Analytical modelling of the electromechanical behaviour of surface-bonded piezoelectric actuators including the adhesive layer

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The behaviour of a piezoelectric actuator is strongly affected by the bonding condition along the interface between the actuator and the host structure. The current paper represents an analytical study of the static effect of the mechanical and geometrical properties of the adhesive layer on the coupled electromechanical behaviour of a thin piezoceramic actuator bonded to an elastic medium. An actuator model with an imperfect adhesive bonding layer, which undergoes a shear deformation, is proposed to simulate the two dimensional electromechanical behaviour of the integrated system. Analytical solution of the problem is provided by solving the resulting integral equations in terms of the interfacial stress. Numerical simulation is conducted to study the effect of the bonding layer upon the actuation process. The effect of interfacial debonding on the response of the layered structure and on the interlaminar strain and stress transfer mechanisms is discussed.

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\section{1. Introduction}

Piezoelectric materials, due to their strong electromechanical coupling characteristics, have been successfully used to control and monitor the static deformation and vibration of various structures \cite{1–3}. To enable the implementation of strain actuators in various structural members without significantly complicating the manufacturing process or changing the mechanical properties of the target structure, piezoelectric sheets, patches and tiles are usually adhesively bonded to the outer faces of the host structure in many practical cases, introducing an additional adhesive layer into the structural assembly \cite{4–7}. Recent studies have shown that the adhesive layer could have significant effect on piezoelectric-based health monitoring \cite{8–13}. Qing et al. \cite{8} presented experimental results on tests performed with piezoelectric sensors mounted on thin aluminum panel by means of adhesive layers of various thicknesses and composition. Their experimental data have shown that the adhesive composition and thickness could significantly affect the amplitude of the acoustic wave propagated into the structure and the measured electromechanical impedance signature of the sensor structure. Later, Dugnani \cite{9} has shown that the shearing of the adhesive layer has considerable effect on the electromechanical impedance signature of a disk-shape piezoelectric sensor. The sensors and the bonder layer undergo degradation due to aging, corrosion, temperature cycling, and vibrations. In recent work of Lin et al. \cite{12} reliability tests on piezoelectric wafer active sensor have been performed to explore the durability and survivability issues associated with various environmental conditions on piezoelectric wafer active sensors for structural health monitoring. Progress in this area has been summarized in a review article by Huang et al. \cite{13}.

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The behaviour of electromechanical structures with bonded piezoelectric materials has been extensively studied. An analytical model that focuses on the strain and stress transfer mechanisms in piezoelectrically activated panels has been presented by Crawley and de Luis [14]. In this analysis, the axial stress in the actuator was assumed to be uniform across its thickness and the host structure was treated as a Bernoulli–Euler beam. The result indicated that, for a perfectly bonded actuator, the shear stress between the actuator and the host beam was transferred mainly at the ends of the actuator. Crawley and Anderson [15] later developed a Bernoulli–Euler model of a piezoelectric actuator by considering the linear stress distribution along its thickness. Im and Atluri [16] further modified the actuator model presented by Crawley and de Luis [14] by considering both the axial and the transverse shear forces in the beam. A refined actuator model based on the plane stress condition was presented for a beam structure with symmetrically surface-bonded actuator patches [17,18]. Wang and Rogers [19] modified the classical laminated plate theory to model actuator-induced bending and extension of laminated plates under static loading. Tauchert [20] further investigated the control of thermal deformation of laminated plates using piezoelectric actuators. Typical examples also include the work by Dimitriadis et al. [21], Tzou and Tseng [22], Mitchell and Reddy [23], Han and Lee [24] and Reddy [25].

Significant attention has also been paid to the modelling of debonding of actuators. It has been shown by Seeley and Chattopadhyay [26] that the control of smart structures can be significantly mispredicted in the presence of debonding. Kim and Jones [27] conducted an analytical and experimental investigation to identify the effects of delamination on smart beams with piezoelectric actuators. They indicated that edge delaminations cause a decrease of the natural frequency but the interior delaminations do not. Wang and Meguid [28] utilized a one-dimensional actuator model with interfacial debonding to analyze debonding effects on embedded and surface-bonded piezoelectric actuators. Tylikowski [29] presented a bending-extensional model of a simply supported laminated beam with debonded piezoelectric actuator elements to study frequency shifts due to actuator debonding. Tong et al. [30] developed analytical models for smart beams with debonded piezoelectric actuators and sensors including adhesive layers, and analyzed effects of debonding on the distributions of strain, stress and displacement. Sun et al. [31] investigated actuator debonding effect on closed-loop vibration control based on the classical beam theory and found that actuator debonding can significantly reduce the control efficiency. A closed-loop-control-based damage detection scheme is presented by Sun and Tong [32] aiming at detecting damage in structures. The results show that even a small edge debonding in a piezoelectric actuator patch can make the sensitive control system unstable, and therefore can be detected.

