THE EFFECT OF IMPERFECT BONDING LAYER ON THE ELECTROMECHANICAL BEHAVIOUR OF SURFACE-BONDED PIEZOELECTRIC SENSORS

Congrui Jin and Xiaodong Wang*

ABSTRACT

The behaviour of a piezoelectric sensor is strongly affected by the bonding condition along the interface between the sensor and the host structure. The current paper represents an analytical study of the static effect of the mechanical and geometrical properties of the adhesive layer on the coupled electromechanical behaviour of a thin piezoceramic sensor bonded to an elastic medium. A sensor model with an imperfect adhesive bonding layer, which undergoes a shear deformation, is proposed to simulate the two dimensional electromechanical behaviour of the integrated system. Analytical solution of the problem is provided by solving the resulting integral equations in terms of the interfacial stress. Numerical simulation is conducted to study the effect of the bonding layer upon the sensing process. The interfacial debonding and its effect on the interlaminar strain and stress transfer mechanisms are discussed in detail.

Keywords: Piezoelectric, Electromechanical, Debonding, Sensor, Modelling

1. INTRODUCTION

Piezoelectric materials, due to their strong electromechanical coupling characteristics, have been successfully used as actuators and sensors (Gandhi and Thompson, 1992; Banks et al., 1996; Tani et al., 1998; Boller, 2000). In the last decades the utilization of networks of piezoelectric sensors/actuators in the design of smart structures for vibration and noise control, as well as damage detection for structural health monitoring applications has been gaining more and more ground. Optimizing the effectiveness and reliability of integrated sensor/actuator systems requires a clear understanding of the sensing/actuating processes and the resulting electromechanical response of the whole structure. When a piezoelectric patch is bonded to a structure as a sensor, the local mechanical deformation will result in an electric voltage across the thickness of the sensor, which can be recorded by data acquisition systems to evaluate the strain/stress level. Ideally, the piezoelectric sensor should not be intrusive, but in reality the existence of a sensor will disturb the mechanical field to be measured. Since the stiffness of some piezoelectric sensors, piezoceramic ones for example, is comparable to that of typical engineering materials, the disturbance from sensors on the measured signal could be significant. In addition, the bonding condition between the sensor and the host structure will also affect the performance of the sensor (Denoyer and Kwak, 1996; Kwak and Sciuilli, 1996; Park et al., 2000; Rabinovitch and Vinson, 2002). It becomes, therefore, an important issue to study the coupled electromechanical behaviour of these sensors with bonding layers to reliably evaluate the relation between the measured signal and the local mechanical deformation.

Due to the presence of material discontinuity between the piezoelectric sensors and the host structure, complicated local electromechanical fields will be generated near the edge of sensors, which will affect the load transfer between the sensor and the host structure, and therefore influence the performance of the sensor. To study the static load transfer between the piezo-electric elements and the host structure, simplified actuator/sensor models have been established. A beam-like structure with surface-bonded and embedded thin-sheet piezoelectric elements is first analyzed to study the load transfer (Crawley and de Luis, 1987). In this analysis, the axial stress in the piezoelectric
elements is assumed to be uniform across their thickness. A Bernoulli–Euler model of a piezoelectric thin-sheet bonded to a beam is further developed by considering the linear stress distribution across the thickness of the piezoelectric element (Crawley and Anderson, 1990). A refined sensor model based on the plane stress condition was presented for a beam structure with symmetrically surface-bonded sensor patches (Lin and Rogers, 1993a,b). Plate and shell models have also been extensively used in modelling the electromechanical behaviour of piezoelectric structures (Dimitriadis et al., 1991; Tzou and Tseng, 1991; Wang and Rogers, 1991; Tauchert, 1992; Mitchell and Reddy, 1995; Han and Lee, 1998; Reddy, 1999). The static local stress field near a thin-sheet piezoelectric element attached to an infinite elastic medium is studied to investigate the stress concentration and the load transfer between the piezoelectric element and the host medium (Wang and Meguid, 2000). Similar analysis is also conducted to determine the static electromechanical field of a piezoelectric layer bonded to an elastic medium with both interfacial and normal stresses being considered (Zhang et al., 2003a,b). Significant attention has also been paid to the modelling of debonding of sensors. Seeley and Chattopadhyay (1996) investigated the effects of debonding between piezoelectric actuators/sensors on the behaviour of smart composite plates by using an FE model based on the “free mode” assumption. Continuity conditions at the delamination junctions were satisfied using the penalty function approach. The results of the numerical model were in good agreement with the experimental data (Seeley and Chattopadhyay, 1998). Tylikowski (2001), Tong et al. (2001) and Sun et al. (2001) presented analytical models for a smart beam with a debonded piezoelectric actuator/sensor. It was assumed that there is no stress transfer between the host beam and the piezoelectric actuator/sensor in the debonding region. Using similar assumptions Luo and Tong (2004) presented a finite element model based on the first-order shear deformation theory. A closed-loop-control-based damage detection scheme is presented by Sun and Tong (2003) aiming at detecting damage in structures. The results show that even a small edge debonding in a piezoelectric sensor patch can make the sensitive control system unstable, and therefore can be detected. Bhalla and Soh (2004) investigated the effect of the shear lag loss on electromechanical impedance measurements, and found that the bonding layer can significantly modify the measured admittance signatures. A useful review is given by Della and Shu (2007).

