Revelation mechanisms and allocative efficiency

Florenz Plassmann*

Department of Economics, State University of New York at Binghamton
Binghamton, NY 13902-6000
fplass@binghamton.edu
Phone: 607-777-4934; Fax: 607-777-4900

Nicolaus Tideman

Department of Economics, Virginia Polytechnic Institute and State University
Blacksburg, VA 24061
ntideman@vt.edu
Phone: 540-231-7592; Fax: 540-231-9288

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Abstract

We show that revelation mechanisms affect owners’ valuations of the assets they own and that different mechanisms provide owners with the incentive to reveal—truthfully—different valuations. Self-assessment of property with compulsory sale at the self-assessed price is the only known mechanism that promotes allocative efficiency by providing efficient incentives for transferring property to those who value it most. We introduce two modifications of the standard self-assessment mechanism that maintain full incentives to invest and raise as much public revenue as can be raised efficiently.

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1. INTRODUCTION

Revelation mechanisms provide an incentive for owners to reveal their subjective monetary valuations of their assets. In assuming that an owner’s valuation is determined exogenously, the literature has not paid attention to the fact that the owner’s valuation, and hence the amount she discloses, is contingent on the type of mechanism used. By modeling explicitly how owners adjust their valuations in response to changes in taxation and possibilities of sale, we show that different revelation mechanisms provide owners with the incentive to reveal—truthfully—different subjective valuations.

One insight from our work is relevant for resolving the allocative inefficiency that results when owners of assets without perfect substitutes expect, with unreasonably high probability, to receive future offers above the discounted value of the flow of benefits from ownership that accrue to them. These owners impair allocative efficiency when they refuse offers above the value to themselves, because they thereby withhold the asset from a person who would receive greater benefits from owning the asset than they do. This inefficiency can arise for all privately held assets without perfect substitutes, but it tends to be most visible when owners hold out for higher offers from developers who seek to assemble multiple properties.

Previous analyses of revelation mechanisms have assumed that owners’ valuations are determined exogenously. We show that the mechanism proposed by Becker et al. (1964), henceforth the BDM mechanism, provides owners with an incentive to reveal valuations that incorporate owners’ expectations regarding future sales, thus failing to resolve the allocative inefficiency that can arise from private ownership. In contrast, with appropriately calibrated taxes, the self-assessment mechanism that is generally attributed to Harberger (1965) provides owners with an incentive to reveal how they would value an asset without regard to their expectations about future sales. We show that, because different tax structures affect the flow of benefits in different ways, the version of self-assessment proposed by Plassmann and Tideman (2008), which incorporates a variable marginal tax rate, leads to a different flow of net benefits—and hence provides owners with an incentive to reveal a different net valuation—than the version proposed by Tideman (1969) and Weyl and Zhang (2016, 2018) that has a constant marginal tax rate. Although, with the right tax rates, both versions of self-assessment motivate owners to reveal their valuations without the markup that reflects their expectations regarding future sales, the absence of a markup does not always lead to allocative efficiency. We show
that, nevertheless, as long as an asset’s value to potential buyers does not depend on who owns the asset, self-assessment at the appropriate tax rate leads to allocative efficiency.

Previous analyses of self-assessment have considered self-assessed taxes that are levied on the entire amount that owners disclose. To alleviate the problem that such taxes distort the owners’ investment decisions, Weyl and Zhang (2016, 2018) propose a self-assessed tax with a low constant rate that balances improvements in allocative efficiency with distortions in investment. We show that exempting new investment from the self-assessed tax provides incentives for efficient investment while maintaining the incentive for truthful revelation of value. But because owners assign higher values to assets that are partially tax-exempt, self-assessment with exemptions leads to an equilibrium with different prices than self-assessment without exemptions. It is straightforward to combine such a self-assessed tax with a separate tax on rent that raises the maximum amount of tax revenue that can be raised efficiently. We show that, because owners assign lower values to assets whose rent is subject to a separate tax, the equilibrium attained under self-assessment with a separate tax on rent differs from the equilibrium that prevails when rent is not taxed separately.

The remainder of the paper is organized as follows. In Section 2, we establish the owner’s valuations under different expectations regarding future sales and future taxes. In Section 3, we show that an appropriately calibrated self-assessed tax provides an owner with an incentive to reveal her valuation as if she intended to keep the asset forever, and that different tax rules lead to different valuations. In Section 4, we emphasize various characteristics of self-assessment, identify a sufficient condition for self-assessment to lead to allocative efficiency, and show that the BDM mechanism does not achieve allocative efficiency. In Sections 5 and 6, we present two extensions of self-assessment that accommodate exemptions for investment and a separate tax on rent. In Section 7, we examine how our earlier results relate to self-assessment with a constant tax rate. Section 8 concludes.

2. THE OWNER’S RESERVATION PRICE

Consider an asset that offers a constant rate of benefits to its owner. These benefits include all monetary benefits that the owner receives in form of rent and interest payments, as well as any subjective benefits that might result from ownership and possession of the asset. Let \( B \) denote
the monetary equivalent of the benefit rate in every future time period. For expositional simplicity, we assume that owners behave as if their lives continue forever and that they expect $B$ to remain unchanged over time, which permits us to derive simple closed-form solutions. In Section 4.3, we argue that our key insights carry over to more general models.

2.1 Valuing assets that owners expect to keep forever
Consider first an owner who expects to own the asset forever. Such an owner values the asset at the present discounted value of the asset’s future benefit stream, or

$$X_{\text{Keep}}^* = \sum_{t=1}^{\infty} \frac{B}{(1+\gamma)^t} = \frac{B}{\gamma},$$  \hspace{1cm} (1)

where $\gamma$ is the owner’s discount rate. The owner would be adequately compensated by $X_{\text{Keep}}^*$ if she were to lose control over the asset, for example, because someone steals or destroys it, although the owner considers such an event—without compensation—to be sufficiently unlikely that she assigns a probability of zero to it. If the owner expects full compensation for such an event at no cost to herself, then the possibility of the event has no effect on her valuation of the asset.

2.2 Valuing assets that owners expect to lose
Consider next an owner who believes that during any future time period after the first, with probability $p$ the asset will cease to provide any benefits to her, possibly because she loses the asset or someone steals it, and that she will receive no compensation for her loss. This owner values the asset at the present discounted value of the asset’s expected future benefit stream, or

$$X_{\text{Lose}}^* = \sum_{t=1}^{\infty} \frac{B}{(1+\gamma)^t} (1-p)^{t-1} = \sum_{t=1}^{\infty} \frac{B}{1-p} \left(\frac{1-p}{1+\gamma}\right)^t = \frac{B}{\gamma+p},$$  \hspace{1cm} (2)

where the term $(1-p)^{t-1}$ represents the discount factor that accommodates the possibility that the asset ceases to provide benefits after period $t$. Equation 2 indicates that an asset that might cease to provide benefits is less valuable to its owner than an asset whose benefit stream the owner expects to continue forever.
2.3 Valuing assets that owners expect to sell

Now consider a situation in which there is no possibility of an uncompensated loss, but there is a possibility that the owner’s benefit stream ceases because she sells the asset. Define $p_S(X)$ as the owner’s belief of the probability of receiving an offer to purchase the asset at a price of $X$ or more within one unit of time, and let $\pi_S(X)$ be $-\frac{dp_S}{dx}$\(^1\). For notational ease, we suppress the argument $X$ and write $p_S$ and $\pi_S$ rather than $p_S(X)$ and $\pi_S(X)$ when there is no risk of misunderstanding. We assume that $p_S$ is time-invariant, so that our framework describes situations with large numbers of potential buyers, in which $p_S$ and $\pi_S$ do not change when the owner receives and decides to reject offers, and in which potential buyers do not revise their offers after observing earlier offers being rejected.\(^2\)

Let $X_{Sell}$ be the lowest offer that the owner will accept, so that the expected proceeds of sale per time period are $E = \int_{X_{Sell}}^{\infty} D\pi_S(D) \, dD$, with $\frac{dE}{dx} = -X_{Sell}\pi_S(X_{Sell})$. The possibility of selling the asset at a price of $X_{Sell}$ or higher raises the expected benefit rate from owning the asset to $B + E$, until it is sold, leading to a present discounted value of expected returns of

$$V = \sum_{t=1}^{\infty} \frac{B+E}{1-p_S} \left( \frac{1-p_S}{y+p_S} \right)^t = \frac{B+E}{y+p_S} = \frac{B}{y} + \frac{E-p_S}{y+p_S}.$$ \hspace{1cm} (3)

The expression $E - p_S \frac{B}{y}$ denotes the expected net gain per period from the possibility of selling the asset. Because the owner will not accept an offer below $X_{Keep}^* = \frac{B}{y}$, the term $E - p_S \frac{B}{y}$ must be non-negative.