Existing studies on the electromechanical behaviour of piezoelectric actuators with imperfect bonding have been mainly confined to the global response of piezoelectric structures. It should be mentioned, however, that as one of the fundamental issues surrounding the effectiveness of a piezoelectric actuator in a smart structure system, the accurate assessment of the local stress distribution around the actuator plays a dominant role in determining the actuation effect being transferred from the actuator to the host structure. The lack of the study on local stress distribution around debonded actuators may be because of the difficulties associated with the complicated electromechanical coupling and geometric configurations of this problem. The objective of the present paper is, therefore, to develop an analytical model to study the coupled electromechanical behaviour of a thin-sheet piezoceramic actuator imperfectly bonded to an elastic half plane under in-plane mechanical and electrical loadings. The current work is an extension of the work presented in Wang and Meguid [28] in which the actuator is characterized by an electroelastic line model with the poling direction being perpendicular to its length. An imperfect adhesive bonding layer between the actuator and the host structure is introduced to study the influence of the mechanical and geometrical properties of the adhesive layer on the coupled electromechanical behaviour of the integrated structure. The emphasis of the current study is on the local stress and strain fields near imperfectly bonded actuators and the load transfer. Numerical simulation is conducted to study the influence of the geometry and the material mismatch of the adhesive layer upon the actuation process. The interfacial debonding and its effect upon the stress distribution and the overall performance of the structure are evaluated in detail.

2. Formulation of the problem

Let us now consider the plane strain problem of a thin piezoceramic actuator sheet bonded to a homogeneous and isotropic elastic half plane through a thin bonding layer, as illustrated in Fig. 1. The host medium is modelled as a half plane to represent the case where it is much thicker than the actuator. The lengths of both the actuator and the bonding layer are denoted as 2a, and the thicknesses of the actuator and the bonding layer are denoted as h and h′, respectively. It is assumed that the poling direction of the actuator is along the z-axis. An electrical field $E_z$ is applied along the poling direction of the actuator by applying a voltage (V) between the upper and the lower electrodes of the actuator with $E_z = V/h = (V - V')/h$. Throughout this paper, superscripts $a$, $b$ and $h$ are used to designate physical quantities that belong to the actuator, the bonding layer and the host, respectively.

2.1. The actuator

Since the present study is focusing on a thin-sheet actuator with relatively small thickness in comparison with its length, $\sigma_z^a$ and $\varepsilon_z^a$ are assumed to be uniform across its thickness, and $\sigma_{yz}^a$ in the actuator can be ignored. Considering the fact that the bonding layer is much thinner than the actuator, it is assumed that the axial stress and deformation are uniform.
across the thickness of the bonding layer as well. The interfacial shear stress between different layers is denoted as $\tau$, as shown in Fig. 1 with $u^+$ and $u^-$ representing the displacements on the upper and lower surface of the bonding layer, respectively.

Based upon these assumptions, the actuator can be modelled as an electroelastic line subjected to the applied electric field and distributed axial force, $\tau/h$, as shown in Fig. 1. The equilibrium equation of the actuator can then be expressed as

$$\frac{d\sigma^a_y(y)}{dy} + \frac{\tau(y)}{h} = 0 \quad (1)$$

Since all the load transferred between the actuator and the bonding layer can be attributed to $\tau$, the two ends of the actuator can be assumed to be traction free, i.e.

$$\sigma^a_y = 0, \quad |y| = a \quad (2)$$

By integrating Eq. (1) and making use of Eq. (2), the axial stress in the actuator can be expressed in terms of the shear stress $\tau$ as

$$\sigma^a_y(y) = -\int_{a}^{y} \frac{\tau(\xi)}{h} \, d\xi \quad (3)$$

with

$$\int_{-a}^{a} \tau(\xi) \, d\xi = 0 \quad (4)$$

The axial stress of the actuator can be expressed in terms of the axial strain $\varepsilon^a_y$ and the electric field $E_z$ by using the following general constitutive relation:

$$\sigma^a_y(y) = E^a \varepsilon^a_y(y) - e^a E_z \quad (5)$$

where $E^a$ and $e^a$ are effective material constants [28].

The resulting axial strain $\varepsilon^a_y$ and displacement $u^a_y$ can then be expressed in terms of $\tau$ as

Fig. 1. Schematics of the actuator configuration.
\[ e_y^x(y) = -\frac{1}{E_h} \int_a^y \tau(\xi) d\xi + \frac{\varepsilon E_a}{E_h}, \quad |y| < a \]  
\[ u_y^x(y) = -\frac{1}{E_h} \int_a^y (y - \xi) \tau(\xi) d\xi + \frac{\varepsilon E_a y}{E_h}, \quad |y| < a \]  

2.2. The bonding layer

The adhesive layer, formed by epoxy or conductive epoxy for example, is generally much more compliant than the host structure. Since the deformation of the actuator is transferred totally through the adhesive layer, the mechanical and geometrical properties of the adhesive layers will have a significant effect upon the performance of the integrated structure.

