Supplementary Material for “Asymmetric Complementary Interface for Directional Adhesion”

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S1  Schematics of the interface as crack propagates in different directions under different loading conditions

Figure S1: Schematics of the interface under pulling along (a) direction 1 and (b) direction 2. The solid line represents the cracked surfaces while the dash line represents the uncracked surfaces. The pattern is not drawn to scale.

Figure S2: Schematics of the interface under symmetric peeling along (a) direction 1 and (b) direction 2. The pattern is not drawn to scale.
Figure S3: Schematics of the interface under asymmetric peeling along (a) direction 1, (b) direction 2 (c) direction 3, (d) direction 4. The pattern is not drawn to scale.
S2 Supporting results for symmetric peeling

Besides the qualitative analysis on directional adhesion in Section 3.2, the stress fields ahead of the crack tips at $B_1^+$ and $B_2^+$ were also investigated under the symmetric peeling condition. Figure S4(a) and (b) respectively show two regions in which the crack tips are at $B_1^+$ and $B_2^+$ in the first period of the pattern. The two boxed regions in Figure S4(a) and (b) are enlarged and shown in Figure S4(c) and (d). These two regions are referred to as region $B_1$ and region $B_2$, respectively. The stress field is extracted at the integration points in two elements $\alpha$ and $\beta$ ahead of the crack tip, and then transformed according to the direction of the future crack surface, shown in Figure S4(e) and (f). $1-2$ and $1'-2'$ are the original and transformed coordinate systems with $1'$-axis coincident with the future crack surface. The angle between $1-2$ and $1'-2'$ systems in regions $B_1$ and $B_2$ are -30° and 90°, respectively. Thus, the stress transformations in the two regions are given by:

In region $B_1$:

\[
\sigma_{2'2'} = \frac{1}{4} \sigma_{11} + \frac{3}{4} \sigma_{22} + \frac{\sqrt{3}}{2} \sigma_{12} \\
\sigma_{1'2'} = -\frac{\sqrt{3}}{4} \sigma_{11} + \frac{\sqrt{3}}{4} \sigma_{22} + \frac{1}{2} \sigma_{12}
\]

(S1)

In region $B_2$:

\[
\sigma_{2'2'} = \sigma_{11} \\
\sigma_{1'2'} = -\sigma_{12}
\]

(S2)

The transformed stress fields in $\alpha$ and $\beta$ are averaged to obtain a representative stress field (denoted by superscript $\text{ave}$) ahead of the crack tip,
Figure S4: Illustration of stress analysis ahead of the crack tip, (a)(c)(e) for symmetric peeling along direction 1, and (b)(d)(f) for symmetric peeling along direction 2.

given in Table S1. Although PDMS simulated here is hyperelastic instead of linearly elastic, an analogy can be drawn to an equivalent elastic problem, where among all the stress components ahead of the crack tip, $\sigma_{ave}^{1/2/2'}$ and $\sigma_{ave}^{1'}$ contribute to Mode I and II stress intensity factors, respectively, while $\sigma_{ave}^{1/1'}$...
makes no contribution. As can be seen, $\sigma_{2'2'}^{ave}$ and $\sigma_{1'2'}^{ave}$ are both greater in region $B_1$ than in region $B_2$, implying greater $R_{min}$ values in the vicinity of kink $B_1$ than in the vicinity of kink $B_2$.

Table S1: Stress components in elements $\alpha$ and $\beta$ in 1–2 and 1′–2′ coordinate systems in regions $B_1$ and $B_2$.