The lowest offer that the owner will accept, $X_{Sell}^*$, solves her first-order condition

$$\frac{dV}{dX_{Sell}} = \frac{(-X_{Sell}\pi_S(y+p_S) - (B+E)(-\pi_S))}{(y+p_S)^2} = 0$$

\(^1\) Note that $\frac{dp_S}{dx} < 0$, so that $\pi_S(X) = -\frac{dp_S}{dx} > 0$ describes the rate at which the probability of an offer increases as $X$ decreases. Because we are considering the probability of an offer within a unit of time and not the rate of offers per unit of time, $p_S$ has an upper bound of 1. Because of the possibility of receiving more than one offer, $\pi_S(X)$ is not quite the same as the rate of offers per dollar at $X$.

\(^2\) One might formalize this assumption through a Poisson arrival process, in which the arrival of a potential buyer does not affect the expected probability of the arrival of another potential buyer, but our argument does not require such formalization.
Equations 3 and 4 indicate that an asset that the owner might sell at a price above \( X_{Keep}^* \) is a composite of the discounted benefit stream of owning the asset forever, \( X_{Keep}^* \), and the value of a lottery with expected net payoff \( E - p_S \frac{B}{Y} \) per period that continues until it pays off.

Allocative efficiency requires that an asset change hands whenever the asset’s value to a potential buyer exceeds the asset’s value to its current owner. If the asset’s market is perfectly competitive, then the marginal asset conveys the same discounted benefit \( \frac{B}{Y} \) to all persons, so that the equilibrium probability of a sale for any asking price above \( \frac{B}{Y} \) is zero. Because the expected proceeds \( E \) from selling the asset under perfect competition are \( p_S \frac{B}{Y} \), the lottery for selling the asset above \( \frac{B}{Y} \) has no value in this case, and all equilibrium trades occur at \( X_{Sell}^* = X_{Keep}^* \).

Because the possibility of selling the asset for an amount above \( X_{Keep}^* \) at some future point makes the asset more valuable to the owner, the owner’s refusal to sell the asset at an amount between \( X_{Keep}^* \) and \( X_{Sell}^* \) does not impair allocative efficiency, as long as the owner has realistic expectations of the probability \( p_S( X_{Sell}^* ) \) of receiving offers above \( X_{Sell}^* \). A refusal to sell impairs allocative efficiency when owners who overestimate \( p_S \) set inflated asking prices. The literature that emerged in the wake of Myerson and Satterthwaite (1983) has investigated the inefficiency in bargaining that may result from asymmetric information. In a one-shot game with a single buyer and a single seller who seek to conceal their true reservation prices, an efficient exchange might not occur because the seller’s inflated asking price exceeds the buyer’s deflated offer. Athey and Miller (2007) and Athey and Segal (2007, 2013) show that the inefficiency resulting from asymmetric information may persist with repeated bilateral trade.

The holdout problem that occurs in situations of land assembly arises from a similar inflated belief regarding the seller’s bargaining power. When a developer seeks to assemble a set of parcels, each owner has an incentive to demand a price that captures the developer’s entire surplus after acquiring the other parcels. If the total price that the owners demand for the set of parcels exceeds the developer’s willingness to pay even though the sum of the amounts \( X_{Sell}^* \) that reflect the benefits that the owners receive from owning their parcels, including the realistic gain
from future offers, is below the developer’s willingness to pay, then the owners’ inflated belief regarding their bargaining power impairs allocative efficiency.

Thus, inflated markups impair allocative efficiency if they lead owners to reject offers between the uninflated and the inflated markup. One possibility of restoring allocative efficiency, if it could be done, would be to educate owners about their true bargaining power. A feasible alternative is to provide owners with the incentive to set no markup, so that \( X_{Sel}^* = X_{Keep}^* \). In Section 3, we show that taxes that are functions of self-assessed valuations can provide owners with an incentive to self-assess their assets without markup, leading to an efficient allocation of assets.

2.4 Valuing assets that are taxed

Assets on which owners must pay taxes are not as valuable as assets not encumbered with taxes.\(^3\) Consider an asset whose owner must pay a recurring lump-sum property tax at a rate of \( T_L \) dollars per unit of time. For an owner who expects to keep the asset forever, the lump-sum stream of tax payments lowers the rate of return from owning the asset to \( B - T_L \), yielding a reservation price of

\[
X_{Keep, T_L}^* = \sum_{t=1}^{\infty} \frac{B - T_L}{(1 + \gamma)^t} = \frac{B}{\gamma} - \frac{T_L}{\gamma} = X_{Keep}^* - \frac{T_L}{\gamma}. \tag{5}
\]

The difference between \( X_{Keep}^* \) and \( X_{Keep, T_L}^* \) is the present discounted value of all future tax payments, providing an example of how an owner’s valuation is contingent on the circumstances that affect the asset’s stream of net benefits.

For owners who intend to sell their assets, the requirement that whoever owns the assets pay a stream of lump sum taxes affects the assets’ rates of return as well as the probability that the assets will be sold at specified prices. Let \( p_{T_L} \) denote the probability per unit of time that the owner expects to receive an offer to buy the taxed asset at a price of \( X \) or more, with \( \pi_{T_L}(X) = - \frac{d p_{T_L}}{dX} \) and expected proceeds \( E_{T_L} = \int_X^{\infty} D \pi_{T_L}(D) \, dD \). The stream of taxes reduces the

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\(^3\) See the literature on the capitalization of property taxes that followed Oates (1969).
expected rate of benefits from owning the asset to \( B + E_{T_L} - T_L \), and reduces the smallest offer
\[ X_{\text{Sell},T_L}^* \geq X_{\text{Keep},T_L}^* \]
that the owner will accept to
\[ X_{\text{Sell},T_L}^* = \frac{B}{\gamma} - \frac{T_L}{\gamma} + \frac{E_T - p_{T_L}B}{\gamma + p_{T_L}} = X_{\text{Keep},T_L}^* + \frac{E_T - p_{T_L}(B - T_L)}{\gamma + p_{T_L}}. \]

Depending on how the lump-sum tax stream affects the frequency and distribution of offers, the tax might lead to the same, a higher, or a lower markup, compared to the case without the tax.