The shear stress \( \tau \) distributed in the layer is determined by the constitutive relation:

\[ \tau(y) = \mu^b e_y^b(y) \]  

where

\[ e_y^b(y) = \frac{u^+(y) - u^-(y)}{h^b} \]  

with \( \mu^b \) and \( e_y^b \) being the shear modulus and the shear strain of the bonding layer, respectively. According to the continuity condition of the displacements, \( u^+ \) and \( u^- \) also represent the longitudinal displacements of the lower surface of the actuator and the upper surface of the host medium, respectively.

2.3. The host medium

Since there is no additional mechanical load applied to the host medium, the stress field generated inside the host structure is only due to the existence of the shear stress caused by the actuator at the top of its surface. Therefore, the boundary condition along the top surface will be

\[ \sigma_{xy}(y, 0) = \begin{cases} -\tau(y) & |y| < a \\ 0 & |y| > a \end{cases} \quad \sigma_{zy}(y, 0) = 0 \]  

Making use of the fundamental solution of a half elastic plane subjected to a concentrated horizontal force [33] and the superposition principle, \( e_y^h \) and \( u_y^h \) resulting from the applied force given by Eq. (10) can be obtained as

\[ e_y^h(y, 0) = \frac{1 - \nu^h}{\pi \mu^h} \int_a^y \frac{\tau(\xi)}{y - \xi} d\xi, \quad |y| < a \]  
\[ u_y^h(y, 0) = \frac{1 - \nu^h}{\pi \mu^h} \int_a^y \int_a^y \frac{\tau(\xi)}{y - \xi} d\xi d\eta, \quad |y| < a \]

where \( \nu^h \) and \( \mu^h \) are the Poisson’s ratio and the shear modulus of the host, respectively.

2.4. Static load transfer in a perfectly bonded piezoelectric actuator

Taking the derivative of both sides of Eq. (8) with respect to \( y \) gives

\[ -\frac{d\tau(y)}{dy} = \mu^b \left[ \frac{e^+(y) - e^-(y)}{h^b} \right] \]  

Substituting Eqs. (6) and (11) into Eq. (13), the following integral equation can be obtained

\[ \frac{1 - \nu^h}{\pi \mu^h} \int_a^y \frac{\tau(\xi)}{y - \xi} d\xi + \frac{1}{E_a} \int_a^y \frac{\tau(\xi)}{y - \xi} d\xi - \frac{h^b d\tau(y)}{\mu^b} = \frac{\varepsilon E_a}{E_h}, \quad |y| < a \]  

from which the shear stress \( \tau \) can be determined.

Eqs. (14) and (4) can be normalized as

\[ \frac{1}{a} \int_{-\eta}^{\eta} \frac{\tau(\zeta)}{\zeta} d\zeta + \frac{1}{a} \int_{-\eta}^{\eta} \frac{\tau(\zeta)}{\zeta} d\zeta - \frac{d\tau(\eta)}{d\eta} = 1, \quad |\eta| < 1 \]

\[ \int_{-\eta}^{\eta} \tau(\zeta) d\zeta = 0 \]

where

\[ \bar{\tau}(\eta) = \frac{\tau(a\eta)}{\tau(a)}, \quad \eta = \frac{y}{a} \]  

and
In the limiting case that the thickness of the bonding layer tends to zero, the third term in Eq. (15) disappears, and Eq. (15) reduces to

\[
\int_{-1}^{1} \tau(\zeta) d\zeta = 0,
\]

which is a singular integral equation of the first kind [28]. The solution of it involves a square-root singularity at \(|\eta| = 1\) [33], which corresponds to the two ends of the actuator. For the case where a bonding layer exists, the solution of Eq. (15) has no singularity at \(|\eta| = 1\), indicating that introducing the bonding layer may lead to lower stress concentration and more uniform distribution of stress.

The general solutions of \(\tau\) in Eq. (15) can be expressed in terms of the following expansions of Chebyshev polynomials

\[
\tau(\eta) = \frac{1}{\sqrt{1 - \eta^2}} \sum_{j=0}^{\infty} d_j T_j(\eta)
\]

where \(T_j\) are Chebyshev polynomials of the first kind with \(T_j(\eta) = \cos(j \theta)\) and \(\cos \theta = \eta\). By truncating the Chebyshev polynomial expansions to the \(N\)th term and considering the boundary conditions at the following collocation points along the actuator:

\[
\eta_k = \cos \frac{k}{N + 1} \pi, \quad k = 1, 2, \ldots, N
\]

Eq. (15) reduces to

\[
\sum_{j=1}^{N} d_j \frac{\sin \left(\frac{jk \pi}{N + 1}\right)}{\sin \left(\frac{k \pi}{N + 1}\right)} \left\{ \frac{1}{q} + \frac{1}{\nu q} \sin \left(\frac{k \pi}{N + 1}\right) \right\} + \frac{\nu'}{\nu q'} \left[ j + \cot \left(\frac{j \pi}{N + 1}\right) \cot \left(\frac{k \pi}{N + 1}\right) \right] = -\frac{1}{\pi}, \quad k = 1, 2, \ldots, N
\]

The unknown coefficients \(d_j\) and the stress field due to the presence of the actuator can be readily determined by using Eq. (21).

