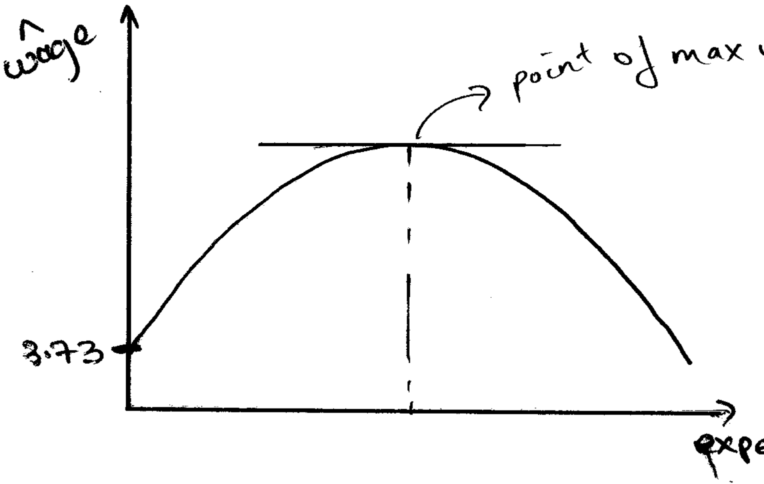
Question 1

a)

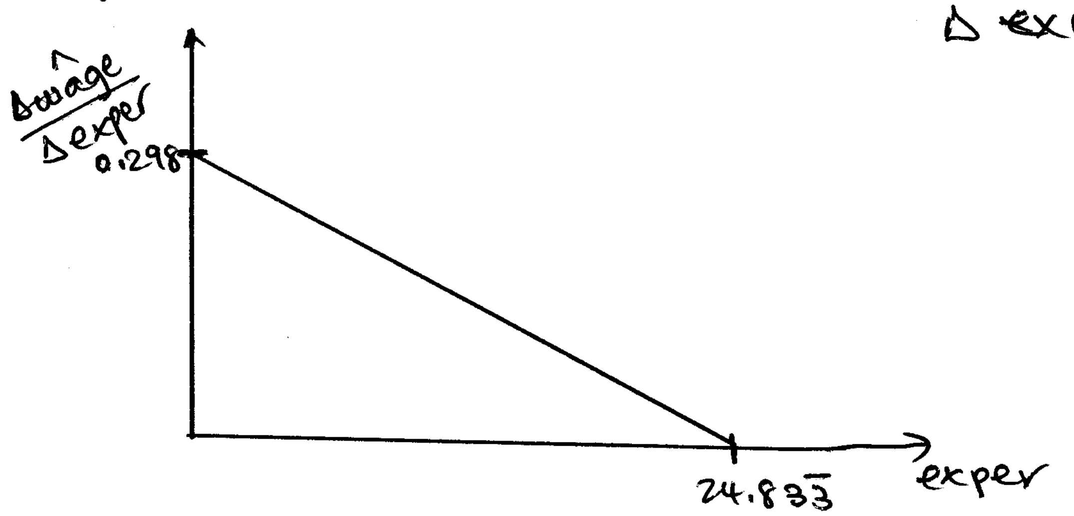


when wage is maximized we have

$$\frac{\Delta \hat{wage}}{\Delta exper} = 0.298 - 2(0.006)\,exper \overset{set}{=} 0$$

$$\Rightarrow exper = \frac{0.298}{2(0.006)} = 24.83\bar{3}$$

b) Marginal return of experience $= \frac{\Delta \hat{wage}}{\Delta exper}$



Yes this is consistent with economic theory, ie. marginal return of exper is decreasing with increasing experience $\Rightarrow$ (diminishing marginal return to experience).

c) This could be answered intuitively :-

$\hat{wage} = 3.73 + 0.298\, exper - 0.006\, exper^2$

What does this intercept represents?

$\Rightarrow$ With no experience ($exper = 0$) your hourly salary is \$3.73. Then what will be your weekly salary, assuming 40 hrs = 1 work week?

It has to be \$3.73 × 40 = \$149.20.

