Answer Key - Spring 04 - Exam 1

① $Y = \beta_0 + \beta_1 X + u$

$n = 500, \quad \sum X = 24000, \quad \sum Y = 10,700$

$\sum (X-\bar{X})^2 = 66,000, \quad \sum (Y-\bar{Y})^2 = 1,398,000$

$\sum (Y-\bar{Y})(X-\bar{X}) = 194,000$

a) $\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} = \frac{10,700}{500} = 21.4 - (2.93\overline{93})\, 48 = -119.691$

$\hat{\beta}_1 = \frac{\sum (X-\bar{X})(Y-\bar{Y})}{\sum (X-\bar{X})^2} = \frac{194,000}{66,000} = 2.93\overline{93}$

b) $\hat{\beta}_1 = \frac{\sum (X-\bar{X})(Y-\bar{Y})}{\sum (X-\bar{X})^2} = \frac{\sum (X-\bar{X})(Y-\bar{Y})}{\sqrt{\sum (X-\bar{X})^2}} \cdot \frac{1}{\sqrt{\sum (X-\bar{X})^2}} \cdot \frac{\sqrt{\sum (Y-\bar{Y})^2}}{\sqrt{\sum (Y-\bar{Y})^2}}$

$= \frac{\sum (X-\bar{X})(Y-\bar{Y})}{\sqrt{\sum (X-\bar{X})^2} \cdot \sqrt{\sum (Y-\bar{Y})^2}} \cdot \frac{\sqrt{\sum (Y-\bar{Y})^2}}{\sqrt{\sum (X-\bar{X})^2}}$

$= r \cdot \sqrt{\frac{Var(Y)}{Var(X)}} \quad \blacksquare$

c) If $\bar{Y}$ is on the line then $\bar{Y} = \hat{\beta}_0 + \hat{\beta}_1 X$ and we want to show that $X = \bar{X}$. So $\bar{Y} = (\bar{Y} - \hat{\beta}_1 \bar{X}) + \hat{\beta}_1 X = \bar{Y} + \hat{\beta}_1 (X - \bar{X})$ so for $\bar{Y} = \bar{Y}$ X must equal $\bar{X}$. Therefore the point $(\bar{X}, \bar{Y})$ is on the line and our regression runs through the mean of our data.

d) $\sum \hat{u} = 0 \Rightarrow \sum(Y - \hat{\beta}_0 - \hat{\beta}_1 X) = \sum(Y - (\bar{Y} - \hat{\beta}_1 \bar{X}) - \hat{\beta}_1 X)$

$$\Rightarrow \sum\left[(Y-\bar{Y}) - \hat{\beta}_1(X-\bar{X})\right] = \sum(Y-\bar{Y}) - \hat{\beta}_1 \sum(X-\bar{X})$$

$$= 0 - 0 = 0 \quad \blacksquare$$

e) $\sum X\hat{u} = 0 \Rightarrow \sum X(Y - \hat{\beta}_0 - \hat{\beta}_1 X) = \sum X(Y-\bar{Y}) - \hat{\beta}_1 \sum(X-\bar{X})X$

$$\hat{\beta}_1 = \frac{\sum(Y-\bar{Y})(X-\bar{X})}{\sum(X-\bar{X})^2} = \frac{\sum(Y-\bar{Y})X}{\sum(X-\bar{X})X}$$

$$\sum X\hat{u} = \sum X(Y-\bar{Y}) - \frac{\sum(Y-\bar{Y})X}{\sum(X-\bar{X})X} \cdot \sum(X-\bar{X})X$$

$$= \sum X(Y-\bar{Y}) - \sum(Y-\bar{Y})X = 0 \quad \blacksquare$$

f) $\sum(X-\bar{X})(Y-\bar{Y}) = \sum(X-\bar{X})Y = \sum(Y-\bar{Y})X$

$$\sum(X-\bar{X})(Y-\bar{Y}) = \sum\left[XY - \bar{X}Y - \bar{Y}X + \bar{X}\bar{Y}\right]$$

$n\bar{X} = \sum X$
$n\bar{Y} = \sum Y$

$$\Rightarrow \sum XY - \bar{X}\sum Y - \bar{Y}\sum X + n\bar{X}\bar{Y}$$

$$\sum XY - \bar{X}(n\bar{Y}) - \bar{Y}(n\bar{X}) + n\bar{X}\bar{Y} = \sum XY - n\bar{X}\bar{Y}$$

$$= \sum XY - \bar{Y}\sum X = \sum X(Y-\bar{Y})$$

$$= \sum XY - \bar{X}\sum Y = \sum Y(X-\bar{X}) \quad \blacksquare$$

g) if $Y = \beta_1 X + U$ compute $\hat{\beta}_1$

$$\hat{\beta}_1 = \frac{\Sigma XY}{\Sigma X^2} = \frac{\Sigma(X-\bar{X})(Y-\bar{Y}) + n\bar{X}\bar{Y}}{\Sigma(X-\bar{X})^2 + n\bar{X}^2} = \frac{194{,}000 + 513{,}600}{66{,}000 + 1{,}152{,}000}$$

$$= \frac{707600}{1{,}218{,}000} = .580952$$

$$\Sigma \hat{U} \neq 0 \Rightarrow \Sigma \hat{U} = \Sigma[Y - \hat{\beta}_1 X] = \Sigma Y - \frac{\Sigma YX}{\Sigma X^2}(\Sigma X)$$

$$= 10{,}700 - .580952(24000)$$

$$= -3242.85 \blacksquare$$

$$\Sigma \hat{U} X = 0 \Rightarrow \Sigma \hat{U} X = \Sigma X(Y - \hat{\beta}_1 X) = \Sigma XY - \hat{\beta}_1 \Sigma X^2$$

$$= \Sigma XY - \frac{\Sigma XY}{\Sigma X^2}(\Sigma X^2)$$

$$= \Sigma XY - \Sigma XY = 0 \blacksquare$$

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a) - β_2 measures the change in new car sales per thousand people given a 1 unit change in the New car price index