### 2.5. Effect of interfacial debonding

The high level of shear stress at the edges of the actuator may result in an unwanted edge debonding as shown in Fig. 2. For a perfectly bonded actuator, when the maximum shear stress is larger than the bonding strength, edge debonding may develop. The debonded part of the actuator has zero boundary stresses. According to the present actuator model, the debonded part at the edge of the actuator will experience no stress. As a result, the effective length of the actuator is reduced from its original length to that of the bonded part only. Therefore, the behaviour of an edge debonded actuator can be simulated by a shorter actuator [28].

Debonding may also occur in the interior of the actuator. Let us consider an actuator occupying the region \(t_i < y < t_f\) which is partially debonded in \(d_i < y < d_f\), as illustrated in Fig. 3a. The debonded part of the actuator can be regarded as a one-dimensional element subjected to an axial stress \(\sigma_a\). By making use of the equilibrium equation (Eq. (1)) and the traction free condition at the two ends of the actuator, the axial stress in the actuator can be expressed in terms of \(\tau\) as
The partially debonded actuator can then be regarded as two "actuators" subjected to an axial stress \( \sigma_d \) at the inner tips of them, as shown in Fig. 3b. The resulting axial strain can then be expressed in terms of \( s \) as

\[
\sigma_d(y) = \begin{cases} 
- \int_{y_1}^{y_t} \frac{\tau(\xi)}{h} \, d\xi & t_1 < y < d_i \\
\sigma_d & d_i < y < d_r \\
\sigma_d - \int_{y_d}^{y_{dr}} \frac{\tau(\xi)}{h} \, d\xi & d_r < y < t_r 
\end{cases}
\]  

(22)

where

\[
\sigma_d = - \int_{t_1}^{d_1} \frac{\tau(\xi)}{h} \, d\xi
\]

(23)

The partially debonded actuator can then be regarded as two "actuators" subjected to an axial stress \( \sigma_d \) at the inner tips of them, as shown in Fig. 3b. The resulting axial strain can then be expressed in terms of \( \tau \) as

\[
\varepsilon^a_y(y) = \begin{cases} 
- \frac{1}{\pi R} \int_{t_1}^{y_t} \tau(\xi) \, d\xi + \frac{E_z}{2} & t_1 < y < d_i \\
\frac{\varepsilon_x + \varepsilon_y}{E} & d_i < y < d_r \\
\frac{1}{\pi R} \left[ \sigma_d h - \int_{y_d}^{y_{dr}} \tau(\xi) \, d\xi \right] + \frac{E_z}{2} \, d_r & d_r < y < t_r 
\end{cases}
\]

(24)

The half plane to which the actuator is bonded is subjected to the following boundary conditions:

\[
\sigma_{yz}(y, 0) = \begin{cases} 
- \tau(y) & t_1 < y < d_i \text{ and } d_i < y < t_r \\
0 & \text{otherwise} 
\end{cases}
\]

(25)

The surface strain in the host medium can be obtained, by making use of the fundamental solution of a half plane subjected to a concentrated surface force, as

\[
\varepsilon_s^y(y, 0) = \frac{1 - \nu^h}{\pi h^2} \left[ \int_{t_1}^{y_t} \frac{\tau(\xi)}{y - \xi} \, d\xi + \int_{d_1}^{y_{dr}} \frac{\tau(\xi)}{y - \xi} \, d\xi \right]
\]

(26)

By making use of the constitutive relation of the bonding layer given by Eqs. (8) and (9), the strains in the actuator and the host medium presented by Eqs. (24) and (26) can be related as

\[
\varepsilon^a_y - \varepsilon^h_y = - \frac{h'}{\mu^s} \frac{d\tau(y)}{dy} \quad t_1 < y < d_i, \quad d_i < y < t_r, \quad z = 0
\]