Algebraically, define

$$\hat{wage}^* = \text{weekly salary}$$
$$\Rightarrow \hat{wage}^* = \hat{wage} \times 40$$

Then

$$\hat{wage} = 3.73 + 0.298\, exper - 0.006\, exper^2$$

$$\Rightarrow \frac{\hat{wage}^*}{40} = 3.73 + 0.298\, exper - 0.006\, exper^2$$

$$\Rightarrow \hat{wage}^* = (3.73 \times 40) + (0.298 \times 40)\, exper - (0.006 \times 40)\, exper^2$$

$$\Rightarrow \hat{wage}^* = 149.2 + 11.92\, exper - 0.24\, exper^2$$

The se. of all ~~the~~ $\hat{\beta}^*$'s will be increased by ~~the~~ factor 40. The ~~the~~ t-value which is

$$t^*_{\hat{\beta}_j} = \frac{\hat{\beta}_j^*}{se(\hat{\beta}_j^*)}$$

will be ~~the~~ same as ~~the~~ old t values. That is t-values are invariant to change in scale.

d) $\hat{wage}^* = wage + 0.27 \Rightarrow wage = wage^* - 0.27$

$\Rightarrow \hat{wage} = 3.73 + 0.298\, exper - 0.006\, exper^2$

$\Rightarrow \hat{wage}^* - 0.27 = 3.73 + 0.298\, exper - 0.006 exper^2$

$\Rightarrow \hat{wage}^* = (3.73 + 0.27) + 0.298\, exper - 0.006\, exper^2$

Only the estimate of the intercept changes, it increases by 0.27.
All the slope coefficients remain the same.

This represents an upward shift of your old regression line.

<u>Question 2</u>

a) $E_i = \beta_0 + \beta_1 Y_i + u_i \ldots (1) \quad \sigma_i^2 = \sigma^2 Y_i^2$

Yes there is a problem with the OLS estimators of β_0 and β_1. They will be unbiased and consistent <u>but</u> inefficient due to the presence of heteroskedasticity. See Gauss Markov Theorem.

b) - Run OLS on the regression (1)

$E_i = \beta_0 + \beta_1 Y_i + u_i$

- obtain estimates of β_0 and β_1 and the residuals $\hat{u}_i$. Square each residual.
- under heteroskedasticity,

$$Var^*(\hat{\beta}_1) = \frac{\sum (x_i - \bar{x})\, \sigma_i^2}{[\sum (x_i - \bar{x})^2]^2}$$

$$\Rightarrow \hat{Var}^*(\hat{\beta}_1) = \frac{\sum (x_i - \bar{x})^2 \hat{u}_i^2}{\left[\sum (x_i - \bar{x})^2\right]^2}$$

Then the heteroskedastic robust standard error of β_1 is $\sqrt{\hat{Var}^*(\hat{\beta}_1)}$

and thus the t-value is such that

$$t\,\hat{\beta}_1 = \frac{\hat{\beta}_1}{\sqrt{\hat{Var}^*(\hat{\beta}_1)}}$$

Recall that under homoskedasticity
$$\hat{Var}(\hat{\beta}_1) = \frac{\hat{\sigma}^2}{\sum (x_i - \bar{x})^2}$$

c) This is very straightforward: you know the functional form of the heteroskedasticity and there are no unknown parameters involved in this functional form

$$\sigma_i^2 = \sigma^2 Y_i^2$$ functional form with no unknown parameters

Then you only need to transform the original model ensuring that the error term is homoskedastic, and then you run ols on the new model:

$$\frac{E_i}{Y_i} = \beta_0^* \cdot 1/Y_i + \beta_1^* \frac{Y_i}{Y_i} + \frac{u_i}{Y_i}$$

New model:

$$\Rightarrow \frac{E_i}{Y_i} = \beta_0^* \cdot 1/Y_i + \beta_1^* + v_i \quad \text{where } v_i = \frac{u_i}{Y_i}$$

Check for homoskedasticity

✓ $E(v_i) = E\left(\frac{u_i}{Y_i}\right) = 1/Y_i\, E(u_i) = 0$

✓ $Var(v_i) = Var\left(\frac{u_i}{Y_i}\right) = 1/Y_i^2\, Var(u_i) = \frac{1}{Y_i^2} \cdot \sigma^2 Y_i^2 = \sigma^2.$

d) - Run ols on ① and obtain residuals $\hat{u}_i$ and fitted values ~~$\hat{E}_i$~~ $\hat{E}_i$.
- Compute $\hat{u}_i^2$, $\hat{E}_i^2$.
- run the regression

$$\hat{u}_i^2 = \delta_0 + \delta_1 \hat{E}_i + \delta_2 \hat{E}_i^2 + \text{error}$$

and note the $R^2_{\hat{u}^2}$ from this regression

- $H_0: \delta_1 = \delta_2 = 0$
  H_1: not H_0

$$F = \frac{R^2_{\hat{u}^2} / 2}{1 - R^2_{\hat{u}^2} / n-3} \sim F(2, n-3)$$

reject at α% if $F > F_\alpha(2, n-3)$

Note: this White's procedure can also be done by replacing the second regression above with

$$\hat{u}_i^2 = \delta_0 + \delta_1 Y_i + \delta_2 Y_i^2 + \text{error}$$

Why? Read pg 268 in Text.