- $1000 \cdot \beta_3$ measures the change in new car sales per thousand people for a 1000 change in per-capita real disposable income

- β_4 & β_5 measure the sensitivity of car sales due to changes in the finance costs and the health of the economy

b) $\bar{R}^2 = 1-(1-R^2)[(n-1)/(n-k-1)]$ $\quad n = 40$

$1-\bar{R}^2 = (1-R^2)[(n-1)/(n-k-1)]$

$\frac{n-k-1}{n-1}(1-\bar{R}^2) = 1-R^2 \Rightarrow R^2 = 1-\frac{n-k-1}{n-1}(1-\bar{R}^2)$

Model A: $1-\frac{40-5}{39}(1-.758) = 1-.217179 = .7828$

Model B: $1-\frac{40-4}{39}(1-.764) = 1-.217846 = .7822$

Model C: $1-\frac{40-3}{39}(1-.565) = 1-.412692 = .5873$

c)

Model A

$H_0: \beta_2 = 0$ $\alpha = .05$

$H_1: \beta_2 \neq 0$ DF = 35

$t = \frac{-.071391}{.03473} = -2.0556$

Model B

$H_0: \beta_2 = 0$ $\alpha = .05$

$H_1: \beta_2 \neq 0$ DF = 36

$t = \frac{-.079342}{.0110220} = -7.20305$

$t_{.025} = 2.021$ (40)

$t_{.025} = 2.042$ (30)

$2.0556 > 2.042$ so we reject the null

β_2 is statistically significant

$7.20305 > 2.042$ so we reject the null

d)

Model A

$H_0: \beta_3 = 0$ $\alpha = .05$

$H_1: \beta_3 > 0$ DF = 35

$t = \frac{.003159}{.001763} = 1.79183$

Model B

$H_0: \beta_3 = 0$ $\alpha = .05$

$H_1: \beta_3 > 0$ DF = 36

$t = \frac{.00356}{.000627} = 5.67783$

$t_{.05} = 1.697$ (30)

$t_{.05} = 1.684$ (40)

reject in both cases

β_3 is statistically significant

e) $t = 2.0556$ @ 35 DF p-value = .047

$t = 7.20305$ @ 36 DF p-value = 0

$t = 1.79183$ @ 35 DF p-value = .041

$t = 5.67783$ @ 36 DF p-value = 0

f) $H_0: \beta_3 = \beta_5 = 0$ $\alpha = .05$

H_1: Not H_0

$$F = \frac{(44.65914 - 23.510464)}{23.510464} \cdot \frac{35}{2} = \frac{21.1487 \cdot 35}{23.510464 \cdot 2} = \frac{740.204}{47.0209}$$

$F = 15.742$

$F_{.05} \approx 3.28$

$F > F_{.05}$ so we would reject the null. Unemployment and Personal income are jointly significant.

g) $H_0: \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$

H_1: Not H_0 $\alpha = .05$

$$F = \frac{.7828/4}{(1-.7828)/35} = \frac{.1957}{.0062}$$

$F = 31.5355$

$F_{.05}(4, 35) \approx 2.65$

reject Null

$H_0: \beta_2 = \beta_3 = \beta_4 = 0$

H_1: Not H_0 $\alpha = .05$

$$F = \frac{.7822/3}{(1-.7822)/36} = \frac{.2607}{.0061}$$

$F = 43.0909$

$F_{.05}(3, 36) \approx 2.88$

reject Null

h) if income were measured in thousands then $Income^* = \frac{Income}{1000}$

so $\beta_3 = 3.159$. It is the only thing that changed in the model

i) $H_0: \beta_4 - 2\beta_5 = 0$ $\alpha = .05$

$H_1: \beta_4 - 2\beta_5 \neq 0$

① Set $\beta_4 = 2\beta_5 + \theta$

② rewrite model: $Numcars = \beta_1 + \beta_2 Price + \beta_3 Income + \theta\, Intrade + \beta_5 (Unemp + 2 \cdot intrate)$

③ Estimate model and find $\hat{\theta}$ and $s.e(\hat{\theta})$

④ Set up t-statistic $\frac{\hat{\theta}}{s.e.(\hat{\theta})}$ call it $t(\hat{\theta})$

⑤ compare $t(\hat{\theta})$ to $t_{.025}(35)$ (since this is two-sided test)

⑥ if $t(\hat{\theta}) > t_{.025}(35) \Rightarrow$ reject

Intuition ⇒ see if interest rate is twice as sensitive on new car sales than unemployment rate.