3. ELIMINATING THE MARKUP THROUGH SELF-ASSESSED TAXES

Consider the requirements that (1) the owner announce, at any time \( t \), a price \( X \) at which she must accept any offer to buy the asset, and (2) she pay a self-assessed \( T_S \) per unit of time on the announced price. Let \( p_{T_S} \) be the owner’s expected probability of selling the asset within one period of time when offered at \( X \), under self-assessed tax \( T_S \). The expected proceeds from a sale are therefore \( E_{T_S} = p_{T_S} X \), and the owner maximizes the present discounted value of the expected returns
\[ V = \sum_{t=1}^{\infty} \frac{B + p_{T_S}X - T_S}{1 - p_{T_S}} \left( \frac{1 - p_{T_S}}{1 + \gamma} \right)^t = \frac{B + p_{T_S}X - T_S}{\gamma + p_{T_S}} = \frac{B}{\gamma} - \frac{T_S}{\gamma} + \frac{p_{T_S}(X - \frac{B}{\gamma} \frac{T_S}{\gamma})}{\gamma + p_{T_S}} \]
by announcing the price \( X_{T_S}^* \) that solves her first-order condition
\[ dV_{T_S} \quad \text{dx} = \left( p_{T_S} + \frac{dp_{T_S}}{dx} \right) \left( \gamma + p_{T_S} \right) - (B + p_{T_S}X - T_S) \left( \frac{dp_{T_S}}{dx} \right) = 0. \]

\[ \iff X_{T_S}^* = \frac{B}{\gamma} - \frac{T_S}{\gamma} + \frac{(\frac{dT_S}{dx} - p_{T_S}) \left( \gamma + p_{T_S} \right)}{\frac{dp_{T_S}}{dx} \gamma}. \]

Inspection of equations 7 and 8 proves

**Theorem 1:** If owners receive the self-assessed amount \( X_{T_S}^* \) in the event of a sale and if the marginal self-assessed tax rate (the percent of the marginal dollar of announced value that is
paid in tax per unit of time) equals the probability per unit of time of selling the asset at $X_{TS}^*$, \\
\text{or } \frac{dT_S}{dX} = p_{TS} \bigg|_{X=X_{TS}^*}, \text{ then owners have an incentive to reveal} \\
(1) truthfully their valuations (see Tideman, 1969, Plassmann and Tideman, 2008)\\n(2) valuations without markup (see Weyl and Zhang, 2016, 2018).\\n
Previous analyses have ignored the fact that the owner’s valuations themselves depend on the structure of the self-assessed tax. We analyze the implications of two different tax structures. The first entails a marginal tax rate that equals the probability of a sale at each dollar amount up to the owner’s announced price, or $\frac{dT_S}{dX} = p_{TS} \forall X \leq X_{TS}^*$, as suggested by Plassmann and Tideman (2008). The second tax structure entails a constant tax rate $\frac{dT_S}{dX} = \tau$, as suggested by Tideman (1969) and Weyl and Zhang (2016, 2018).

3.1 A self-assessed marginal tax rate equal to the probability of a sale at each dollar amount up to the owner’s announced price

A self-assessed marginal tax rate that equals the probability of a sale at each dollar amount up to the owner’s announced price, $\frac{dT_S}{dX} = p_{TS} \forall X \leq X_{TS}^*$, implies the self-assessed tax schedule

$$T_{V0} = -A + \int_{0}^{X_{TS}^*} p_{TV_0}(X) dX,$$

(9)

where $A$ is an adjustment term that does not depend on $X$, and $p_{TV_0}(X)$ is the expected probability of selling the asset at $X$, given the self-assessed tax $T_{V0}$. The subscript $V0$ indicates that the marginal tax rate ‘v’aries with $X$ and that the tax is levied on the asset’s entire value, starting at zero. Note that $p_{TV_0}(X)$ incorporates offers from potential buyers who value the asset above $X$, even though they have to pay only $X$ to the owner. Using $T_{V0}$ in the owner’s first-order condition 8 yields the owner’s optimal announcement

$$X_{TV_0}^* = X_{Kee}^* - \frac{T_{V0}}{\gamma},$$

(10)

that is, the present value of the difference between the benefit per period from owning the asset and her continuous tax payment, if the owner intends to keep the asset forever.
Note that the self-assessed tax $T_{v0}$ does not provide the owner with an incentive to announce either $X_{Keep,T_L}^*$ or $X_{Keep}^*$. This result is not surprising, because an asset that is taxed at a rate that varies with the probability of a sale conveys a different stream of monetary benefits than an asset on which the owner pays either a lump-sum tax or no tax. Nevertheless, after accounting for the difference in burdens between self-assessed tax $T_{v0}$ and lump-sum tax $T$, the relationship between $X_{T_{v0}}^*$ and $X_{Keep}^*$ is the same as the relationship between $X_{Keep,T_L}^*$ and $X_{Keep}^*$. Thus, the self-assessed tax $T_{v0}$ eliminates the markup that arises from the expectation of selling the asset. We address the conditions under which $T_{v0}$ achieves allocative efficiency in Section 4.4.

3.2 A constant tax rate

If estimating and administering separate schedules $T_{v0}$ for owners of different assets is too cumbersome or impossible, then the government might set a common tax rate $\tau$, leading to the tax structure $T_{c0} = \tau X$. The subscript $C0$ indicates that the tax rate $\tau$ is ‘c’onstant and that it is levied on the entire self-assessed value, starting at zero. Given the probability $p_{T_{c0}}$ of selling the asset at the owner’s self-assessed amount under the self-assessed tax $T_{c0}$, the owner’s solution to first-order condition 8 is

$$X_{T_{c0}}^* = \frac{B}{y + \tau} + \frac{(\tau - p_{T_{c0}})(y + p_{T_{c0}})}{\frac{dp_{T_{c0}}}{dx}(y + \tau)},$$

(11)

where the first term represents the present value of the benefit that the owner receives from owning forever an asset that is taxed at a constant rate $\tau$, and the second term represents either a markup (if $\tau < p_{T_{c0}}$) or a discount (if $\tau > p_{T_{c0}}$).\(^4\)

When the constant tax rate happens to equal the probability of a sale at the owner’s reservation price, or $\tau = p_{T_{c0}}\big|_{X = X_{T_{c0}}^*}$, then equation 11 simplifies to

$$X_{T_{c0}}^* = \frac{B}{y + p_{T_{c0}}}.$$

(12)

\(^4\) Recall that $\frac{dp_{T_{c0}}}{dx} < 0$. 

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Thus, the owner announces the present value of the monetary benefits that she expects to receive from the asset, given the requirement that she sell the asset at the announced value and her expectation that such a sale occurs with probability \( p_{T_{C_0}} = \tau \).

Note that \( X_{T_{C_0}}^* \) is generally not equal to \( X_{T_{V_0}}^* \) even if \( \tau = p_{T_{C_0}} \big|_{X=X_{T_{C_0}}^*} \), because assets that are taxed at constant rates convey different streams of monetary benefits to their owners than assets that are taxed at variable marginal rates. Nevertheless, if \( \tau = p_{T_{C_0}} \big|_{X=X_{T_{C_0}}^*} \), then the self-assessed tax \( T_{C_0} \) also eliminates any markup that arises from the expectation of selling the asset—although the resulting equilibrium is the one that prevails under \( T_{C_0} \) rather than the one that prevails under \( T_{V_0} \).

4. DISCUSSION

4.1 Why does the self-assessed tax provide an incentive to the owner to assess the asset’s value without markup?

Equation 4 defines the owner’s markup \( \frac{E - p_{S_T} B}{Y + p_{S}} \) as the discounted difference between the expected proceeds from selling the asset and the expected lost benefit of keeping the asset forever. Equation 7 shows that a self-assessed tax at the appropriate rate eliminates the markup by lowering the expected proceeds from a sale so that they equal the expected benefit of keeping the asset forever. Assigning to owners the self-assessed amount \( X_{T_{S}}^* \) rather than the buyer’s offer \( D \) in the event of a sale prevents owners from benefitting from offers that exceed \( X_{T_{S}}^* \); an owner’s self-assessment decision therefore becomes equivalent to identifying how much insurance \( X_{T_{S}}^* \) she should buy at premium \( T_{S} \) against a loss whose probability of occurrence declines with the amount of insurance bought.\(^5\) Owners have an incentive to insure their assets at amounts equal to their loss under insurance, \( \frac{B}{Y} - T_{S} \), if the premium for the last dollar insured, \( \frac{dT_{S}}{dx} \), equals the expected probability of losing this dollar, \( p_{T_{S}} \). A marginal self-assessed tax (= premium) rate

\(^5\) Note that self-assessment is not equivalent to government-provided insurance, because the buyer rather than the government pays the insured amount \( X \) when the owner loses the asset.
\[
\frac{d\tau_s}{dx} = p_{\tau_s}\bigg|_{x=x^*_\tau_s}
\]
therefore provides owners with an incentive to insure their assets without markup.