(27)
Substituting Eqs. (24) and (26) into Eq. (27) gives
\[
1 - \frac{\nu h}{\pi \mu h} \int_0^{d_l} \frac{\tau(\xi)}{y - \xi} \, d\xi + 1 - \frac{\nu h}{\pi \mu h} \int_{d_l}^{d_i} \frac{\tau(\xi)}{y - \xi} \, d\xi - \frac{h'}{\mu h} \int_0^{d_i} \tau(\xi) \, d\xi = \frac{\sigma_0 d}{E} \quad t_i < y < d_i
\]
and
\[
1 - \frac{\nu h}{\pi \mu h} \int_0^{d_l} \frac{\tau(\xi)}{y - \xi} \, d\xi + 1 - \frac{\nu h}{\pi \mu h} \int_{d_l}^{d_i} \frac{\tau(\xi)}{y - \xi} \, d\xi \quad d_r < y < t_r
\]
The axial stress \(\sigma_a\) in Eq. (28) need to be determined by considering the deformation of the debonded part of the actuator. From Eqs. (8) and (9) it can be obtained that
\[
u h (d_r - d_l) = \frac{\sigma_0 d'}{E} (d_r - d_l)
\]
Integrating Eqs. (24) and (26) gives
\[
u h (d_r - d_l) = \frac{\sigma_0 d'}{E} (d_r - d_l)
\]
Substituting Eqs. (30) and (31) into Eq. (29), the following equation can be obtained, which provides an additional condition for determining \(\sigma_a\).
\[
1 - \frac{\nu h}{\pi \mu h} \int_0^{d_l} \left[ \int_{d_l}^{d_i} \frac{\tau(\xi)}{y - \xi} \, d\xi + \int_{d_i}^{d_i} \frac{\tau(\xi)}{y - \xi} \, d\xi \right] \, dy + \frac{h'}{\mu h} \int_0^{d_i} \tau(\xi) \, d\xi = \frac{\sigma_0 d}{E} (d_r - d_l)
\]
Eqs. (23), (28) and (32) can be normalized as
\[
\frac{1}{q} \int_{-1}^{1} \tau'(\eta) \, d\eta = 1, \quad \frac{1}{q} \int_{-1}^{1} \tau'(\eta) \, d\eta = 1
\]
and
\[
\nu h \int_{-1}^{1} \tau'(\eta) \, d\eta = \sigma_0 \nu l
\]
where the normalized stresses are given by
\[
\tau'(\eta_l) = \frac{\tau(a_l \eta_l + y_l)}{\varepsilon_0 E_z}, \quad \tau'(\eta_i) = \frac{\tau(a_i \eta_l + y_i)}{\varepsilon_0 E_z}, \quad \sigma_0 = \frac{\sigma_0 d}{E_0 E_z}
\]
with
\[
\eta_l = \frac{a_l}{a_0}, \quad \eta_i = \frac{a_i}{a_0}, \quad \eta_i = \frac{a_i}{a_0}, \quad \nu = \frac{b}{a_0}, \quad \nu = \frac{b}{a_0}, \quad \nu = \frac{b}{a_0}, \quad a = \frac{1}{2}(t_r - t_i), \quad a_i = \frac{1}{2}(d_i - d_l), \quad a_r = \frac{1}{2}(t_r - d_l), \quad y_i = \frac{1}{2}(d_i + t_i), \quad y_r = \frac{1}{2}(t_r + d_l), \quad \eta_i = 2 \nu \frac{y_i - y_r}{r_p}, \quad \eta_i = -2 \nu \frac{y_i - y_r}{r_p} + 1
\]
The general solutions of \(\tau'\) and \(\tau''\) in Eqs. (33)–(36) can then be expressed in terms of the following expansions of Chebyshev polynomials
\[
\tau'(\eta_l) = \frac{1}{\sqrt{1 - \eta_l^2}} \sum_{j=0}^{\infty} d_j T_j(\eta_l), \quad \tau'(\eta_i) = \frac{1}{\sqrt{1 - \eta_i^2}} \sum_{j=0}^{\infty} d_j T_j(\eta_i)
\]
If the Chebyshev polynomial expansions are truncated to the Nth term, and Eq. (33) is satisfied at the following collocation points at each bonded segment of the actuator given by
\[ \eta_{ik} = \eta_{rk} = \eta_{k} = \cos \frac{k}{N+1} \pi, \quad k = 1, 2, \ldots, N \tag{40} \]

Eqs. (33) and (34) reduce to

\[
\sum_{j=1}^{N} d_j^* \left( \frac{\sin \left( j \frac{\pi}{N} \right)}{\sin \left( \frac{k \pi}{N} \right)} \right) \left( \frac{1}{q} + \frac{1}{\sin \left( \frac{k \pi}{N} \right)} \sin \left( \frac{k \pi}{N+1} \right) \right) \left( \frac{\nu_j}{\pi q} + \frac{\nu_j}{\sin \left( \frac{k \pi}{N+1} \right)} \right) \left( \frac{\nu_j}{\sin \left( \frac{k \pi}{N+1} \right)} \right) \\
+ \frac{\sigma^*}{\pi q} \left[ \left( \frac{1}{q} + \frac{1}{\sin \left( \frac{k \pi}{N+1} \right)} \right) \sin \left( \frac{k \pi}{N+1} \right) \right] + \frac{\nu_j}{\sin \left( \frac{k \pi}{N+1} \right)} \sin \left( \frac{k \pi}{N+1} \right) - \frac{1}{\pi}, \quad k = 1, 2, \ldots, N \tag{41} \]