The objective of the present paper is to provide a comprehensive theoretical study of the effect of imperfect bonding on the electromechanical behaviour of surface-bonded piezoelectric sensors. The current work is an extension of the work presented in Han et al. (2008) in which an integrated model containing a piezoelectric thin-sheet sensor, a viscoelastic bonding layer and an elastic medium (host) is proposed to evaluate its dynamic property under different loading frequencies. Although the effect of the geometrical and material property of the system has been investigated intensively, less effort has been devoted to the study of the debonded sensors. In this paper, we focus on the effect of interfacial debonding in the static case where the loading frequency is so low that the typical wavelength of the incident wave is much longer than the length of the sensor. The imperfect bonding and its effect upon the strain distribution and the overall performance of the structure is evaluated in detail. Numerical simulation is conducted to simulate the effect of the geometrical and material property of the system, especially that of the imperfect bonding layer upon the coupled response of the sensors.

2. FORMULATION OF THE PROBLEM

Let us now consider the plane strain problem of a thin piezoceramic sensor sheet bonded to a homogeneous and isotropic elastic half plane through a thin bonding layer, as illustrated in Fig. 1. The host medium is modelled as a half plane to represent the case that it is much thicker than the sensor. The lengths of both the sensor and the bonding layer are denoted as $2a$, and the thicknesses of the sensor and the bonding layer are denoted as $h$ and $h'$, respectively. It is assumed that the poling direction of the sensor is along the $z$-axis. Throughout this paper superscripts $s$, $b$ and $h$ are used to designate physical quantities that belong to the sensor, the bonding layer and the host, respectively.

2.1. The Sensor

This study will focus on a thin-sheet sensor, with relatively small thickness in comparison with its length. Therefore, the axial stress and strain can be assumed to be uniform across the thickness of the sensor. Since the thickness of the bonding layer is usually smaller than that of the sensor, the same assumption is also used for the bonding layer.
Based upon these assumptions, the sensor can be modelled as an electroelastic line subjected to the distributed axial force, $\tau / h$, where $\tau$ is the shear stress at the bonding layer. The equilibrium equation of the sensor can then be expressed as

$$\frac{d\sigma_y(y)}{dy} + \frac{\tau(y)}{h} = 0$$

(1)

The electromechanical behaviour of the piezoelectric sensor can be described by

$$\sigma_y = E^e \epsilon_y - e^e E_z, \quad D_z = e^e \epsilon_y + \lambda^e E_z$$

(2)

where $\lambda^e$, $E^e$ and $e^e$ are effective material constants (Wang and Meguid, 2000). $D_z$ and $E_z$ represent the electric displacement and the electric field intensity, respectively.

Since all the load transferred between the sensor and the bonding layer can be attributed to $\tau$, the two ends of the sensor can be assumed to be traction free, i.e. $\sigma_y = 0$ at $|y| = a$. In addition, the sensor will be assumed to operate in an open-loop mode with no external charge supplied to it (Lee and Moon, 1989). Therefore, the electric displacement across the sensor will be zero, i.e. $D_z = 0$.

The resulting axial strain $\epsilon_y$ and displacement $u_y$ can then be expressed in terms of $\tau$ as

$$\epsilon_y(y) = -\frac{1}{E^h h} \int_{-a}^{y} \tau(\xi) d\xi, \quad u_y(y) = -\frac{1}{E^h h} \int_{-a}^{y} (y - \xi) \tau(\xi) d\xi, \quad |y| < a$$

(3)

where $E^h = E^e + (e^e)^2 / \lambda^e$.

2.2. The Bonding Layer

The bonding layer is the medium between the sensor and the host structure. Its shear modulus, thickness, and coefficient of viscosity, will govern the property of the layer. The shear stress $\tau$ distributed in the layer is determined by the constitutive relation:

$$-\tau(y) = \mu^h \epsilon_y^b(y)$$

(4)

where

$$\epsilon_y^b(y) = \frac{u^+(y) - u^-(y)}{h'}$$

(5)
with $\mu^b$ and $\varepsilon^b_\tau$ being the shear modulus and the shear strain of the bonding layer, respectively. $u^+$ and $u^-$ represent the longitudinal displacements of the lower surface of the sensor and the upper surface of the host medium, respectively.

### 2.3. The Host Medium

The stress field generated inside the host medium can be divided into two parts (Han et al., 2008). The first is caused by the incident wave in the host medium with a traction free boundary and the second is caused by surface shear stress $\tau$ resulted from the sensor. For the second subproblem, the displacement induced by $\tau$ can be determined by using the boundary condition along the top surface, which can be expressed as

$$\sigma_{yz}(y,0) = \begin{cases} -\tau(y) & |y| < a \\ 0 & |y| > a \end{cases}, \quad \sigma_{zz}(y,0) = 0 \quad (6)$$

Making use of the fundamental solution of a half elastic plane subjected to a concentrated horizontal force (Muskhelishvili, 1953) and the superposition principle, $\varepsilon^h_y$ and $u^h_y$ resulting from the applied force given by Eq. (6) can be obtained.

