This supplement describes our algorithm to determine the shortest walking
distance between two parcels. The street map of downtown Portland is a combination of
two grids that meet at an angle of 22.319 degrees (0.38954 radians) at Ankeny Street (see
Figure 1 below). Each parcel has \((x, y)\) coordinates in a coordinate system that has the
same orientation as the upper grid of the map, with its origin in the upper left corner of
the map, and positive coordinates to the right and down. The \(y\) coordinate of Ankeny
Street is defined as \(H\) (the first “horizontal” street). For each parcel that is located in the
lower (tilted) section of the grid, the Cartesian coordinates \((x^*, y^*)\) in a tilted coordinate
system are determined as \((x^*, y^*) = (r \cos \theta, r \sin \theta)\), where \(r = \sqrt{x^2 + y^2}\) and 
\(\theta = \arctan(y/x) - 0.38954\).

Two parcels \(A\) and \(B\) can be (a) both in the upper (non-tilted) part of the grid, or
(b) both in the lower (tilted) part of the grid, or (c) one parcel can be in the upper and the
other in the lower part of the grid. Assume first that both parcels are corner parcels.
(a) Both parcels are in the upper part of the grid (i.e. \(y_A < H\) and \(y_B < H\)), so the distance
between them is determined as \(|x_A - x_B| + |y_A - y_B|\).
(b) Both parcels are in the lower part of the grid (i.e. \(y_A > H\) and \(y_B > H\)), so the distance
between them is determined as \(|x_A^* - x_B^*| + |y_A^* - y_B^*|\).
(c) Assume that parcel A is in the upper grid (\( y_A < Y \)) and parcel B is in the lower grid (\( y_B > Y \)). Define the point C as the point on Ankeny street that is exactly to the south of Parcel A, so that \((x_C, y_C) = (x_A, H)\) and \((x_C^*, y_C^*) = (r \cos \theta, r \sin \theta)\), where
\[
r = \sqrt{x_C^2 + y_C^2} \quad \text{and} \quad \theta = \arctan(y_C/x_C) - 0.38954.
\]

Now there are three possibilities (see Figure 1 below):

(I) Parcel \( B_1 \): \(x_B^* < x_C^*\) (i.e. the \(x\)-coordinate of parcel B in the tilted grid is to the left of the \(x\)-coordinate of point C). The point \( D_1 \) where one reaches Ankeny street driving north from parcel B is defined as \((x_{D1}, y_{D1}) = (x_B + (y_B - y_C) \tan(0.38954), y_C)\) and \((x_{D1}^*, y_{D1}^*) = (r \cos \theta, r \sin \theta)\), where \( r = \sqrt{x_D^2 + y_D^2} \) and \( \theta = \arctan(y_{D1}/x_{D1}) - 0.38954 \). The distance from parcel A to parcel B is \(y_C - y_A + x_A - x_{D1} + y_B^* - y_{D1}^*\).

(II) Parcel \( B_2 \): \(y_B^* < y_C^*\) (i.e. the \(y\)-coordinate of parcel B in the tilted grid is to the right of the \(y\)-coordinate of point C). The point \( D_2 \) where one reaches Ankeny street driving west from parcel B is defined as \((x_{D2}, y_{D2}) = (x_B - (y_B - y_C)/\tan(0.38954), y_C)\) and \((x_{D2}^*, y_{D2}^*) = (r \cos \theta, r \sin \theta)\), where \( r = \sqrt{x_D^2 + y_D^2} \) and \( \theta = \arctan(y_{D2}/x_{D2}) - 0.38954 \). The distance from parcel A to parcel B is \(y_C - y_A + x_A - x_{D2} + y_B^* - x_{D2}^*\).

(III) Parcel \( B_3 \): \(x_B^* > x_C^*\) and \(y_B^* > y_C^*\). The distance between parcel A and parcel B is \(y_C - y_A + x_B^* - x_C^* + y_B^* - y_C^*\).

Finally, there is the possibility that both parcels A and B are in the same line of blocks in one dimension and not in the other (parcel \( A_1 \) and parcel \( B_4 \)), in which case backtracking may be necessary to correct the distances calculated in (a), (b), and (c). Backtracking is never necessary for parcels that cover a whole block, for corner parcels, or if one parcel is on an East-West street and the other is on a North-South street. If both
parcels are on streets with the same orientation and neither covers a whole block or is a corner lot, backtracking is necessary if the parcels are on different streets and the street numbers have the same number of hundreds. The amount of backtracking is twice the minimum distance from the location of one of the parcels to a street corner.

Figure 1. The intersection of the two grids at Ankeny Street in downtown Portland, Oregon