4.2 Assessing the asset without markup requires that in the event of a sale, owners do not receive the amount offered by the buyer

A necessary condition for the owner to find it optimal to assess the asset without markup is that in the event of a sale, owners receive the self-assessed amount rather than the amount \(D\) that the buyer is willing to pay. To understand why, consider an alternative mechanism under which owners must self-assess their assets and pay a (not necessarily self-assessed) tax \(T\), but now the self-assessed amount is disclosed only to the government. The government then approves a sale when an offer \(\bar{D}\) (made to the government) exceeds the self-assessed amount, in which case the buyer receives the asset and the owner receives \(\bar{D}\), rather than the self-assessed amount.

Let \(p_T(X)\) be the probability of selling the asset within one period of time when offered at a price of \(X\), with \(\pi_T(X) = \frac{d p_T}{dx}\), and expected proceeds of \(E_T = \int_X^\infty D\pi_T(D)\ dD\). An owner who receives \(D\) rather than the self-assessed amount in the event of a sale has an expected benefit rate of \(B + E_T - T\), and she solves her first-order condition

\[
\frac{\partial E}{\partial x} \frac{d\tau}{dx} (\gamma + p_T) - (B - E_T - T) \frac{d p_T}{d x} = 0
\]

by announcing 6

\[
X^*_T = \frac{B}{\gamma} - \frac{T}{\gamma} + \frac{E_T - p_T (B/T) \gamma}{\gamma + p_T} + \frac{d T}{d x} \frac{1}{\gamma + p_T}.
\]

The owner has an incentive to announce neither a markup nor a discount if the last two terms in equation 14 sum to zero; this is achieved by the tax

\[
T = B - \frac{\gamma}{p_T} \left( \frac{d T}{d x} \frac{1}{\gamma + p_T} (\gamma + p_T) - E \right),
\]

which is a function of \(B\) and \(\gamma\), among other parameters. But the government cannot set such a tax because \(B\) and \(\gamma\) are the owner’s private information. This proves

\[6\] Recall that \(\frac{\partial E_T}{\partial x} = -X\pi_T(X)\) and \(\frac{d p_T}{d x} = -\pi_T(X)\).
Theorem 2: If owners receive the offered amount $D$ in the event of a sale, then the government lacks sufficient information to specify a tax that will provide owners with an incentive to self-assess the asset without markup.

Theorem 2 indicates that the BDM mechanism proposed by Becker et al. (1964) fails to provide owners with such an incentive. When adapted to the current framework, the BDM mechanism requires that the owner disclose the self-assessed amount only to the government and that the owner receive $D$ when the government approves a sale.\(^7\) However, the BDM mechanism sets no tax. For $T = 0$, equation 14 coincides with equation 4, indicating that the reservation price that the owner announces includes the full markup. Thus, while the BDM mechanism provides an incentive to the owner to reveal her reservation price, $X_{sell}^*$, it does not prevent the allocative inefficiency that can occur when the owner is overly optimistic regarding the distribution of future offers.

Equation 8 indicates that, if owners are to receive their announced values when they lose control of their assets, any revelation mechanism that provides owners with an incentive to self-assess their assets without markup must require owners to pay a tax rate of $\frac{dT_s}{dX} = p_{rs} \big|_{X = X_{rs}^*}$ on the marginal dollar that they announce, that is, it must be equivalent to efficient self-assessment at the margin. We are not aware of any revelation mechanism other than self-assessment that eliminates markups for privately owned assets whose values are known only to their owners. The mechanism proposed by Niou and Tan (1994) requires that governments be able to learn $X_{keep}^*$ by means other than asking the asset’s owner, for example, through an audit, albeit at a cost that the government prefers not to pay. But we know of no audit procedure that would provide information about $X_{keep}^*$. Mumy (1981) proposed a revelation mechanism that provides multiple owners with the incentive to reveal the values of their indistinguishable individual assets when only the joint value of these assets is known. As with Niou and Tan’s mechanism, Mumy’s mechanism requires that it be possible to determine the values of the assets objectively.\(^8\) Because owners are assumed to not assign any subjective value over and above the observable

\(^7\) The original BDM mechanism is a one-shot game. For this continuously repeated game, we assume that the frequency of offers at each price is time-invariant. Chambers and Lambert (2017) develop a dynamic application of the BDM mechanism that incorporates learning.

\(^8\) Mumy describes the mechanism in the context of a sea captain who must refund the contents of a money deposit box to his passengers after these contents have been mixed up during a storm.
market value to such indistinguishable units, the value of individual units is not in dispute, so that the existence of a markup does not pose any difficulties within Mumy’s framework.

4.3 Assessing assets without markup requires that owners update their self-assessments when their valuations change

To obtain simple closed-form solutions, we assume that owners expect the rate of benefit $B$ and the frequency of offers at each price to remain constant over time. The intuition offered in Sections 4.1 and 4.2 suggests that our results continue to hold if the rate of benefit as well as the probability of a sale vary over time. Adapting equations 1 and 4 to situations when benefits and offers may vary over time yields $X^*_\text{keep} = \sum_{t=1}^{\infty} \frac{B_t}{(1+y)^t}$ as the reservation price of an owner who intends to keep her assets forever, and, defining $p_1 = 0$,

$$X^*_\text{sell} = V = \sum_{t=1}^{\infty} \frac{(B_t + E_t) \prod_{r=1}^{t} (1-p_r)}{(1+y)^t} > X^*_\text{keep} \quad (16)$$

as the reservation price of an owner who is prepared to sell her asset, where $p_t$ is the probability of selling the asset at a price of at least $X_t$ at time $t$, and $E_t = \int_{X_t}^{\infty} D \pi_t(D) \, dD$ describes the expected proceeds of sale at time $t$, with $\pi_t(X) = -\frac{dp_t}{dX}$. Regardless of whether benefits and offers are constant or vary over time, a self-assessed tax that prevents owners from benefitting from offers that exceed the announced value makes the owner’s self-assessment decision equivalent to the decision of how much loss insurance to buy. Thus, an owner who expects to pay, at any time $t$, a premium that equals the probability of a loss at that time, $p_t$, has an incentive to buy insurance against her true loss, that is, self-assess her property without markup. Eliminating the markup therefore requires that owners be allowed to re-assess their assets whenever they adjust their expectations of $B_t$ or $\pi_t(X)$ for any $t$.

4.4 When does self-assessment lead to allocative efficiency?

Because the self-assessed tax is a function of the owner’s subjective benefit stream $B$, a potential buyer who values the asset higher than the owner pays a higher tax than the owner if he acquires the asset. Although one might think that this change in the tax with a change in the ownership would be an indication of allocative inefficiency, this is not generally the case.
Person-specific taxes might lead to allocative inefficiency when they reflect different selling opportunities for different owners. For example, consider a system of person-specific taxes that are contingent on ownership and that correspond to individual selling opportunities. Assume that an owner’s benefit from owning the asset is smaller than the corresponding benefit of a potential owner if he were to acquire the asset, but that the owner’s tax burden is smaller than the potential buyer’s burden, so that the owner’s after-tax benefit from owning the asset exceeds that of the potential buyer. Even though allocative efficiency requires that the asset be transferred to the potential buyer, the differential tax reduces the potential buyer’s highest offer below the owner’s after-tax benefit of keeping the asset, and neither bargaining nor voluntary elimination of the markup by a public-spirited owner can achieve allocative efficiency.