For the static case where a constant strain $\varepsilon^{IN}_y$ is applied along the $y$-direction at infinity, the total strain and displacement can then be obtained by superimposing the solutions of both parts, which can be expressed as

$$\varepsilon^h_y(y,0) = \frac{1 - \nu^h}{\pi \mu^h} \int_{-a}^{a} \frac{\tau(\xi)}{y - \xi} d\xi + \varepsilon^{IN}_y, \quad |y| < a \quad (7)$$

$$u^h_y(y,0) = \frac{1 - \nu^h}{\pi \mu^h} \int_{-a}^{a} \tau(\xi) d\xi + \int_{-a}^{a} \frac{\tau(\xi)}{y - \xi} d\xi dy + u^{IN}_y, \quad |y| < a \quad (8)$$

where $\nu^h$ and $\mu^h$ are the Poisson’s ratio and the shear modulus of the host, respectively.

### 2.4. Static Load Transfer in a Perfectly Bonded Piezo-electric Sensor

Substituting Eqs. (3) and (7) into Eq. (4), the following integral equation can be obtained

$$\frac{1 - \nu^h}{\pi \mu^h} \int_{-a}^{a} \frac{\tau(\xi)}{y - \xi} d\xi + \frac{1}{h E^s_h} \int_{-a}^{a} \tau(\xi) d\xi - \frac{h'}{\mu^h} \frac{d\tau(y)}{dy} = -\varepsilon^{IN}_y, \quad |y| < a \quad (9)$$

which can be normalized as

$$\int_{-a}^{a} \tau(\xi) d\xi = 0$$

$$\frac{1}{q} \int_{-1}^{1} \tau(\xi) d\xi + \frac{1}{q'} \int_{-1}^{1} \tau(\xi) d\xi - \frac{\nu'}{q'} \frac{d\tau(\eta)}{d\eta} = -\varepsilon^{IN}_y, \quad 0 < \eta < 1 \quad (10)$$

where

$$\tau(\eta) = \frac{\tau(a\eta)}{E^s_h}, \quad \eta = \frac{y}{a}, \quad q = \frac{\pi E^h}{2[1 - (\nu^h)^2]E^s_h}, \quad q' = \frac{h'}{E^s_h}, \quad \nu' = \frac{h'}{a}$$

The general solutions of $\tau$ in Eq. (10) can be expressed in terms of the following expansions of Chebyshev polynomials.
\[
\bar{\tau}(\eta) = \frac{1}{\sqrt{1-\eta^2}} \sum_{j=0}^{\infty} d_j T_j(\eta)
\]

where \(T_j\) are Chebyshev polynomials of the first kind with \(T(\eta) = \cos(j\theta)\) and \(\cos \theta = \eta\). By truncating the Chebyshev polynomial expansions to the \(N\)th term and considering the boundary conditions at the following collocation points along the sensor

\[
\eta_k = \cos\left(\frac{k}{N+1}\pi\right), \quad k = 1, 2, \ldots, N
\]

Eq. (10) reduces to

\[
\sum_{j=1}^{N} d_j \frac{\sin\left(\frac{j}{N+1}\pi\right)}{\sin\left(\frac{k}{N+1}\pi\right)} \left\{\frac{1}{q} + \frac{1}{\sqrt{\nu}} \sin\left(\frac{k}{N+1}\pi\right)\right\} = \frac{\varepsilon_{\pi}^{in}}{\pi}, \quad k = 1, 2, \ldots, N
\]

The unknown coefficients \(d_j\) can be readily determined by using Eq. (14). It can then be used to determine the axial displacement and the axial strain of the sensor.

2.5. Effect of Interfacial Debonding

The high level of shear stress at the edges of the sensor may result in an unwanted edge debonding as shown in Fig. 2. For a perfectly bonded sensor, when the maximum shear stress is larger than the bonding strength, edge debonding may develop. The debonded part of the sensor has zero boundary stresses. According to the present sensor model, the debonded part at the edge of the sensor will experience no stress. As a result, the effective length of the sensor is reduced from its original length to that of the bonded part only. Therefore, the behaviour of an edge debonded sensor can be simulated by a shorter sensor.

![Figure 2: Schematics of Edge Debonding](image-url)
Debonding may also occur in the interior of the sensor. Let us consider a sensor occupying the region \( t_i < y < t_r \) which is partially debonded in \( d_l < y < d_r \) as illustrated in Fig. 3. The partially debonded sensor can then be regarded as two “sensors” subjected to an axial stress \( \sigma_d \) at the inner tips of them. The resulting axial strain can then be expressed in terms of \( \tau \) as

\[
\varepsilon'_y(y) = \begin{cases} 
\frac{1}{E_h h} \int_{d_l}^{y} \tau(\xi) d\xi & t_i < y < d_i \\
\frac{\sigma_d}{E_s} & d_i < y < d_r \\
\frac{1}{E_h h} \left[ \sigma_d h - \int_{d_i}^{y} \tau(\xi) d\xi \right] & d_r < y < t_r
\end{cases}
\]  

(15)

where

\[
\sigma_d = -\int_{d_i}^{d_r} \frac{\tau(\xi)}{h} d\xi
\]  

(16)

The half plane to which the sensor is bonded is subjected to the following boundary conditions:

\[
\sigma_{\gamma\gamma}(y, 0) = \begin{cases} 
-\tau(y) & t_i < y < d_i \text{ and } d_r < y < t_r \\
0 & \text{otherwise}
\end{cases}
\]  

(17)