\[
- \sigma^* \left[ \frac{1}{q} \ln \left( \frac{\nu_j}{\sin \left( \frac{k \pi}{N+1} \right)} \right) - \frac{1}{q} \ln \left( \frac{\nu_j}{\sin \left( \frac{k \pi}{N+1} \right)} \right) - \frac{1}{q} \ln \left( \frac{\nu_j}{\sin \left( \frac{k \pi}{N+1} \right)} \right) + 2 \left( \frac{1}{q} - \frac{1}{\sin \left( \frac{k \pi}{N+1} \right)} \right) \right] + \frac{\nu_j}{\sin \left( \frac{k \pi}{N+1} \right)} \sin \left( \frac{k \pi}{N+1} \right) \\
+ \sum_{j=1}^{N} d_j^* \left( \frac{\pi}{j} \right) \left( \frac{\sin \left( j \frac{\pi}{N} \right)}{\sin \left( \frac{k \pi}{N} \right)} \right) \left( \frac{\nu_j}{\sin \left( \frac{k \pi}{N+1} \right)} \right) \left( \frac{\nu_j}{\sin \left( \frac{k \pi}{N+1} \right)} \right) \\
- \pi \sum_{j=1}^{N} d_j^* \left( \frac{\sin \left( j \frac{\pi}{N} \right)}{\sin \left( \frac{k \pi}{N} \right)} \right) \left( \frac{\nu_j}{\sin \left( \frac{k \pi}{N+1} \right)} \right) \left( \frac{\nu_j}{\sin \left( \frac{k \pi}{N+1} \right)} \right) \left( \frac{\nu_j}{\sin \left( \frac{k \pi}{N+1} \right)} \right) = \frac{2}{q} \left( \frac{1}{q} - \frac{1}{\sin \left( \frac{k \pi}{N+1} \right)} \right) \sin \left( \frac{k \pi}{N+1} \right) \sin \left( \frac{k \pi}{N+1} \right) \tag{43} \]

where

\[ \eta_{ik} = \nu_j \left( \frac{2}{q} - \frac{1}{q} \right) \left( \frac{1}{\sin \left( \frac{k \pi}{N+1} \right)} \right) \cos \left( \frac{k}{N+1} \pi \right) \tag{44} \]

\[ \eta_{rk} = -\nu_j \left( \frac{2}{q} - \frac{1}{q} \right) \left( \frac{1}{\sin \left( \frac{k \pi}{N+1} \right)} \right) \cos \left( \frac{k}{N+1} \pi \right) \tag{45} \]

The unknown coefficients \( d_j^* \), \( d_j^* \), and \( \sigma^* \) can be determined by solving Eqs. (41)–(43).

3. Numerical results and discussion

In this section, the results of numerical simulation of the influence of the property of the bonding layer under different material combinations and actuator geometries on the electromechanical behaviour of the integrated system are presented. The attention is focused on the shear stress distribution along the bonding layer. The convergence of the solution using Chebyshev polynomials has been carefully evaluated. The number of terms of Chebyshev polynomials is selected to be 64, with which the convergence of the results for all the cases considered is ensured.

The normalized interfacial shear stress \( \tau' = \tau' E_z \) is an important indicator of the actuation efficiency, and represents the load transfer between the actuator and the host medium. Fig. 4 shows typical shear stress distributions of a surface bonded actuator for \( q = 2.28, \nu = 0.2 \) and \( q = 0.011 \) under two different bonding conditions: \( \nu' = 0 \) and \( \nu' = 0.01 \). To verify the validity of the present actuator model to predict the interfacial stress distribution, the ANSYS software was used to numerically analyze the stress field of the same problem using the real geometric configuration of the actuator. The comparison shows a limited discrepancy between the finite element and analytical results near the tips of the actuator. This discrepancy is caused by the use of different actuator models in the analyses. In the analytical work, one-dimensional representation was used to model the behaviour of the actuator, while in the finite element analysis a two-dimensional model was used. The current one-dimensional actuator model can be used to predict interfacial shear stress away from the tips of the actuator (two times
of the thickness of the actuator, for example). Note that most sheet-actuators have high length-to-thickness ratio, and in these cases the current explicit model can predict the load transfer between the actuator and the host structure analytically in an efficient manner.

### 3.1. Actuator with a uniform interfacial layer

As shown in Eq. (17), $q$ represents the material combination of the actuator and the host structure, $v$ represents actuator geometry, $q'$ represents material property of the bonding layer and $v'$ represents the geometry of the bonding layer. Carefully examining Eq. (15) indicates that the normalized interfacial shear stress is governed by three parameters: $q$, $v$ and $v'/q'$. In the following discussion, the values of $q$, $v$ and $v'$ are varied to evaluate different responses of the structure.

The piezoelectric material considered in the following examples is a typical piezoceramics, whose properties are given in Table 1 [34]. The properties of the bonding layer [6] and the host medium [35] are given in Table 2. From these material constants it can be determined that $q = 1.0$ and $q' = 0.011$. The geometry of the actuator is assumed to be $a = 1.0 \text{ cm}$ and $h = 200–2000 \mu \text{m}$. The length of the bonding layer is the same as that of the actuator.