However, as long as the asset’s valuation by potential buyers does not depend on who owns the asset, a system of self-assessed, person-specific taxes that eliminate the markup also lead to allocative efficiency. To understand why, consider Figure 1, which shows probability schedule $p_{TV0} = \int_{X_{TV0}}^{\infty} \pi(D)dD$ as the solid curved line, that is, as a functional relationship between any self-assessed amount $X_{TV0}$ under tax schedule $TV0$ and the probability $p_{TV0}$ of a sale at the self-assessed amount. Equation 10 defines the owner’s announcement, $X_{TV0}^*$. Because the horizontal axis extends over the range $[0, 1]$, the owner’s valuation $X_{TV0}^*$ can also be expressed as the rectangle formed by the four points $0X_{TV0}^*a1$. The owner’s self-assessed tax is the horizontally striped area underneath schedule $p_{TV0}$ up to $X_{TV0}^*$. Equation 10 indicates that the owner’s benefit stream from owning the asset forever in the absence of a self-assessed tax, $X_{Keep}^*$, can be expressed as the sum of the owner’s announcement $X_{TV0}^*$ and the present value of all future taxes. Figure 1 shows $X_{Keep}^*$ as rectangle $0X_{Keep}^*b1$, and the present value of all future taxes as rectangle $X_{TV0}^*X_{Keep}^*ba$.

--- Figure 1 about here ---

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9 The figure represents a functional relationship in which the variable measured on the horizontal axis is a function of the variable measured on the vertical axis, in the same way as economists are accustomed to represent demand curves.

10 In Section 4.5, we address the concern that the self-assessed tax may be unacceptably large.
Consider a potential buyer whose benefit stream from owning the asset forever, $Y_{Keep}^\ast$, exceeds that of the owner, so that allocative efficiency requires transferring the asset to the potential buyer. Assume that the asset’s value to other potential buyers is independent of who owns the asset, so that probability schedule $p_{Tv_0}$ and tax schedule $Tv_0$ apply regardless of ownership. Let $Y_{Tv_0}^\ast$ be the potential buyer’s valuation under tax schedule $Tv_0$, as defined by equation 10. Figure 1 shows the potential buyer’s benefit stream from owning the asset forever, in the absence of a self-assessed tax, as rectangle $0Y_{Tv_0}^\ast d1$, and the potential buyer’s valuation under the self-assessed tax as rectangle $0Y_{Tv_0}^\ast c1$. If the potential buyer acquires the asset, then his self-assessed tax is the sum of the horizontally and vertically striped areas underneath schedule $p_{Tv_0}$ up to $Y_{Tv_0}^\ast$.

Figure 1 shows that, if $p_{Tv_0}$ is independent of ownership and $Y_{Keep}^\ast > X_{Keep}^\ast$, the potential buyer’s valuation $Y_{Tv_0}^\ast$ exceeds the owner’s reservation price $X_{Tv_0}^\ast$ so that the potential buyer will acquire the asset under self-assessment. Conversely, $Y_{Keep}^\ast < X_{Keep}^\ast$ implies $Y_{Tv_0}^\ast < X_{Tv_0}^\ast$ (exchange all $X$ and $Y$ in Figure 1), so that the potential buyer will not acquire the asset under self-assessment. This provides a sufficient condition for self-assessment to achieve allocative efficiency, which we summarize in11

**Theorem 3:** If the probability-of-sale schedule remains unchanged when the asset changes hands, then the self-assessed tax promotes allocative efficiency in ownership.

In Section 5, we show that investment does not change an owner’s tax burden if the cost of the new capital is exempt from the self-assessed tax and neither the owner nor potential buyers develop subjective attractions to the new capital. Theorem 3 therefore does not require that the potential buyer intend to maintain the owner’s use of the asset.

As a corollary to Theorem 3, the self-assessed tax fails to achieve allocative efficiency if a change in ownership changes the probability schedule and hence the potential buyer’s self-assessed tax in one of the following ways: either $Y_{Keep}^\ast > X_{Keep}^\ast$ leads to $Y_{Tv_0}^\ast < X_{Tv_0}^\ast$ and the potential buyer fails to acquire the asset when he should become the new owner, or $Y_{Keep}^\ast <$

---

11 The proof follows directly from Figure 1: because schedule $p_{Tv_0}$ applies to the owner as well as the potential buyer, $Y_{Tv_0}^\ast < X_{Tv_0}^\ast$ implies that the potential buyer pays a lower tax than the owner if he acquires the asset. Thus the combination $Y_{Keep}^\ast > X_{Keep}^\ast$ and $Y_{Tv_0}^\ast < X_{Tv_0}^\ast$ is impossible, because it would imply $Y_{Keep}^\ast - Y_{Tv_0}^\ast > X_{Keep}^\ast - X_{Tv_0}^\ast$, indicating that the potential buyer’s tax exceeds the owner’s tax.
leads to $Y^*_{TV} > X^*_{TV}$ so that the potential buyer acquires the asset when it should remain the property of the current owner. In terms of Figure 1, these outcomes require separate probability schedules that apply to the potential buyer and that are either sufficiently far above the owner’s probability schedule $p_{TV}$ for a higher tax on the potential buyer to prevent an efficient exchange, or sufficiently far below $p_{TV}$ for a lower tax on the potential buyer to induce an inefficient exchange.

4.5 Allocative efficiency is achieved by taxing the marginal dollar appropriately

Equation 8 indicates that truthful self-assessment requires accurate taxation of the marginal dollar rather than accurate taxation of the entire self-assessed amount. Thus, exempting some fraction of the self-assessed amount from the tax can reduce the owner’s self-assessed tax burden without impairing allocative efficiency, as illustrated in Figure 2.

--- Figure 2 about here ---

Like Figure 1, Figure 2 shows probability schedule $p_{TV}$ as the solid curved line and the owner’s valuation $X^*_{TV}$ as the rectangle formed by the four points $0X^*_{TV}a1$. The owner’s self-assessed tax is the striped area underneath schedule $p_{TV}$ up to $X^*_{TV}$, indicating that the owner’s self-assessed tax burden can be large in relation to the owner’s valuation.

For any announced value $X_{TV}$, the area underneath probability schedule $p_{TV}$ up to $X_{TV}$ represents the lower limit of the potential buyers’ valuations of the asset. For most assets, it is reasonable to assume that a buyer will purchase the asset with certainty if the owner’s self-assessed price is sufficiently low. Let $X^L_{TV}$ be the highest price at which a buyer purchases the asset with certainty under tax schedule $T_{V0}$. Since allocative efficiency is not improved by requiring owners to pay self-assessed taxes on prices at which their assets sell with certainty, the lower limit of the integral in equation 9 that defines $T_{V0}$ can be set equal to $X^L_{TV}$, rather than 0, without loss of efficiency.

An asset encumbered with a lower tax burden has a higher probability of being purchased at any announced price, so the schedule $p_{TVL}$ that describes the probabilities of sale when part of the self-assessed value is tax exempt exceeds the probability schedule $p_{TV}$, for all announced
values. The subscript $VL$ indicates that the tax rate ‘v’aries with the announced value and that it is levied only on values above a ‘l’ower limit above zero. Let $X_{T_{VL}}^L$ denote the highest price at which the asset sells with certainty when only announced values above $X_{T_{VL}}^L$ are taxed, so that exempting all values from the self-assessed tax at which the asset sells with certainty leads to tax schedule

$$T_{VL} = -A + \begin{cases} \int_{X_{T_{VL}}^L}^{X_{T_{VL}}^*} p_{T_{VL}}(X) dX & \text{if } X_{T_{VL}}^* > X_{T_{VL}}^L \\ 0 & \text{if } X_{T_{VL}}^* \leq X_{T_{VL}}^L \end{cases}. \quad (17)$$

where $X_{T_{VL}}^*$ is the value that the owner announces. Figure 2 shows probability schedule $p_{T_{VL}}$ as the thick dashed curved line, the price $X_{T_{VL}}^L$ at which the asset sells with certainty under tax schedule $T_{VL}$ as the thick dashed straight line, and the integral in equation 17 as the solid grey area for a particular value of $X_{T_{VL}}^*$.  