The surface strain in the host medium can be obtained by making use of the fundamental solution of a half plane subjected to a concentrated surface force. For the case where a constant strain \( \varepsilon_{y}^{\text{inh}} \) is applied along the \( y \)-direction at infinity, the total strain and displacement can then be obtained by superimposing the solutions of both parts, which can be expressed as

\[
\varepsilon_y(y, 0) = \frac{1 - V^h}{\pi \mu^h} \left[ \int_{d_i}^{t_i} \frac{\tau(\xi)}{y - \xi} d\xi + \int_{d_i}^{t_r} \frac{\tau(\xi)}{y - \xi} d\xi \right] + \varepsilon_{y}^{\text{inh}}
\]  

(18)

The strains in the sensor and the host medium presented by Eqs. (15) and (18) can be related as
The axial stress $\sigma_d$ in Eq. (19) need to be determined by considering the deformation of the debonded part of the sensor. From Eqs. (4) and (5) it can be obtained that

$$u'_s(d_r) - u'_s(d_l) = u'_b(d_r) - u'_b(d_l) + \frac{h'}{\mu'} [\tau(d_r) - \tau(d_l)]$$

(20)

Integrating Eqs. (15) and (18) and making use of Eq. (20) give the following equation, which provides an additional condition for determining $\sigma_d$

$$\frac{1 - v^b}{\pi \mu^b} \int_{d_l}^{d_r} \int_{y - \xi}^{y} \tau(\xi) \, d\xi \, dy + \frac{1 - v^h}{\pi \mu^h} \int_{d_l}^{d_r} \int_{y - \xi}^{y} \tau(\xi) \, d\xi \, dy + \frac{h'}{\mu'} \int_{d_l}^{d_r} \tau(y) \, dy = \left( \frac{\sigma_d}{E^s} - \epsilon_{y}^{IN} \right) (d_r - d_l)$$

(21)

Eqs. (16), (19) and (21) can be normalized as

$$\frac{1}{q} \int_{-1}^{1} \frac{\tau'(\xi) \, d\xi}{\eta_l - \xi} + \frac{1}{q} \int_{-1}^{1} \frac{\tau'(\xi) \, d\xi}{\eta_r - \xi}$$

$$+ \left[ \frac{-v'_l}{q'} \frac{d\tau'(\eta_l)}{d\eta_l} + \frac{1}{v'_l} \int_{-1}^{1} \tau'(\xi) \, d\xi = -\epsilon_{y}^{IN} \right] \left| \eta_l \right| < 1$$

$$- \frac{-v'_r}{q'} \frac{d\tau'(\eta_r)}{d\eta_r} - \frac{1}{v'_r} \int_{-1}^{1} \tau'(\xi) \, d\xi = -\epsilon_{y}^{IN} \right] \left| \eta_r \right| < 1$$

(22)

and

$$\frac{1}{v'_l} \int_{-1}^{1} \frac{\tau'(\xi) \, d\xi}{\eta_l - \xi} + \frac{1}{v'_l} \int_{-1}^{1} \frac{\tau'(\xi) \, d\xi}{\eta_r - \xi} \, d\eta_l + \frac{q v'_r}{q'} \tau'(1) - \frac{q v'_r}{q'} \tau'(-1)$$

$$= 2q \left( \frac{1}{v} - \frac{1}{v'_l} - \frac{1}{v'_r} \right) \left( \sigma' - \epsilon_{y}^{IN} \right)$$

(23)

where the normalized stresses are given by

$$\tau'(\eta_l) = \frac{\tau(a, \eta_l + y_l)}{E^s}, \quad \tau'(\eta_r) = \frac{\tau(a, \eta_r + y_r)}{E^s}, \quad \sigma' = \frac{\sigma_d}{E^s}$$

(25)

with
\[ \eta_v = \frac{y = y_i}{a_i}, \quad \eta_r = \frac{y = y_r}{a_r} \]
\[ v = \frac{h}{a}, \quad v_r = \frac{h_r}{a_r}, \quad v = \frac{h}{a} \]
\[ v' = \frac{h'}{a}, \quad v'_r = \frac{h'}{a_r}, \quad v' = \frac{h'}{a} \]
\[ a = \frac{1}{2} (t_r - t_l), \quad a_i = \frac{1}{2} (d_i - t_l), \quad a_r = \frac{1}{2} (t_r - d_r) \]
\[ y_i = \frac{1}{2} (d_i + t_l), \quad y_r = \frac{1}{2} (t_r + d_r) \]
\[ \eta^*_v = 2v_r \frac{v_r - v}{v_r v} - 1, \quad \eta^*_r = -2v_r \frac{v_r - v}{v_r v} + 1 \]

The general solutions of \( \tau' \) and \( \tau' \) in Eqs. (22)–(24) can then be expressed in terms of the following expansions of Chebyshev polynomials

\[ \tau' (\eta_v) = \frac{1}{\sqrt{1 - \eta_v^2}} \sum_{j=0}^{\infty} d'_j T_j (\eta_v), \quad \tau' (\eta_r) = \frac{1}{\sqrt{1 - \eta_r^2}} \sum_{j=0}^{\infty} d'_j T_j (\eta_r) \]  

(27)

