Econ 466 Final Exam Answer key

② $Y = \beta_0 + \beta_1 X + \beta_2 D_2 + \beta_3 D_3 + U$

(+24)

a) β_1 is the average pay ~~increase~~ decrease for having another year of experience in the workforce, ceterus paribus

β_2 is the expected wage differential b/w Harvard & Binghamton MBA's holding experience fixed.

β_3 is the expected wage differential b/w Wharton & Binghamton MBA's holding experience fixed.

I would expect both β_2 & β_3 to be positive.

To test $H_0: \beta_2 = 0$ against $H_1: \beta_2 \neq 0$ I would use a simple t-test @ the 5% level. To test $H_0: \beta_2 = \beta_3$ against $H_1: \beta_2 \neq \beta_3$ I would reparameterize the model as $Y = \beta_0 + \beta_1 X + \beta_3(D_2 + D_3) + \delta D_2 + v$ where $\delta = \beta_2 - \beta_3$. I would then test the hypothesis that $H_0: \delta = 0$ against $H_1: \delta \neq 0$ @ the 5% level.

b) Now we have the alternate model: $Y = \beta_0 + \beta_1 X + \beta_2 D_2 + \beta_3 D_3 + \beta_4 X \cdot D_2 + \beta_5 X \cdot D_3 + v$

This model is more general than the first one since it does not place the strict assumption of a constant and equal ~~increase~~ decrease in expected wages for another year of experience across MBA programs. This model allows for the fact that a Harvard MBA may get more out of his/her education to really benefit from another year of experience on the job market.

β_2 & β_3 have the same interpretation as in part a) but now we cannot evaluate β_1 by itself due to the lack of a ceterus paribus interpretation.

β_1 represents the expected increase/decrease in annual income for a Binghamton MBA for another year of job experience.

β_4 represents the expected incremental change beyond that

(+ job experience)
of what a Binghamton MBA makes, for Harvard MBAs.
β_5 has the sam interpretation as β_4 but now refers to a Wharton MBAS incremental change beyond a Binghamton MBA's for another year of experience
To test $H_0: \beta_4 = \beta_5$ against $H_1: \beta_4 \neq \beta_5$ I would transform the model as $Y = \beta_0 + \beta_1 X + \beta_2 D_2 + \beta_3 D_3 + \beta_5 X^*(D_2 + D_3) + \gamma X^* D_3 + w$ where $\gamma = \beta_4 - \beta_5$.
I would then test $H_0: \gamma = 0$ against $H_1: \gamma \neq 0$ using a t-test of γ @ the 5% level. If I were to accept the hypothesis that $\beta_4 = \beta_5$ then this model would suggest that while both Harvard & Wharton MBA's have an additional effect on income through experience, over Binghamton MBAs, this additional effect is no different when comparing b/w Harvard & Wharton. In essence Harvard & Wharton are better than Binghamton, but they are equivalently better when considering a Binghamton MBA.

(2) $Y_t = \beta_0 + \beta_1 X_t + U_t$ where $U_t = \rho U_{t-2} + \varepsilon_t$ $\varepsilon_t \sim iid(0, \sigma_\varepsilon^2)$

(+15) 10 observations and ρ is known.

a) To transform the data so that the Cochrane-Orcutt procedure could be used, we have $Y_t^* = Y_t - \rho Y_{t-2}$, $X_t^* = X_t - \rho X_{t-2}$, $\beta_0^* = (1-\rho)\beta_0$ and we have to drop the first 2 observations of both Y_t & X_t since Y_{-1} Y_0 & X_0 X_{-1} do not exist. Our model is $Y_t^* = \beta_0^* + \beta_1 X_t^* + \varepsilon_t$ (8 obs)
b) If ρ was not known I would estimate the original model by OLS and find the residuals $(\hat{U}_t)$. I would then run the auxilary regression $\hat{U}_t = \rho \hat{U}_{t-2} + \varepsilon_t$ by OLS with 8 observations to obtain an estimate of ρ. I would then use this estimate to transform the data as in part a) and then run OLS to the transformed model in a) w/ 8 obs to find β_0 & β_1.

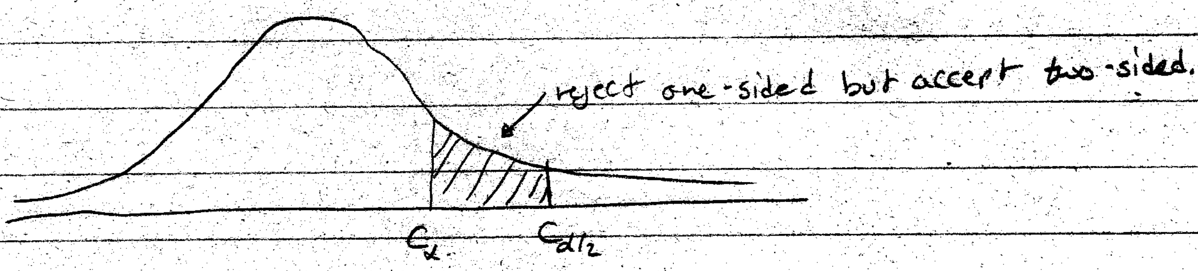
c) To test the model for autocorrelation we would want to test the hypothesis $H_0: \hat{\rho}=0$ against $H_1: \hat{\rho} \neq 0$ using a t-test from our auxilary regression in part b). This test should probably be @ the 10 or 15% level due to the fact that there are so few observations. $df = 10 - 3 = 7$