e) FGLS: you know the functional form of the heteroskedasticity (Y_i^θ) but it contains an unknown parameter (θ) $\Rightarrow$ you first estimate θ and then transform the original model.

$$E_i = \beta_0 + \beta_1 Y_i + u_i \longrightarrow \text{model ①}$$

$$u_i^2 = \sigma^2 Y_i^{\theta}$$

$$\Rightarrow \log u_i^2 = \log \sigma^2 + \theta \log Y_i$$

$$\Rightarrow \log u_i^2 = \gamma_0 + \theta \log Y_i$$

- run model ① and obtain $\hat{u}_i$. Compute $\hat{u}_i^2$.
- run the regression

$$\log(\hat{u}_i^2) = \gamma_0 + \theta \overbrace{\log Y_i}^{\text{consider this as a new X variable}} \text{ to obtain } \hat{\theta}$$

- transform model ① as

$$\frac{E_i}{\sqrt{Y_i^{\hat{\theta}}}} = \beta_0^* \cdot \frac{1}{\sqrt{Y_i^{\hat{\theta}}}} + \beta_1^* \frac{Y_i}{\sqrt{Y_i^{\hat{\theta}}}} + \frac{u_i}{\sqrt{Y_i^{\hat{\theta}}}}$$

model ③.

Model ③ is now homoskedastic.

- run ols on model ③ to get estimates of β_0^* and β_1^*.

Question 3

a) Define: V_t – % votes received by Democratic candidate in State t

U_t – unemployment rate in State t

$$F = \begin{cases} 1 & \text{if female} \\ 0 & \text{if male} \end{cases}$$

$$B = \begin{cases} 1 & \text{if Bill Clinton appeared at campaign in State } t \\ 0 & \text{otherwise} \end{cases}$$

Model 1:

$$V_t = \cancel{\beta}\alpha_0 + \alpha_1 U_t + \alpha_2 F + u$$

Model 2:

$$V_t = \beta_0 + \beta_1 U_t + \beta_2 F + \beta_3 B + v$$

Model 3:

$$V_t = \gamma_0 + \gamma_1 U_t + \gamma_2 F + \gamma_3 B + \gamma_4 F \cdot B + \omega$$

reference group: $\bar{M}, \bar{B}$

b) In Model 3 :-

γ_4 – slope differential

γ_3 – differential effect of Bill Clinton's appearance

γ_2 – differential effect of being a female voter

γ_1 – the effect of a 1% increase in unemployment rate in state t on the % votes received by Democratic candidate in that state.

γ_0 – % of male voters that candidate in state t received when Bill Clinton did not appear at that campaign, ceteris paribus.

– other interpretations we accepted !!

c) <u>Model 2</u>:

(i) $H_0: \beta_1 = 0$, $H_1: \beta_1 \neq 0$

(ii) $H_0: \beta_3 = 0$, $H_1: \beta_3 \neq 0$

(iii) Not applicable

Model 3:

(i) $H_0: \gamma_1 = 0$, $H_1: \gamma_1 \neq 0$

(ii) $H_0: \gamma_3 = \gamma_4 = 0$, H_1: not H_0

(iii) $H_0: \gamma_2 + \gamma_4 = 0$, $H_1: \gamma_2 + \gamma_4 \neq 0$.

	Female	Male
Bill	$\gamma_0 + \gamma_2 + \gamma_3 + \gamma_4$	$\gamma_0 + \gamma_3$
No Bill	$\gamma_0 + \gamma_2$	γ_0

d) Model 4:

$V_t = \delta_0 + \delta_1 D_1 + \delta_2 D_2 + \delta_3 D_3 + \delta_4 U_t + \text{error}$

where
$D_1 = F \cdot B$
$D_2 = M \cdot B$
$D_3 = F \cdot \bar{B}$
$M \cdot \bar{B}$ - reference group

	Female	Male
Bill	$\delta_0 + \delta_1$	$\delta_0 + \delta_2$
No Bill	$\delta_0 + \delta_3$	δ_0

Comparing Model 3 and 4, we see that

$\gamma_0 = \delta_0$

$\gamma_2 = \delta_3$

$\gamma_3 = \delta_2$

$\gamma_2 + \gamma_3 + \gamma_4 = \delta_1$

$\delta_4 = \gamma_1 \rightarrow$ unemployment coefficient

(e) $H_0: \delta_1 = \delta_2$

$H_1: \delta_1 \neq \delta_2$