If the Chebyshev polynomial expansions are truncated to the \( N \)th term, and Eq. (22) is satisfied at the following collocation points at each bonded segment of the sensor given by

\[ \eta_k = \eta_{ak} = \eta_{ck} = \cos \left( \frac{k}{N+1} \pi \right), \quad k = 1, 2, \ldots, N \]  

(28)

Eqs. (22) and (23) reduce to

\[ \sum_{j=1}^{N} d'_j \frac{\sin \left( \frac{j}{N+1} \pi \right)}{\sin \left( \frac{k}{N+1} \pi \right)} \left\{ \frac{1}{q} + \frac{1}{v \pi j} \sin \left( \frac{k}{N+1} \pi \right) \right\} + \frac{v'_i}{v q'} \left[ \frac{j + \cot \left( \frac{j}{N+1} \pi \right) \cot \left( \frac{k}{N+1} \pi \right)}{\sin \left( \frac{k}{N+1} \pi \right)} \right\} \]

\[ + \sum_{j=1}^{N} d'_j \frac{\left[ (\eta^*_v)^2 - 1 \right] - \left[ \eta^*_v \right]}{q \sqrt{\left( \eta^*_v \right)^2 - 1}} + \frac{\sigma^i}{\pi} \left\{ \frac{v_r - \frac{1}{q \sqrt{(\eta^*_r)^2 - 1}}} {\sin \left( \frac{k}{N+1} \pi \right)} + \frac{v'_i}{v q'} \left[ \frac{j + \cot \left( \frac{j}{N+1} \pi \right) \cot \left( \frac{k}{N+1} \pi \right)}{\sin \left( \frac{k}{N+1} \pi \right)} \right] \right\} \]

\[ \sum_{j=1}^{N} d'_j \frac{\sin \left( \frac{j}{N+1} \pi \right)}{\sin \left( \frac{k}{N+1} \pi \right)} \left\{ \frac{1}{q} + \frac{1}{v \pi j} \sin \left( \frac{k}{N+1} \pi \right) \right\} + \frac{v'_i}{v q'} \left[ \frac{j + \cot \left( \frac{j}{N+1} \pi \right) \cot \left( \frac{k}{N+1} \pi \right)}{\sin \left( \frac{k}{N+1} \pi \right)} \right\} \]  

(29)
\[-\sum_{j=1}^{N} d'_j (-1)^j \left[ \frac{\sqrt{(\eta'_{jk})^2 - 1}}{q \sqrt{(\eta_{jk})^2 - 1}} \right] + \frac{\sigma^*}{\pi} \left[ \frac{v_j}{q \sqrt{(\eta_{jk})^2 - 1}} + k N + 1 + \frac{v'_j v_r}{\pi q' \sin^2 \left( \frac{k}{N+1} \right)} \right] = \frac{\epsilon_{jN}^\prime}{\pi}, k = 1, 2, ..., N \quad (30)\]

\[-\sigma^* \left[ \frac{1}{q} \ln \left| \eta'_b + \sqrt{\eta'_b^2 - 1} \right| - \frac{1}{q} \ln \left| \eta'_b + \sqrt{\eta'_b^2 - 1} \right| + 2 \left( \frac{1}{v} - \frac{1}{v'} \right) \right] + \frac{v'_j + v'_r}{\pi q' \sin \left( \frac{1}{N+1} \right)} \]

\[+ \sum_{j=1}^{N} \frac{d'_j}{v_j} \left[ \pi (-1)^j \left[ \frac{\sqrt{(\eta'_{jk})^2 - 1}}{q \sqrt{(\eta_{jk})^2 - 1}} \right] + \frac{\epsilon_{jN}^\prime}{\pi} \right] \]

\[+ \sum_{j=1}^{N} \frac{d'_j}{v_r} \left[ -\pi \left[ \frac{\sqrt{(\eta'_{rk})^2 - 1}}{q \sqrt{(\eta_{rk})^2 - 1}} \right] d \eta_{rk} = (-1) \frac{v'_r}{q' \sin \left( \frac{1}{N+1} \right)} \right] \]

\[= -2 \left( \frac{1}{v} - \frac{1}{v'} \right) \epsilon_{jN}^\prime \quad (31)\]

where

\[\eta'_b = v_j \left[ \frac{2}{v} - \frac{1}{v_j} - \frac{1}{v_r} \cos \left( \frac{k}{N+1} \right) \right] \quad (32)\]

\[\eta'_r = v_r \left[ \frac{2}{v} - \frac{1}{v_j} - \frac{1}{v_r} \cos \left( \frac{k}{N+1} \right) \right] \quad (33)\]

The unknown coefficients \(d'_j, d'_r\) and \(\sigma^*\) can be determined by solving Eqs. (29)–(31), which can then be used to determine the axial displacement and the axial strain of the debonded sensor.

3. NUMERICAL RESULTS AND DISCUSSION

In this section, the results of numerical simulation of the influence of the property of the bonding layer under different material combinations and sensor geometries on the electromechanical behaviour of the integrated system are presented. The attention will be focused on the strain distribution along the sensor, which represents the load transfer between the sensor and the host medium. The convergence of the solution using Chebyshev polynomials has been carefully evaluated. The number of terms of Chebyshev polynomials is selected to be 64, with which the convergence of the results for all the cases considered is ensured.