<table>
<thead>
<tr>
<th>Stress ($\times 10^{-3}MPa$)</th>
<th>$\alpha_{B_1}$</th>
<th>$\beta_{B_1}$</th>
<th>$\alpha_{B_2}$</th>
<th>$\beta_{B_2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{11}$</td>
<td>18.19</td>
<td>6.43</td>
<td>5.40</td>
<td>2.17</td>
</tr>
<tr>
<td>$\sigma_{22}$</td>
<td>12.35</td>
<td>17.48</td>
<td>14.28</td>
<td>12.35</td>
</tr>
<tr>
<td>$\sigma_{12}$</td>
<td>-3.35</td>
<td>-9.10</td>
<td>4.09</td>
<td>-4.43</td>
</tr>
<tr>
<td>$\sigma_{1'1'}$</td>
<td>19.63</td>
<td>17.08</td>
<td>14.28</td>
<td>12.35</td>
</tr>
<tr>
<td>$\sigma_{2'2'}$</td>
<td>10.91</td>
<td>6.83</td>
<td>5.40</td>
<td>2.17</td>
</tr>
<tr>
<td>$\sigma_{1'2'}$</td>
<td>0.85</td>
<td>-9.34</td>
<td>-4.09</td>
<td>4.43</td>
</tr>
<tr>
<td>$\sigma_{1'2'}^{ave}$</td>
<td>18.36</td>
<td>13.32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{1'1'}^{ave}$</td>
<td>18.36</td>
<td>13.32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{2'2'}^{ave}$</td>
<td>8.85</td>
<td>3.79</td>
<td></td>
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</tr>
<tr>
<td>$\sigma_{1'2'}^{ave}$</td>
<td>-4.25</td>
<td>0.17</td>
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S3  Supporting results for asymmetric peeling

This section presents supporting results for asymmetric peeling. Figure S5(a) and (b) shows the schematics of the interface for peeling directions 1 and 2. Figure S5(c) and (d) show how $R$ changes with the normalized apparent crack tip location $x/\lambda_x$. Figure S5(e) and (f) show enlarged regions in (c) and (d). The corresponding results for peeling directions 3 and 4 are shown in the left and right panels of Figure S6, respectively. Upon examining the figures, one similarity among the results from the four peeling directions is the discontinuities in the $R$ curve at “kink” where the crack propagation changes direction. On the other hand, more differences can be observed. Firstly, $R$ curves for peeling directions 1 and 4 show periodic patterns, i.e., the range of variation in $R$ is approximately the same for different periods. But this does not hold for peeling directions 2 and 3. For these two directions, there is an ascending trend in the $R$ curves indicated by the blue arrows in Figure S5(d) and Figure S6(c).

To explain this observation, Figure S7(a) shows the schematic of the interface when the crack tip accesses $2 < x/\lambda_x < 3$ in asymmetric peeling direction 2. Figure S7(b) enlarges the boxed region in Figure S7(a). Superscripts $u$ and $l$ are used to distinguish the “kinks” on the upper and lower surfaces. Figure S7(c) shows the same enlarged region but is an image from ABAQUS, with the $\sigma_{11}$ distribution. Clearly, the upper part is bent and the compressive contact between corner $C_2^u$ and surface $B_2'C_2'$ results in stress
concentration in the highlighted region in Figure S7(b), indicated by the negative $\sigma_{11}$ in the same region in Figure S7(c). This leads to a considerable amount of strain energy stored in the model. As the crack further advances, the compressive contact between $C^u_2$ and $B^l_2C^u_2$ becomes weaker, and the stored strain energy is released, giving rise to the ascending trend in the $R$ curve. Likewise, in the case of peeling direction 3 as illustrated in Figure S8, the contact between $B^l_3$ and $A^u_3B^u_3$ gives rise to the ascending trend in the $R$ curve. The ascending trend was not observed in cases of peeling directions 1 and 4 because under those loading conditions the vertical surfaces separate and there are no compressive contacts between them causing additional stored strain energy.
Figure S5: Energy release rate ratio $R$ for two different asymmetric peeling directions, left panel: peeling direction 1; right panel: peeling direction 2. (a) and (b) are schematics of the interface. (c) and (d) are $R$ versus the normalized apparent crack tip location $x/\lambda_x$. (e) and (f) are $R$ versus the normalized actual crack tip location $a/\lambda_a$, for the boxed regions in (c) and (d) respectively.
Figure S6: Energy release rate ratio $R$ for two different asymmetric peeling directions, left panel: peeling direction 3; right panel: peeling direction 4. (a) and (b) are schematics of the interface. (c) and (d) are $R$ versus the normalized apparent crack tip location $x/\lambda_x$. (e) and (f) are $R$ versus the normalized actual crack tip location $a/\lambda_a$, for the boxed regions in (c) and (d) respectively.
Figure S7: (a) Schematics of the interface in asymmetric peeling direction 2 as the crack tip accesses $2 < x/\lambda_x < 3$ (b) enlarged view of the boxed region in (a). (c) the same enlarged region from ABAQUS with the $\sigma_{11}$ stress distribution.
Figure S8: (a) Schematic of the interface in asymmetric peeling direction 3 as the crack tip accesses $1 < x/\lambda_x < 2$. (b) enlarged view of the boxed region in (a). (c) the same enlarged region from ABAQUS with the $\sigma_{11}$ stress distribution.
S4  Additional discussion on the effect of $H/l$ and $\theta$