Since the distribution of $\tau'$ is anti-symmetrical about the middle point of $y = 0$, the stress distribution along only half of the actuator will be presented. The results for shear stress distribution along the bonding interface for $q = 1.0$ and $q' = 0.011$ with two bonding conditions, $v' = 0$ and $v' = 0.01$, are shown in Figs. 5 and 6, respectively. As shown in Fig. 5, the increase in the $v$ value makes shear stress distribution less concentrated around the tips of the actuator. Increase of shear stress level along the actuator due to increasing $v$ value is also observed. When a bonding layer is included, as shown in Fig. 6, the stress

### Table 1
Material Properties of the Piezoelectric Actuator.

<table>
<thead>
<tr>
<th>Elastic stiffness parameters</th>
<th>$c_{11}$</th>
<th>$c_{12}$</th>
<th>$c_{13}$</th>
<th>$c_{33}$</th>
<th>$c_{44}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Pa)</td>
<td>$13.9 \times 10^{10}$</td>
<td>$6.78 \times 10^{10}$</td>
<td>$7.43 \times 10^{10}$</td>
<td>$11.5 \times 10^{10}$</td>
<td>$2.56 \times 10^{10}$</td>
</tr>
<tr>
<td>Piezoelectric constants</td>
<td>$\varepsilon_{31}$</td>
<td>$\varepsilon_{33}$</td>
<td>$\varepsilon_{15}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(C/m²)</td>
<td>$-5.2$</td>
<td>$15.1$</td>
<td>$12.7$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dielectric constants</td>
<td>$\varepsilon_{11}$</td>
<td>$\varepsilon_{33}$</td>
<td>$6.45 \times 10^{-9}$</td>
<td>$5.62 \times 10^{-9}$</td>
<td></td>
</tr>
<tr>
<td>(C/Vm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 2
Material properties of the host medium and the bonding layer.

<table>
<thead>
<tr>
<th>Host medium</th>
<th>Poisson’s ratio $\nu^h$</th>
<th>$5.27 \times 10^{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bonding layer</td>
<td>Shear modulus $\mu^b$ (Pa)</td>
<td>$1.0 \times 10^9$</td>
</tr>
</tbody>
</table>
concentration around the tip of the actuator is dramatically reduced, yet on the other hand the shear stress level along the actuator is increased, as compared with the corresponding curve in Fig. 5.

To study the effect of material mismatch $q$, the material constants of the actuator are fixed, and the shear modulus of the host medium is changed to achieve different material combinations. $q = 0.1, 0.5, 1.0, 2.0$ and $5.0$ are chosen, and two bonding conditions, $v' = 0$ and $v' = 0.01$, are considered to investigate the effect of the material combination on the load transfer between the actuator and the host structure. The normalized shear stress distribution curves under these two bonding conditions are shown in Figs. 7 and 8, respectively. In Fig. 7, with the increase of $q$, an increase of the shear stress level at the tips of the actuator can be observed. Fig. 8 shows that when the bonding layer is included, the increase of $q$ results in an increase of the shear stress level along the entire length of the actuator, and the shear stress distribution becomes less concentrated around the tips of the actuator. Figs. 5–8 indicate that the material mismatch and geometrical property of the actuator and the host structure show more significant effect on the shear stress distribution when the bonding layer is included, at least for the currently considered bonding layer. Proper selection of material combination and actuator geometry, if possible, will increase the actuation efficiency of the actuator.

Fig. 9 shows a typical shear stress distribution along a surface bonded actuator for $q = 1.0, v = 0.05$ and $q' = 0.011$. High stress concentration around the tip of the actuator is observed when $v' = 0$. Comparing the five curves in this figure, we can clearly see that, as the $v'$ value increases from 0 to 0.02, the shear stress level along the actuator between the interval of $y/a = 0.0 \sim 0.9$ shows very limited changes, but the stress concentration around the tip of the actuator, where $y/a = 1.0$, decreases significantly.
Fig. 7. Interfacial shear stress distribution ($\nu = 0.05$ and $\nu^* = 0$).

Fig. 8. Interfacial shear stress distribution ($q' = 0.011$, $\nu = 0.05$ and $\nu^* = 0.01$).

Fig. 9. Interfacial shear stress distribution ($q = 1.0$, $q' = 0.011$ and $\nu = 0.05$).
3.2. Interfacial debonding

3.2.1. Edge debonding

For a perfectly bonded actuator, when the maximum shear stress is larger than the bonding strength, edge debonding will initiate and grow to a length $d$, as shown in Fig. 2. As mentioned in Section 2.5, a debonded actuator can be regarded as a shorter actuator. The effective length of the actuator is reduced from its original length $2a$ to $2a_{\text{eff}} = 2a - d$.

To evaluate the effect of the thickness of the bonding layer in edge debonding problem, Fig. 10 shows the normalized shear stress $\tau^*\eta_{\text{ge}}$ at the edge point $\eta_{\text{ge}} = y/a = -1$ as a function of $d/a$ for $q = 1.0$, $q' = 0.011$ and $\nu = 0.05$. As expected when the debonding continues developing, the edge stresses decrease. When the edge stresses decrease to the values below the interfacial toughness, the debonding will stop growing. This is the self-arresting mechanism, which has been discussed by Wang and Meguid [28] and Luo and Tong [36]. Fig. 10 shows that the increase of the debonding length, from $d = 0$ to $d = a$ for example, will decrease the shear stress $\tau^*\eta_{\text{ge}}$ by up to 3% for $\nu = 0.007$ and 12% for $\nu = 0.025$, indicating that with the increase of the thickness of the bonding layer, the self-arresting effect becomes more obvious.