The piezoelectric material considered in the following examples is a typical piezoceramics, whose properties are given in Table 1 (Park, 1990). The properties of the bonding layer (Park et al., 2000) and the host medium (Wang and Huang, 2006) are given in Table 2. From these material constants it can be determined that \(q = 0.3928\) and \(q' = 0.0083\). The geometry of the sensor is assumed to be \(a = 1:0 \text{ cm} \) and \(h = 500–2000 \text{ µm}\). The length of the bonding layer is the same as that of the sensor.
Table 1
Material Properties of the Piezoelectric Sensor

<table>
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<tr>
<th>Property</th>
<th>Unit</th>
<th>Elastic</th>
<th>Piezoelectric</th>
<th>Dielectric</th>
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<td></td>
<td></td>
<td>$c_{11}$</td>
<td>$c_{12}$</td>
<td>$c_{13}$</td>
</tr>
<tr>
<td>Elastic</td>
<td>(Pa)</td>
<td>$13.9 \times 10^{10}$</td>
<td>$6.78 \times 10^{10}$</td>
<td>$7.43 \times 10^{10}$</td>
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Table 2
Material Properties of the Host Medium and the Bonding Layer

<table>
<thead>
<tr>
<th>Layer</th>
<th>Property</th>
<th>Unit</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Host medium</td>
<td>Young’s modulus</td>
<td>$E_h$ (Pa)</td>
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<td></td>
<td>Poisson’s ratio</td>
<td>$\nu_h$</td>
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</tr>
<tr>
<td>Bonding layer</td>
<td>Shear modulus</td>
<td>$\mu'$ (Pa)</td>
<td>$1.0 \times 10^{9}$</td>
</tr>
</tbody>
</table>

3.1. Sensor with a Uniform Interfacial Layer

3.1.1. Strain Distribution Along the Sensor

The relation between the sensor strain $\varepsilon_y^s$ and the strain to be measured $\varepsilon_y^{IN}$ can be evaluated using the amplitude of a static strain ratio, $k(y)$, defined as

$$k(y) = \frac{\varepsilon_y^s(y)}{\varepsilon_y^{IN}} , \quad |y/a| < 1$$

which represents the percentage of deformation transferred from the host medium to the sensor. Ideally, if the sensor does not disturb the incident field the value should be one. The change of $\cdot$ represents the intrusive effect of the sensor on the original incident field. As shown in Eq. (11), $q$ represents the material combination of the sensor and the host structure, $v$ represents sensor geometry, $q'$ represents material property of the bonding layer and $v'$ represents the geometry of the bonding layer. Carefully examining Eq. (10) indicates that the strain distribution is governed by three parameters: $q$, $v$ and $v' = q'$. In the following discussion, the values of $q$, $v$ and $v'$ are varied to evaluate different responses of the structure.

Fig. 4 shows the effect of $v = h / a$ upon the amplitude of the strain ratio $A = |k(y)|$ along the sensor with $q = 0.3928$ and $v' = 0$. Significant effect of $v$ upon the strain ratio is observed, with lower value of $v$ corresponding to relatively higher strain ratio, indicating that sensors with higher length-to-thickness ratio have a higher sensitivity. To validate the model, a numerical analysis using commercially available finite element software ANSYS is conducted. Both the sensor and the host medium are modelled using PLANE13 elements, which can simulate coupled electromechanical fields. A comparison of the result from the current model with that of the finite element simulation is also made in Fig. 4. Excellent agreement is observed for $v = 0.05$, showing the feasibility and accuracy of the proposed sensor model. The corresponding result for the case where the bonding condition is $v' = 0.015$ is shown in Fig. 5. Again, significant effect of $v$ is observed. In comparison with the results shown in Fig. 4, the strain transfer ratio is much lower for a given $v$ especially near the ends of the sensor.

To study the effect of material mismatch $q$, the material constants of the sensor are fixed, and the shear modulus of the host medium is changed to achieve different material combinations. For the convenience of calculation, $q = 0.2, 0.5, 1.0, 2.0, 5.0$ and $10.0$, are chosen, and two bonding conditions, $v' = 0$ and $v' = 0.01$, are considered to investigate the effect of the material combination on the load transfer between the sensor and the host structure. Fig. 6 shows the strain distribution along the sensor for different $q$ values when $v' = 0$. With the decrease of the
Figure 4: Amplitude of Strain ratio ($q = 0.3928$ and $v' = 0$). Solutions Obtained from Current Model are Shown as Lines and FEA as Data Points.

Figure 5: Amplitude of Strain Ratio ($q = 0.3928$, $q' = 0.0083$ and $v' = 0.015$)
stiffness of the sensor (increasing \( q \)), the amplitude of the normalized strain increases. When \( q \) reaches 5.0, the strain distribution curve becomes very flat, approaching 1.0 in most part of the sensor. This is because the disturbance of a soft sensor is relatively insignificant. It can be observed that when \( q > 5.0 \), the strain of the sensor provides a good prediction of the applied strain to be measured. For \( q < 5.0 \), however, significant difference between them is observed. We might as well consider two extreme cases where the sensor is either very stiff or very soft compared with the host medium. For a stiff sensor \((q \to 0, \ q / \nu \to 0 \text{ is actually needed})\), the governing equation for the interfacial shear stress, Eq. (9) reduces to

\[
\frac{1 - \nu^2}{\pi \mu^b} \int_y^a \frac{\tau(\xi)}{y - \xi} \, d\xi = -\epsilon_y^{IN}, \quad |y| < a
\]  