An owner who announces $X_{T_{VL}}^* \leq X_{T_{VL}}^L$ pays no tax, but allocative efficiency is not impaired because the asset changes hands with certainty. Because an owner who announces $X_{T_{VL}}^* > X_{T_{VL}}^L$ pays $dT_{VL}/dx = p_{T_{VL}}$ on the last dollar of value she announces, first-order condition 8 indicates that she has an incentive to announce

$$X_{T_{VL}}^* = X_{Keep} - \frac{T_{VL}}{\gamma}. \quad (18)$$

Thus, $T_{VL}$ eliminates any markup and motivates allocative efficiency. If $X_{T_{V0}}^L > 0$, then $T_{VL} < T_{V0}$ and therefore $X_{T_{VL}}^* > X_{T_{V0}}^*$ because assets encumbered with smaller tax burdens are more valuable to their owners.

The general threshold value $X^L$ at which an asset sells with certainty represents the amount for which there is agreement that the asset is worth at least that much. In a perfectly competitive market in which the marginal asset conveys the same benefit $B$ to all persons in equilibrium, no buyer offers an amount above $X^L$, so that all equilibrium trades occur at the efficient price $X^L = X_{Sell} = X_{Keep}$. For reproducible assets, $X^L$ is the asset’s cost of production minus any depreciation. For non-reproducible assets, $X^L$ is the highest value at which the asset sells with certainty. The amount by which the owner’s reservation price without markup exceeds $X^L$ represents the owner’s subjective valuation—that is, the amount by which the owner values the asset, over and above the amount at which potential buyers are willing to acquire the asset. Thus, $T_{VL}$ is imposed only on the owner’s subjective valuation, over and above the asset’s
objectively quantifiable value under self-assessed tax $T_{VL}$, to the extent that there are others who also value the asset beyond its objectively quantifiable value. If the owner is the only person who values the asset at an amount above the threshold, then the probability of selling the asset at a price above the threshold is zero, and the efficient self-assessed tax is zero as well.

4.6 Applications of self-assessment

Truthful self-assessment requires that the government and the owner agree on the probability of a sale at the value that the owner announces. It is not necessary that they agree on the entire probability schedule because, as the analysis in Section 4.5 indicates, tax rates that differ from the probability of a sale at amounts other than the amount announced by the owner do not affect her incentive to be truthful. It is also not necessary that the government and the owner know the correct probability, but only that the owner believes that the government has used the correct probability as the marginal tax rate.

Setting the appropriate tax rate is straightforward if the probability of a sale is under the government’s direct control, as can be true when the government decides which properties it might take under Eminent Domain.\textsuperscript{12} For the assessment of real estate in general, the techniques of assessors and real estate agents permit estimation of the values that persons other than the owners might assign to individual properties, which can be used to estimate the probabilities of sales.\textsuperscript{13} In addition to improving the efficiency of real estate transactions in general, self-assessment can help to resolve the holdout problem that frequently arises during urban redevelopment.\textsuperscript{14} Weyl and Zhang (2018) propose self-assessment to improve the efficiency of sales of licenses for the use of public resources. The existence of multiple identical licenses for the same resource makes it comparatively easy to estimate the probability of sales at different prices. Posner and Weyl (2017) provide detailed discussions of these and other possible

\textsuperscript{12} See Tideman and Plassmann (2005).

\textsuperscript{13} We suspect that most of the complaints about real estate assessment arise when assessors seek to determine an owner’s reservation price, which includes the owner’s subjective valuation. We expect that many owners would be more likely to agree with estimates of the probability of sales, which depend on the estimates of the valuations of persons other than the owner.

\textsuperscript{14} See Plassmann and Tideman (2011).
applications of self-assessment, including applications to business assets, corporate acquisitions, personal property, as well as private applications like “sharing economy” platforms.

Still, the adoption of owner-specific tax schedules may be impractical or impossible in some cases. In Section 7, we therefore discuss some characteristics of the self-assessed tax $T_{c0}$ that assigns the same tax rate to all owners.

5. Extension 1: ACCOMMODATING EFFICIENT INVESTMENT UNDER SELF-ASSESSMENT

Owners make efficient investment decisions if investing does not change their tax burdens. Weyl and Zhang (2016, 2018) show that the self-assessed tax $T_{v0}$ without exemption leads to inefficient investment decisions, because investment raises the owner’s valuation and hence alters the owner’s tax burden. However, the self-assessed tax $T_{vl}$ with exemption for the objectively quantifiable value of assets provides efficient investment incentives because it exempts new capital. This result holds as long as (1) the owner does not form any subjective attachment to her new capital, and (2) the investment does not alter the asset’s attractiveness to potential buyers.

To understand why, consider Figure 3, which shows how investment affects the owner’s tax burden under tax schedule $T_{VL}$. The solid line $p_{TV}$ indicates the probability schedule of sales at different self-assessed amounts in the absence of investment. Given the owner’s announcement $x_{TV}^*$, her self-assessed tax is the horizontally-striped area. The dashed line $p_{TVL}$ indicates the probability schedule of sales after investment. Given the owner’s announcement $x_{TVL}^{**}$, her self-assessed tax after investment is the solid grey area. The difference between the exempt amounts, $X_{TVL}^L$ and $X_{TVL}^{UL}$, represents the objectively quantifiable value of the new capital. For example, if an asset that would have sold with certainty for any amount below $X_{TVL}^L = $100,000 prior to investment sells with certainty for any amount below $X_{TVL}^{UL} = $600,000 after investment, then the new capital is worth $X_{TVL}^{UL} - X_{TVL}^L = $500,000.

--- Figure 3 about here ---
Investment changes the owner’s tax burden per period by the difference between the striped area and the solid area in Figure 3, or

$$\Delta T_{VL} = T'_{VL} - T_{VL}$$

$$= \int_{X^L_{T_{VL}}}^{X^U_{T_{VL}}} p^L_{T_{VL}}(X) dX - \int_{X^L_{T_{VL}}}^{X^U_{T_{VL}}} p_{T_{VL}}(X) dX$$

$$= \int_{X^L_{T_{VL}}}^{X^U_{T_{VL}}} \left( 1 - \Pi_{T_{VL}}'(X) \right) dX - \int_{X^L_{T_{VL}}}^{X^U_{T_{VL}}} \left( 1 - \Pi_{T_{VL}}(X) \right) dX$$

$$= \left( (X^U_{T_{VL}} - X^L_{T_{VL}}) - (X^U_{T_{VL}} - X^L_{T_{VL}}) \right) - \left( \int_{X^L_{T_{VL}}}^{X^U_{T_{VL}}} \Pi_{T_{VL}}'(X) dX - \int_{X^L_{T_{VL}}}^{X^U_{T_{VL}}} \Pi_{T_{VL}}(X) dX \right).$$

(19)

where $$\Pi_{T_{VL}}(X) = 1 - p_{T_{VL}}(X)$$ is the probability of not selling the asset during any time period under the self-assessed tax $$T_{VL}$$. The first term in the last line of equation 19 represents the change in the owner’s subjective attachment to the new capital. For example, an owner who self-assesses the asset at $$X^L_{T_{VL}} = 150,000$$ prior to investment and at $$X^U_{T_{VL}} = 660,000$$ after making an investment that increases the value of the asset to potential buyers by $$X^U_{T_{VL}} - X^L_{T_{VL}} = 500,000$$ has formed an additional subjective attachment worth $$(X^U_{T_{VL}} - X^L_{T_{VL}}) - (X^U_{T_{VL}} - X^L_{T_{VL}}) = 10,000$$ to the improved asset.