3.2.2. Central debonding

Local stress concentration and/or weak interfacial bonding may also result in debonding in the interior of the actuator as shown in Fig. 3. To simulate this situation, the actuator considered is assumed symmetrically debonded in $\eta < d$, i.e. $t_r = -t_r = a$ and $d_r = -d_r = d$. The effective length of the actuator is reduced from its original length $2a$ to $2a_{\text{eff}} = 2(a - d)$.
**Fig. 12.** Interfacial shear stress distribution \((q = 1.0, q_0 = 0.011, \nu = 0.05 \text{ and } \nu' = 0.01)\).

**Fig. 13.** Interfacial shear stress distribution \((q = 1.0, d/a = 0.6 \text{ and } \nu' = 0)\).

**Fig. 14.** Interfacial shear stress distribution \((q = 1.0, q' = 0.011, d/a = 0.6 \text{ and } \nu = 0.01)\).
Shear stress redistributions along actuators with central debondings for $q = 1.0$, $q' = 0.011$ and $v = 0.05$ with two bonding conditions, $v' = 0$ and $v' = 0.01$, are shown in Figs. 11 and 12, respectively. Fig. 11 shows that when the bonding layer is ignored shear stresses concentrate near the debonding edge, particularly for larger debonding lengths; it may be detrimental to the bonding strength. Fig. 11 also shows that central debondings mainly cause the stress redistribution near the debonding area, and does not significantly affect stresses far away from the debonding. However, when the bonding layer is included as illustrated in Fig. 12, shear stresses do not concentrate in the debonding edge and show a much more uniform distribution.

The results for shear stress redistribution along a surface bonded actuator for $q = 1.0$, $q' = 0.011$ and $d/a = 0.6$ under two bonding conditions, $v' = 0$ and $v' = 0.01$, are shown in Figs. 13 and 14, respectively. High stress concentration at the debonding edge and increase of shear stress level along the actuator due to increasing $v$ value are observed in Fig. 13. When a bonding layer is included, as shown in Fig. 14, the stress concentration around the tip of the actuator is dramatically reduced, and the parameter $v$ has more effect on the shear stress distribution.

Fig. 15 shows a typical shear stress redistribution along a surface bonded actuator for $q = 1.0$, $q' = 0.011$, $v = 0.05$ and $d/a = 0.6$. The normalized shear stresses at the edge points $\eta_{el} = y/a = d/a$ and $\eta_{er} = y/a = 1$ as a function of $d/a$ are shown in Figs. 16 and 17, respectively. It is observed that the shear stress at the debonding edge is singular when $v' = 0$, yet as the $v'$ value increases from 0 to 0.02, the concentration of the shear stress at the debonding edge is significantly reduced. When the debonding occurs in $d/a < 0.7$, the shear stresses at the debonding edges, $\tau'(\eta_{el})$ and $\tau'(\eta_{er})$, show a very limited change. When the interior debondings are developed in range of $0.7 < d/a < 1$, stress redistribution causes the shear stresses at the edge
points to increase significantly. \( \tau'(\eta_{cr}) \), for example, increases by up to 27% for \( v' = 0.002 \) and 55% for \( v' = 0.02 \) at \( d/a = 0.9 \). The corresponding results of the axial stress in the debonded part of the actuator, \( \sigma^* \), as a function of \( d/a \) is shown in Fig. 18. The thickness of the bonding layer does show significant effect upon the axial stress \( \sigma^* \) in the actuator. It should be mentioned, however, when the thickness of the layer are small compared with the thickness of the actuator, \( d/a \) shows insignificant effect until the debonding approaches the tip of the actuator. It is also interesting to observe that with the effect of the bonding layer, the axial stress in the debonded part of the actuator is more sensitive to the change of the debonding length, \( d/a \), while the shear stress at the edge of the actuator is less sensitive to \( d/a \) in comparison with the case where no bonding layer exists.

4. Concluding remarks

The focus of this paper is on the study of the effect of the material and geometric properties of the actuator and the bonding layer on the load transfer from the actuator to the host medium. A general analytical solution is provided to the coupled electromechanical behaviour of a piezoelectric actuator bonded to a host through an adhesive bonding layer under plane electric loading. The validity of the present model has been demonstrated by application to specific examples and comparison with the corresponding results obtained from Finite Element method. The numerical investigation of the influence of the geometry and the material mismatch of the adhesive layer upon the response of the coupled structure is provided. Both edge and central debonding and their effect upon the stress distribution in the composite structure are discussed. The simulation results indicate that the increase of the bonding layer thickness will increase the shear stress distribution level along the
internal of the actuator, and decrease the strain concentration at the tips of the actuator. Besides, the material combination of the actuator and the host structure needs to be carefully selected in order to improve actuator efficiency.

References