which can be solved analytically and results in the following strain ratio \( k(y) = q(1 - \eta^2)\nu^2 / \nu \). In this case, the strain ratio will be very small but the distribution of it along the sensor can be predicted. When the sensor is very soft \((q \to \infty)\), Eq. (9) reduces to

\[
\frac{1}{hE^b} \int_{-a}^y \tau(\xi) d\xi = -\epsilon_y^{IN}, \quad |y| < a
\]  

which results in \( k(y) = 1 \). Theoretically, this is the perfect situation for sensing. The corresponding result for the case where the bonding condition is \( \nu' = 0.01 \) is shown in Fig. 7. The strain distribution curves approach 1.0 only in the middle part of the sensor. This indicates that when the bonding layer is included, the disturbance of the sensor is significant even for large \( q \).

Fig. 8 shows the amplitude of the strain ratio along the sensor for different bonding layer thickness \( \nu' = 0, 0.002, 0.004, 0.008 \) and 0.016 with \( q = 0.3928 \). A significant deduction of the strain ratio occurs with increasing bonding layer thickness. At \( y / a = 0.75 \) for example, a deduction of 20% of the strain ratio is observed when \( \nu' = 0.008 \) in comparison with the case \( \nu' = 0 \). The corresponding result for \( q = 10.0 \) is shown in Fig. 9. It is observed that the strain ratio is less than unity near the ends of the sensor. The length of this zone depends on \( \nu' / q' \), as shown in Eq. (10), which in turn depends on the stiffness and thickness of the bond layer. As \( \mu^b \) increases and \( h' \) reduces, \( \nu' = q' \) reduces, and the deformation is effectively transferred even over very small zones near the ends of the sensor. Considering the fact that \( q = 10.0 \) corresponds to a very soft sensor, the current result indicates that, even when the host structure is much stiffer than the sensor, the effect of the bonding layer on the load transfer will still be very important.

### 3.1.2. Output Voltage of the Sensor

According to the relationship between voltage and electric field intensity, the voltage distribution along the sensor can be determined as

\[
V(y) = -\int_0^h E_z(y) \, dz = \frac{e^s h}{\lambda^s} \frac{\partial u_y^s}{\partial y}
\]  

When the upper and lower surfaces of the sensor form two electrodes, the total resulting voltage across the sensor can be obtained by averaging the voltage across the sensor obtained before, i.e.

\[
V_{\text{out}} = \frac{1}{2a} \int_{-a}^a V(y) \, dy = \frac{e^s h}{2a \lambda^s} \left[ u_y^s(a) - u_y^s(-a) \right]
\]  

\( V_{\text{out}} \) is an important parameter of the piezoelectric sensor, which will be measured in a real system. The higher the \( V_{\text{out}} \) value, the stronger the signal detected by the sensor is.

Fig. 10 shows the variation of the normalized output voltage of the sensor with the thickness of the bonding layer and material combinations. The amplitude of the incident strain field is kept to be the same and the output voltage is normalized by \( (V^\text{out})_0 \), i.e. \( B = |V_{\text{out}}| / (V_{\text{out}})_0 \), where \( (V_{\text{out}})_0 \) is the corresponding result for \( q = 10.0 \) at \( \nu' = 0 \). It is shown that for a fixed value of \( q \), the output voltage decreases with an increase in the thickness of the bonding
Figure 6: Amplitude of Strain Ratio ($v = 0.05$ and $v' = 0$)

Figure 7: Amplitude of Strain Ratio ($v = 0.05$, $q' = 0.0083$ and $v' = 0.01$)
Figure 8: Amplitude of Strain Ratio \((q = 0.3928, q' = 0.0083\) and \(v = 0.05\))

Figure 9: Amplitude of Strain Ratio \((q = 10.0, q' = 0.0083\) and \(v = 0.05\))
Figure 10: Normalized Voltage Distribution with Different Thicknesses of the Bonding Layer and Material Combinations ($q' = 0.0083$ and $v = 0.05$)

Figure 11: Normalized Voltage Distribution with Different Thicknesses of the Bonding Layer and Sensor Geometries ($q = 0.3928$ and $q' = 0.0083$)
layer; for a fixed value of \( \nu' \), the output voltage increases with increasing \( q \). The increase of \( \nu' \), from 0.01 to 0.03 for example, will decrease the output voltage by up to 15.15% for \( q = 0.2 \) and 22.52% for \( q = 10.0 \), indicating that with the increase of \( q \), the effect of the thickness of the bonding layer becomes more obvious.

Fig. 11 shows the variation of the normalized output voltage with the thickness of the bonding layer and sensor geometries. The output voltage is normalized by the corresponding result for \( \nu = 0.2 \) at \( \nu' = 0 \). It is shown that with a given \( \nu \), the output voltage decreases with increasing \( \nu' \); with a given \( \nu' \), the output voltage decreases with decreasing \( \nu \).

### 3.2. Interfacial Debonding

#### 3.2.1. Edge Debonding

For a perfectly bonded sensor, when the maximum shear stress is larger than the bonding strength, edge debonding will initiate and grow to a length \( d \), as shown in Fig. 2. As mentioned in Section 2.5, a debonded sensor can be regarded as a shorter sensor. The effective length of the sensor is reduced from its original length \( 2a \) to \( 2a_{\text{eff}} = 2a - d \).

