)}{1-\alpha-\beta}} s_k^{\frac{\alpha}{1-\alpha-\beta}} s_h^{\frac{\beta}{1-\alpha-\beta}}$$

What about $\frac{Y}{L}^*$? Multiply both sides by $A = A_0 e^{gt}$

$$\frac{Y}{L}^* = A_0 e^{gt} (n+g+\delta)^{\frac{-(\alpha+\beta)}{1-\alpha-\beta}} s_k^{\frac{\alpha}{1-\alpha-\beta}} s_h^{\frac{\beta}{1-\alpha-\beta}}$$

Take logs:

$$y^* - l^* = a_0 + gt + \ldots\ldots$$

Solow Model

Adding Human Capital (cont.)

Mankiw-Romer-Weil model:

$$y^* - l^* = a_0 + gt + \frac{\alpha}{1-\alpha-\beta} \ln s_K - \frac{\alpha+\beta}{1-\alpha-\beta} \ln(n+g+\delta) + \frac{\beta}{1-\alpha-\beta} \ln s_h$$

Solow:

$$y^* - l^* = a_0 + gt + \frac{\alpha}{1-\alpha} \ln s_K - \frac{\alpha}{1-\alpha-\beta} \ln(n+g+\delta)$$

This can explain why estimated coefficients on s_K & $(n+g+\delta)$ were <u>bigger</u> than predicted by Solow model

- s_h might be positively correlated with s_K (across countries)
- Even if s_h uncorrelated with s_K, MRM predicts bigger coefficients.

Expressing Model in terms of Human Capital Level

From $h^* = (\quad)^{\frac{1}{1-\alpha-\beta}}$, get $s_h = h^{*\frac{1-\alpha-\beta}{1-\alpha}} (n+g+\delta)^{\frac{1}{1-\alpha}} s_K^{-\frac{\alpha}{1-\alpha}}$

substitute into $\frac{Y}{L}^* =$ etc. & take logs to get

$$y^* - l^* = a_0 + gt + \frac{\alpha}{1-\alpha} \ln s_K - \frac{\alpha}{1-\alpha} \ln(n+g+\delta) + \underbrace{\frac{\beta}{1-\alpha} \ln(h^*)}_{\text{omitted variable}}$$

recall $h^* = \left(\frac{s_K^{\alpha} s_h^{1-\alpha}}{n+g+\delta}\right)^{\frac{1}{1-\alpha-\beta}}$

$\left.\begin{array}{l} + \text{ correlated w/ } s_K \\ - \text{ correlated w/ } n \end{array}\right\}$ biasses coefficients on s_K & n

MRW (1992)

Data & estimation

$$\text{SCHOOL} = X\, s_H$$

$$\Rightarrow \frac{Y^*}{L} = A_0 e^{gt} (n+g+\delta)^{-\frac{(\alpha+\beta)}{1-\alpha-\beta}} \; s_K^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{1}{X}\text{SCHOOL}\right)^{\frac{\beta}{1-\alpha-\beta}}$$

take logs

$$(y-l)^* = \text{-------------------} + \left(\frac{\beta}{1-\alpha-\beta}\right)\ln\left(\frac{1}{X}\right) + \left(\frac{\beta}{1-\alpha-\beta}\right)\ln \text{SCHOOL}$$

$$= a_0 + gt - \frac{\beta}{1-\alpha-\beta}\ln X + \underbrace{\frac{\alpha}{1-\alpha-\beta}}\ln s_K - \underbrace{\frac{\alpha+\beta}{1-\alpha-\beta}}\ln(n+g+\delta) + \underbrace{\frac{\beta}{1-\alpha-\beta}}\ln SCH$$

$$y-l = \underbrace{\qquad\qquad}_{\text{constant}} + B_1 \ln s_K - B_2 \ln(n+g+\delta) + B_3 \ln SCH$$

Regression where residual includes $\underbrace{(y-l)-(y-l)^*}_{\text{deviation from LRE}}$

"Implied values of α and β":

if $B_2 = \dfrac{\alpha+\beta}{1-(\alpha+\beta)}$

$$B_2[1-(\alpha+\beta)] = \alpha+\beta$$

$$B_2 - B_2(\alpha+\beta) = \alpha+\beta$$

$$\frac{B_2}{\alpha+\beta} - B_2 = 1$$

$$\frac{B_2}{\alpha+\beta} = 1 + B_2$$

$$\alpha+\beta = \frac{B_2}{1+B_2}$$

$$B_1 = \frac{\alpha}{1-(\alpha+\beta)}$$

$$B_1 = \frac{\alpha}{1-\frac{B_2}{1+B_2}}$$

$$\alpha = B_1\left(1-\frac{B_2}{1+B_2}\right)$$

MRW (1992)

Data & estimation (cont.)

"Restricted regression"

$$(y-l)^* = \ldots\ldots + \underbrace{\frac{\alpha}{1-\alpha-\beta}}_{\beta_3}\left(\ln s_k - \ln(n+g+\delta)\right) + \underbrace{\frac{\beta}{1-\alpha-\beta}}_{\beta_4}\left(\ln s_{CH} - \ln(n+g+\delta)\right)$$

Implied values of α and β:

$$\beta_3 + \beta_4 = \frac{\alpha+\beta}{1-(\alpha+\beta)} \qquad \beta_3 = \frac{\alpha}{1-(\alpha+\beta)}$$

$$\Rightarrow \widehat{(\alpha+\beta)} = \frac{\hat{\beta}_3 + \hat{\beta}_4}{1+\hat{\beta}_3+\hat{\beta}_4} \qquad \alpha = \beta_3\left[1-(\alpha+\beta)\right]$$

$$\hat{\alpha} = \hat{\beta}_3\left[1-\widehat{(\alpha+\beta)}\right]$$

MRM find $\hat{\alpha} \approx 0.30$

recall NIPAs + ($r = MPK$) $\rightarrow \alpha \approx 1/3$

SOLOW MODEL

ADDING HUMAN CAPITAL (MRM, 1992)

Implication for capital migration

Why do s_K, s_h vary so much across countries?
government policy

Why doesn't capital migrate to countries where
s_K small relative to n?

In LRSS, $MPK = \alpha k^{*\alpha-1} h^{*\beta}$

$$= \alpha \frac{n+g+\delta}{s_K}$$

so return to capital should be higher there.

→ "Expropriation risk"
government policy

this "puzzle" same as for ordinary Solow model

Conclusions (if you believe paper)

- No need for "endogenous growth" models.
Diminishing returns to reproducible factor is realistic

- Differences in $\frac{Y}{L}$ across countries reflect differences in capital, physical <u>and</u> human

MKW (1992)

Criticisms of argument

Perhaps
MKW's "human capital" variable is strongly correlated with other things that

- aren't really human capital
- affect output realized from given inputs of human capital & physical capital)

Why this might be true

- estimated coefficient on SCHOOL is "too large" to be effect of human capital alone
  (like MKW's argument about coeffs on s_K in model omitting h)
- variations across countries in implied measure of "human capital" is too great to be human capital.
  (differences in years of school & school quality aren't that great)

Possible other things: "institutions"
Bad institutions divert inputs (labor, capital, human capital) into activities with high private returns
no contribution to output