This section presents additional analyses on the effect of $H/l$ and $\theta$. To understand the effect of $H/l$ shown in Figure 12(a) in the main text, the model is partitioned into “bulk” and “interface” regions, as shown in Figure S9. Here, the interface region ranges from $\frac{l}{4}$ above the top of the teeth to $\frac{l}{4}$ below the bottom of the teeth (see Figure 1(c) for the definition of $l$).

Following the definition in Eqn.(1), $G_{\text{pattern}}$ can now be written as:

$$G_{\text{pattern}} = \frac{U_{\text{bulk}}(a) + U_{\text{intf}}(a) - U_{\text{bulk}}(a + \Delta a) - U_{\text{intf}}(a + \Delta a)}{\Delta a}$$

$$= \frac{U_{\text{bulk}}(a) - U_{\text{bulk}}(a + \Delta a)}{G_{\text{bulk}}} + \frac{U_{\text{intf}}(a) - U_{\text{intf}}(a + \Delta a)}{G_{\text{intf}}}$$

(S3)

where the subscripts “bulk” and “intf” respectively stand for the bulk and the interface. Although the partition of the model is somewhat arbitrary, $G_{\text{intf}}$ and $G_{\text{bulk}}$ roughly capture the relaxation of the materials, upon crack propagation, near the interface and away from the interface. Both are affected by the patterns on the interface and the overall geometry of the model.

Then the local minima of the energy release rate ratio $R_{\text{min}}$ can be written as:

$$R_{\text{min}} = R_{\text{min,bulk}} + R_{\text{min,intf}}$$

(S4)
where \( R_{\text{min,bulk}} = \frac{G_{\text{bulk}}}{G_{\text{flat}}} \) and \( R_{\text{min, intf}} = \frac{G_{\text{intf}}}{G_{\text{flat}}} \) at \( x/\lambda_x \) where \( R_{\text{min}} \) occurs.

Figure S9: Schematic of bulk and interface partition

Figure S10(a) and (b) respectively show \( R_{\text{min, intf}} \) and \( R_{\text{min, bulk}} \) for the two peeling directions. Along peeling direction 1, as \( H/l \) increases \( R_{\text{min, bulk}} \) increases while \( R_{\text{min, intf}} \) decreases. This is due to the increase in the volume fraction of the bulk so that \( G_{\text{bulk}}/G_{\text{flat}} \) increases and \( G_{\text{intf}}/G_{\text{flat}} \) decreases. It is also clear that the changes in \( R_{\text{min, bulk}} \) and \( R_{\text{min, intf}} \) are of similar magnitude, which together with the opposite trend leads to the insensitivity of \( F \) to \( H/l \) along direction 1 (Figure 12(a) in the main text).