The second term in the last line of equation 19 measures variations between the curvatures of $$\Pi_{T_{VL}}'(D)$$ and $$\Pi_{T_{VL}}'(D)$$, arising from differences in the asset’s value before and after investment to potential buyers. For example, if an asset that would have sold for $150,000 with a probability of 50 percent per year prior to a $500,000 investment sells for $650,000 with only a probability of 40 percent per year after investment, then investment has lowered the asset’s value to potential buyers.

Equation 19 indicates that if objective changes in value are exempt from the self-assessed tax, then investment changes the owner’s self-assessed tax bill only when such investment affects either the owner’s subjective attachment to the asset or the asset’s subjective value to potential buyers. In this case, the magnitude of the striped area differs from that of the solid area in Figure 2, and the self-assessed tax affects the owner’s decision to invest. In the absence of such subjective changes in the asset’s value to the owner and to potential buyers, investment
does not alter the owner’s self-assessed tax bill and therefore does not distort the owner’s interest to invest efficiently.

Weyl and Zhang (2016) suggest that investment might be not observable, making it impossible to exempt the value of capital from the self-assessed tax.\(^\text{15}\) We argue that the issue is not as problematic as Weyl and Zhang suggest, because existing income tax systems that allow depreciation on investments as a deduction already confront the problem of accounting for investments and depreciation. To adjust the lower limit, the same administrative rules that are currently used to identify investment and depreciation under income taxes can be used with a self-assessed tax.\(^\text{16}\) An owner has an incentive to invest efficiently as long as the exemption permitted by the tax schedule corresponds to her expectation regarding the asset’s actual depreciated value.

Weyl and Zhang (2016, p.16) further suggest that, if the government can observe a (noisy) signal of the owner’s investment, then the government can offer to owners investment subsidies that counterbalance the distortion from a self-assessed tax without exemptions. Such investment subsidies can achieve the same result as exemptions, but care is required to provide appropriate incentives for efficient investment. The idea of investment subsidies is related to the investment tax credit (ITC) that existed in various forms in US tax law between 1962 and 1986.\(^\text{17}\) The ITC distorted investment decisions with respect to investments with different depreciation rates, because it did not vary with the length of time over which the new capital depreciates, and it therefore induced larger reductions in the required rate of return for capital that depreciates faster.\(^\text{18}\)

When assets are subject to an income tax rather than a property tax, new capital can be effectively exempted from the income tax by permitting it to be fully written off at the time of investment. When a full write-off of new capital at the time of investment is not allowed, the negative impact of taxation can be non-distorting with respect to assets with different rates of

\(^{15}\) We thank Glen Weyl for reminding us that he and Anthony Zhang had addressed the issues of observability and investment subsidies in their paper.

\(^{16}\) However, under an income tax, taxpayers have an incentive to overstate depreciation, in order to increase their tax deductions, while under a self-assessed tax, they have an incentive to understate depreciation, in order to increase their tax exemptions.

\(^{17}\) See, for example, Chirinko (2000).

\(^{18}\) See Jeremias (1979).
depreciation (although still discouraging investment to some extent) if some standard fraction of the investment is written off at purchase and the remainder is written off in the pattern of actual depreciation.\textsuperscript{19} Correspondingly, a self-assessed tax is non-distorting among assets with different rates of depreciation if a standard fraction of the true value of capital is exempt from the self-assessed tax in every period, and the tax does not discourage investment at all if that fraction is 100%. Thus, an investment subsidy at the time of investment does not distort among assets with different patterns of depreciation if and only if it equals some standard fraction of the present value of the tax that the owner would pay if there were no subsidy.

6. EXTENSION 2: RAISING PUBLIC REVENUE EFFICIENTLY UNDER SELF-ASSESSMENT

Governments can raise public revenue efficiently through taxes that either (1) are fixed in magnitude so that owners cannot alter them by changing their behavior, or (2) correct inefficient behavior by charging owners some or all of the social costs of their actions that they otherwise would not bear. A self-assessed tax is an example of the latter when it provides owners with an incentive to avoid all markups, including those that are inflated by overly optimistic beliefs regarding the distribution of future offers, so that assets change hands efficiently.

Revenue can be raised efficiently by combining the self-assessed tax with a tax on that part of an asset’s benefit that is not the result of human effort, that is, a tax on the asset’s rent. The magnitude of an asset’s rent is defined as the opportunity cost of leaving the asset unused, if it were in an unimproved condition.\textsuperscript{20}

Because owners cannot affect the opportunity cost of leaving the unimproved asset unused, a tax on rent, \( T_R \), has the same effect on the owner’s valuation as the lump-sum tax \( T_L \) that we introduced in Section 2.\textsuperscript{21} A tax that combines collection of rent with a self-assessed tax

\textsuperscript{19} See Tideman (1975), Sunley (1978), and Bradford (1980).

\textsuperscript{20} This definition accommodates situations where the return in a particular year depends on the time when redevelopment occurred.

\textsuperscript{21} This is true as long as the tax rate does not exceed 100%. Because of the necessity of relying on third-party assessment to determine the magnitude of an asset’s rent and because (a) assessment is never perfect and (b) the social cost of exceeding 100% and consequently discouraging investment is greater than the social cost of underassessing by the same percentage and consequently losing tax revenue, any actual tax on rent will sensibly aim to collect an amount that is less than the full rent.
that exempts the asset’s objectively quantifiable value of capital provides the owner with an incentive to assess the asset at

\[
X_{T_{VL}}^* = \frac{B}{\gamma} - \frac{T_R}{\gamma} - \frac{T_{VL}}{\gamma} = X_{\text{Keep}, T_R}^* - \frac{T_{VL}}{\gamma},
\]

(20)

thereby eliminating any markup and motivating allocative efficiency, while raising the maximum amount of public revenue from the asset that can be raised efficiently.

7. PRACTICAL SELF-ASSESSMENT

Weyl and Zhang (2016) propose the self-assessed tax \( T_{c0} \) with a (low) constant tax rate to ameliorate the inefficiency in investment that accompanies \( T_{V0} \). While the inefficiency in investment can be avoided by adopting self-assessed tax \( T_{VL} \), the adoption of owner-specific tax schedules that are defined by the probability of a sale at different owner-announced values may be impractical, and it might be impossible for the government to identify tax schedules that correspond to the owner’s beliefs regarding the frequency of offers at different prices. This makes it worthwhile to consider a self-assessed tax with a constant tax rate that applies to all owners.

Begin with the case without self-assessment and with an owner who expects to sell her asset at a price of \( X \) or higher according to probability schedule \( p_S = \int_X^\infty \pi(D)dD \). Figure 4 shows probability schedule \( p_S \) as the dotted curved line and the owner’s reservation price \( X_{\text{sell}}^* \) as the black dot on the vertical axis.

--- Figure 4 about here ---

Now consider the case when the owner pays a self-assessed tax \( T_{CL} \) with constant tax rate \( \tau \) that is applied to announced values above a threshold \( X_{T_{CL}}^L \), or

\[
T_{CL} = \begin{cases} 
\tau(X - X_{T_{CL}}^L) & \text{if } X > X_{T_{CL}}^L \\
0 & \text{if } X \leq X_{T_{CL}}^L
\end{cases}
\]

(21)

As before, we assume that the threshold equals the amount at which the asset sells with certainty, that is, self-assessed tax \( T_{CL} \) is imposed only on the owner’s subjective valuation, over and above
the largest amount \( X_{CL}^l \) at which others are willing to acquire the asset with certainty. The subscript CL indicates that the tax rate is ‘c’onstant and that it is levied only on values above a ‘lower’ limit above zero. The solid curved line in Figure 4 shows probability schedule \( p_{T_{CL}} \) that describes the probability of receiving offers under self-assessed tax \( T_{CL} \) with constant tax rate \( \tau \).