To evaluate the effect of the thickness of the bonding layer in edge debonding problem, Fig. 12 shows the normalized output voltage as a function of \( d/a \) for \( q = 0.3928 \), \( q' = 0.0083 \) and \( \nu = 0.05 \). The output voltage is normalized by the corresponding result for \( \nu' = 0 \) at \( d = 0 \). As expected when the debonding continues developing, the output voltage decreases. It is also shown that the increase of the debonding length, from \( d = 0 \) to \( d = a \) for example, will decrease the output voltage by up to 28.9% for \( \nu' = 0.002 \) and 42.4% for \( \nu' = 0.016 \), indicating that the effect of edge debonding becomes more significant with the increase of the thickness of the bonding layer.

#### 3.2.2. Central Debonding

Local stress concentration and/or weak interfacial bonding may also result in debonding in the interior of the sensor as shown in Fig. 3. To simulate this situation, the sensor considered is assumed symmetrically debonded in \( |y| < d \), i.e. \( t_r = -t_l = a \) and \( d_r = -d_l = d \). The effective length of the sensor is reduced from its original length \( 2a \) to \( 2a_{\text{eff}} = 2(a - d) \).

Strain redistributions along sensors with central debondings for \( q = 2.0 \), \( q' = 0.0083 \) and \( \nu = 0.05 \) with two bonding conditions, \( \nu' = 0 \) and \( \nu' = 0.01 \), are shown in Figs. 13 and 14, respectively. Fig. 13 shows that when the bonding layer is absent the central debonding shows insignificant effect on the load transfer between the debonded sensor and the host structure. It is also shown that central debondings mainly cause the strain redistribution in the debonding area, and does not significantly affect strain ratio far away from the debonding edge. However, when the bonding layer is included as illustrated in Fig. 14, the effect of debonding on the strain redistribution becomes much more obvious.

To investigate the effect of the material combination on the performance of the debonded sensor, the results for strain redistribution for \( \nu = 0.05 \), \( q' = 0.0083 \) and \( d/a = 0.6 \) under two bonding conditions, \( \nu' = 0 \) and \( \nu' = 0.01 \), are shown in Figs. 15 and 16, respectively. Fig. 15 shows that the central debonding has more effect on the strain redistribution for smaller \( q \) values. When a bonding layer is included, as shown in Fig. 16, the effect of debonding becomes significant for any value of \( q \). It is also observed that the strain ratio along the undeboinded part of the sensor is obviously increased.

The strain ratio along the debonded sensor for different bonding layer thickness for \( q = 10.0 \) is shown in Fig. 17. It is observed that even for a very soft sensor, the effect of central debonding will still be very important when the thickness of the bonding layer is significant.

To evaluate the variation of the output voltage of the sensor with the debonding length and the influence of the thickness of the bonding layers, Fig. 18 shows the normalized output voltage as a function of \( d/a \) for \( q = 0.3928 \), \( q' = 0.0083 \) and \( \nu = 0.05 \). The output voltage is normalized by the corresponding result for \( \nu' = 0.001 \) at \( d = 0 \). It is shown that the thickness of the bonding layer does show significant effect upon the normalized output voltage. It should be mentioned, however, when the thickness of the layer is small compared with the thickness of the sensor, \( d/a \) shows insignificant effect until the debonding approaches the tip of the sensor.
Figure 12: Normalized Output Voltage as a Function of Debonding Length ($q = 0.3928$, $q' = 0.0083$ and $v = 0.05$)

Figure 13: Amplitude of Strain Ratio ($q = 2.0$, $v = 0.05$ and $v' = 0$)
Figure 14: Amplitude of Strain Ratio ($q = 2.0$, $v = 0.05$, $q' = 0.0083$ and $v' = 0.01$)

Figure 15: Amplitude of Strain Ratio ($v = 0.05$ and $v' = 0$). Red lines: Undebonded cases ($d/a = 0$). Blue lines: Debonded Cases ($d/a = 0.6$)
Figure 16: Amplitude of Strain Ratio ($v = 0.05$, $q' = 0.0083$ and $v' = 0.01$) Red lines: Undebonded Cases ($d / a = 0$). Blue lines: debonded cases ($d / a = 0.6$)

Figure 17: Amplitude of Strain Ratio ($q = 10.0$, $q' = 0.0083$ and $v = 0.05$). Red lines: Undebonded Cases ($d / a = 0$). Blue Lines: Debonded Cases ($d / a = 0.6$)
4. CONCLUDING REMARKS
The focus of this paper is on the study of the effect of the material and geometric properties of the sensor and the bonding layer on the load transfer from the sensor to the host medium. A general analytical solution is provided to the coupled electromechanical behaviour of a piezoelectric sensor bonded to a host through an adhesive bonding layer when the loading frequency is so low that the typical wavelength of the incident wave is much longer than the length of the sensor. The validity of the present model has been demonstrated by application to specific examples and comparison with the corresponding results obtained from Finite Element method. The numerical investigation of the influence of the geometry and the material mismatch of the adhesive layer upon the response of the coupled structure is provided. Both edge and central debonding and their effect upon the strain distribution in the composite structure are discussed.

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References
The Effect of Imperfect Bonding Layer on the Electromechanical Behaviour of Surface-Bonded Piezoelectric Sensors