Along peeling direction 2, as \( H/l \) increases again \( R_{\text{min, bulk}} \) increases while \( R_{\text{min, intf}} \) decreases. However, different from direction 1 the change is small for \( R_{\text{min, intf}} \) but larger for \( R_{\text{min, bulk}} \). This results in \( R_{\text{min}} \) following the same trend as that of the bulk. Also, \( R_{\text{min, intf}} \) along direction 2 at each location is negative, i.e., \( G_{\text{intf}} < 0 \), indicating more stored energy as crack propagates. This is due to the large deformation that takes place at the teeth when
the crack tip is near $B$, which is found insignificant along direction 1 but
significant along direction 2 (comparing Figures 9 and 10 in Section 3.2 of
the main text). The negative value of $R_{\text{min,intf}}$ contributes to the small
magnitude of $R_{\text{min}}$ and hence more sensitivity of $F(=\frac{1}{R_{\text{min}}})$ to $H/l$.

To understand the observation in Figure 12(b), three auxiliary simulations
are performed to investigate the deformation of the upper and lower surfaces
when the crack reaches $B^+$ under three loading conditions: peeling only the
top surface, only the bottom surface, or both. The deformed $S_{ru}^d$ and $S_{rd}^d$
are shown in Figure S11 for peeling direction 1, along with their undeformed
shapes for comparison. Similarly, the deformed $S_{vd}^u$ and $S_{vd}^d$ are shown in
Figure S12 for peeling direction 2, along with the undeformed shapes. Figure
S13 shows $G_{\text{pattern}}$ evaluated at $B_1^+$ and $B_2^+$ under symmetric peeling, plotted
against the four $\theta$ values.

As discussed in Section 3.2, when the crack tip is at $B_1^+$, peeling either the
upper or lower part contributes to crack propagation and the contribution
from peeling the upper part is greater. Upon examining Figure S11, the
crack opening before the crack tip decreases monotonically with increasing
$\theta$, whether the peeling is applied only on the top edge, only on the bottom
dge, or both edges. This is consistent with the trend of $G_{\text{pattern}}$ represented
by the dashed cyan line in Figure S13.

Also discussed in Section 3.2, when the crack tip is at $B_2^+$, peeling the
lower part contributes to crack propagation whereas peeling the upper part
contributes to crack-trapping. This is confirmed by examining Figure S12
Figure S10: $R_{\text{min,\,bulk}}$ and $R_{\text{min,\,intf}}$ plotted against the apparent crack tip location for peeling directions (a) 1 and (b) 2.

where peeling only the upper part causes $S_{vd}^l$ to penetrate into $S_{vd}^u$, and peeling only the lower part causes $S_{vd}^l$ and $S_{vd}^u$ to separate. In addition, in Figure S12(b) the crack opening decreases considerably with increasing $\theta$. The dom-
inant role of peeling the lower part can be seen from Figure S12(c) where the crack opening under symmetric peeling also decreases monotonically with increasing $\theta$, which is consistent with the trend of $G_{\text{pattern}}$ represented by the dashed blue line in Figure S13. Furthermore, the crack trapping effect caused by peeling the upper part contributes to the small magnitude of $G_{\text{pattern}}$ along direction 2 and hence more sensitivity of $F$ to $\theta$.

Figure S11: Deformations of $S_{\text{ur}d}$ and $S_{\text{lr}d}$ with their undeformed shapes for peeling (a) only the upper part, (b) only the lower part and (c) both parts, along direction 1.
Figure S12: Deformations of $S_{vd}^u$ and $S_{vd}^l$ with their undeformed shapes for peeling (a) only the upper part, (b) only the lower part and (c) both parts, along direction 2.

S5 Results under plane strain

All of our FE simulations presented in the main texts are performed by assuming plane stress. A separate FE simulation is performed with the same geometry and loading conditions as in Section 3.2 but using the plane strain assumption. The results are shown in Figure S14, along with those using the plane stress assumption for comparison. The results are similar both qualitatively and quantitatively, indicting the insensitivity to the choice of...
Figure S13: $G_{\text{pattern}}$ at $B_1^+$ and $B_2^+$ under symmetric peeling in directions 1 and 2 respectively, plotted against $\theta$.

plane stress or plane strain assumptions.
Figure S14: Adhesion enhancement factor $F$ for symmetric peeling with plane stress and strain assumptions. The geometry and loading conditions of the model are the same as in Section 3.2.