The difference between the optimal announced value \( X_{T_{CL}} \) as determined in equation 11 and \( X_{Sell}^* \) is the sum of the (reduction in the present value of the net benefit stream that is caused by the requirement to pay the self-assessed tax) and (the markup under \( X_{Sell}^* \) that is eliminated by the self-assessed tax) and the (markup or discount that arises under \( T_{CL} \) when \( \tau \neq p_{T_{CL}} \), or

\[
X_{T_{CL}} - X_{Sell}^* = -\frac{\tau B}{y + \tau} - \frac{E - pS}{y + pS} + \frac{(\tau - p_{T_{CL}})(y + p_{T_{CL}})}{(y + \tau) \frac{dp_{T_{CL}}}{dx}} < 0 .
\] (22)

An owner who is subject to a constant self-assessed tax rate has an incentive to self-assess her asset at, below, or above its value to her if the tax rate is equal to, above, or below the probability of sale at the value of the asset to her, given the tax. If—by chance—the tax rate \( \tau \) equals the probability of a sale \( p_{T_{CL}} \) at the value \( X_{T_{CL}}^* \) that the owner announces, then the last term in equation 22 is zero and \( X_{T_{CL}}^* \) differs from \( X_{Sell}^* \) by the reduction in asset value caused by the burden of the self-assessed tax and the eliminated markup that would exist under \( X_{Sell}^* \). Thus, as determined in equation 12, the owner’s announcement equals the present discounted value of her after-tax benefit stream, \( \frac{B}{y + \tau} \), and hence carries no markup. Figure 4 shows the owner’s tax burden \( T_{CL} = \tau(X_{T_{CL}}^* - X_{T_{CL}}^L) \) as the combined area of the horizontally and vertically striped rectangles.

For a truthful report, \( X_{T_{CL}}^* \), to be in the interest of the owner, the vertical line at the tax rate, \( \tau \), must intersect the solid curve line representing \( p_{T_{CL}} \) exactly at \( X_{T_{CL}}^* \). But this will not always happen. The tax rate \( \tau \) might exceed the probability of a sale at the value that the owner announces, say, \( X_{T_{CL}}^{**} \), which occurs when the asset’s benefit rate to the owner exceeds the benefit rate \( B \) that defines \( X_{T_{CL}}^* \), making it in the owner’s interest to buy more “insurance” against being bought out than she would buy if her benefits were \( B \), even though the price of the insurance
exceeds the risk.\footnote{Probability schedule $p_{T_{CL}}$ is defined for tax rate $\tau$, and it is therefore the same for $B^{***}$ and $B$. Because the valuation of the asset by potential buyers remains unchanged, the larger benefit stream $B^{***} - B$ must accrue to the asset’s current owner only (that is, be entirely subjective) but not to potential buyers. The last term in equation 21 is negative when $\tau > p_{T_{CL}}$ and hence describes a discount because $\frac{dP_{T_{CL}}}{dx} < 0$.} Figure 4 shows the tax burden $T_{CL}^{***} = \tau (X_{T_{CL}}^{***} - X_{T_{CL}}^{L})$ of the owner who reports $X_{T_{CL}}^{***}$ as the combined area of the horizontally striped rectangle, the vertically striped rectangle, and the solid grey rectangle.

The tax rate $\tau$ might also be below the probability of a sale, if the owner announces, say $X_{T_{CL}}^{***}$. Because $X_{T_{CL}}^{***} < X_{T_{CL}}^{*}$, the owner’s benefit rate that leads to a report of $X_{T_{CL}}^{***}$ must be below the benefit rate $B$ that leads to $X_{T_{CL}}^{*}$. Such an owner finds insurance cheap at the margin and announces a value that involves a positive markup. Figure 4 shows the owner’s tax burden $T_{CL}^{***} = \tau (X_{T_{CL}}^{***} - X_{T_{CL}}^{L})$ as the area of the horizontally striped rectangle.

The two extensions to self-assessment that we introduced in Sections 5 and 6 carry over to the tax $T_{CL}$. Raising public revenue efficiently through a separate tax on rent $T_R$ reduces the owner’s valuation of the asset and hence reduces her announcement by $\frac{T_R}{y+\epsilon}$, but the tax on rent does not affect the owner’s incentives to announce a markup, a discount, or no markup, as specified by the second term in equation 11. Similarly, investment changes the owner’s tax burden by

\[
\Delta T_{CL} = T'_{CL} - T_{CL} = \tau \left( X_{T_{CL}}^{*} - X_{T_{CL}}^{L} \right) - \tau \left( X_{T_{CL}}^{*} - X_{T_{CL}}^{L} \right)
= \tau \left( \left( X_{T_{CL}}^{*} - X_{T_{CL}}^{*} \right) - \left( X_{T_{CL}}^{L} - X_{T_{CL}}^{L} \right) \right),
\]

where $X_{T_{CL}}^{*}$ denotes the owner’s self-assessed value after investment, $X_{T_{CL}}^{L}$ denotes the adjusted threshold, and $T_{CL}'$ denotes the owner’s corresponding self-assessed tax. If the threshold $X_{T_{CL}}^{L}$ accommodates all changes in the asset’s objectively quantifiable value and if owners do not form subjective attachments to their new capital, then the self-assessed tax $T_{CL}'$ has the same tax burden as $T_{CL}$ and therefore does not deter efficient investments. Because the constant tax rate does not depend on the probability of a sale, changes in the asset’s subjective value to potential buyers have no effect on the owner’s tax burden.
8. CONCLUSION

Assets that are taxed are less valuable than assets that are not taxed. A self-assessed tax that requires that the owner sell the asset at the self-assessed price and whose tax rate equals the probability of a sale at the owner’s self-assessed value reduces the owner’s valuation by exactly the amount of the owner’s markup that might prevent allocative efficiency in the absence of self-assessment when overly optimistic owners set unreasonably high markups because of inflated beliefs about the probability of receiving profitable offers. Such a self-assessed tax achieves allocative efficiency as long as the asset’s subjective valuation by potential buyers does not depend on who owns the asset. In contrast, revelation mechanisms that assign to owners, in the event of a sale, the amount offered by the buyer rather than the amount at which the owner herself values the asset do not induce the owner to report her value of continued ownership and therefore risk not achieving allocative efficiency. Our analysis emphasizes the importance of modeling the owner’s valuation explicitly, rather than relying on an assumption that an asset’s value to its owner is determined exogenously.

Exempting an asset’s objectively quantifiable value from the self-assessed tax restores incentives for efficient investment, and combining the self-assessed tax with a separate tax on rent allows governments to raise public revenue efficiently without sacrificing allocative efficiency. While a self-assessed tax whose marginal tax rate varies with the probability of a sale may be impractical or impossible, it is straightforward to implement self-assessment with a constant tax rate. A self-assessed tax whose tax rate exceeds the probability of a sale reduces allocative efficiency by inducing owners to offer to sell their assets below the reservation prices that reflect the stream of benefits that they expect to receive from the assets, but allocative efficiency is improved, compared to no tax, whenever a tax at a rate at or below the probability of a sale is introduced. In addition, adopting a regime of self-assessment with a low constant tax rate provides owner-valuation information that can be expected to reduce disagreements, in comparison with conventional third-party assessments, in situations that involve land assembly and eminent domain.
REFERENCES:


Figure 1. The self-assessed tax $T_{v_0}$ paid by the owner or by a potential buyer, if he acquires the asset

Figure 2. The self-assessed taxes $T_{v_0}$ and $T_{v_L}$
Figure 3. The self-assessed tax burden with and without investment

Figure 4. The self-assessed tax $T_{CL}$ with constant tax rate $\tau$