3
+20

a) If a Dummy variable is (0, 2) instead of (0, 1) the estimated coefficients value will be halved because we are scaling the variable by 2. $\boxed{\text{TRUE}}$

b) As the sample size gets larger the ability to determine a coefficients true value should be clearer. If the null is $\beta_j = 0$ against the alternative $\beta_j \neq 0$ then as the sample gets larger our t-statistic should reflect more accurately the value of β_j in the population. If β_j truly is zero then $t_{\hat{\beta}_j}$ should be getting smaller, sco as to ensure that we are failing to reject. $\boxed{\text{FALSE}}$

c) If one rejects the null under a one-sided alternative with a t-test lets say @ the 5% level switching to a two-sided test @ the same level then one could only reject the null if the p-value for the one sided statistic was less than or equal to 2.5%. $\boxed{\text{FALSE}}$



d) The first regression may be better than the second regression but because of the higher R^2. The first model uses Y and the second uses $\ln Y$. Since the two regressands are different the R^2s from the two models are not comparable. My concluding criterion is wrong. $\boxed{\text{FALSE}}$

4 +10 Bonus $Y=\beta_0+\beta_1 X+\beta_2 Z+u$ and you have data on Y & X but not Z. To obtain a consistent estimate of β_1 you can either run the regression w/o Z if there is evidence that X & Z are independent or you can find a variable that suffices as a good proxy for Z.

5 +16 $E=\beta_0+\beta_1 Y+\beta_2 F+u$ and $\hat{\beta}_1<0$.

a) $\hat{\beta}_1$ should be positive since food in general is not an inferior good. Therefore as $Y\uparrow$ so too should E. Families w/ more money may not eat more food, but they probably eat more expensive food.

b) if $var(u_i)=\sigma^2 F_i$ then to eliminate heteroskedasticity you would run OLS on the model $\frac{E}{\sqrt{F}}=\beta_0\left(\frac{1}{\sqrt{F}}\right)+\beta_1\left(\frac{Y}{\sqrt{F}}\right)+\beta_2\sqrt{F}+\varepsilon$

c) To test to see if my friend is right I can run OLS to the original model model and obtain the squared residuals ($\hat{u}^2$). Then I can run can auxilary regression $\hat{u}^2=\alpha_0+\alpha_1 F+\varepsilon$ and perform a t-test on α_1. The null is $H_0: \alpha_1=0$ against $H_1: \alpha_1\neq 0$. Depending on the sample size, I would pick an appropriate level for the test.

$\alpha=.05$ or $\alpha=.10$

(6) Demand : $Q = \alpha_1 P + \alpha_2 z_1 + u_1$ Q & P are endogenous.

Supply : $Q = \beta_1 P + \beta_2 z_2 + u_2$

(+25)

a) The demand equation is identified because z_2 is not in the demand equation but is in the supply equation.
The supply equation is identified because z_1 is not in the supply equation but is in the demand equation.

b) Solve supply for P

$$\frac{Q}{\beta_1} - \frac{\beta_2}{\beta_1} z_2 - \frac{u_2}{\beta_1} = P$$

Plug P into the demand equation

$$Q = \frac{\alpha_1}{\beta_1} Q - \frac{\alpha_1 \beta_2}{\beta_1} z_2 - \frac{\alpha_1}{\beta_1} u_2 + \alpha_2 z_1 + u_1$$

$$Q\left(1 - \frac{\alpha_1}{\beta_1}\right) = \alpha_2 z_1 - \frac{\alpha_1 \beta_2}{\beta_1} z_2 - \frac{\alpha_1}{\beta_1} u_2 + u_1$$

$$Q = \frac{\beta_1 \alpha_2}{\beta_1 - \alpha_1} z_1 - \frac{\alpha_1 \beta_2}{\beta_1 - \alpha_1} z_2 - \frac{\alpha_1}{\beta_1 - \alpha_1} u_2 + \frac{\beta_1}{\beta_1 - \alpha_1} u_1 = \Pi_{11} z_1 + \Pi_{12} z_2 + v_1$$

Plug Q back into P

$$P = \frac{\alpha_2}{\beta_1 - \alpha_1} z_1 - \frac{\alpha_1 \beta_2}{\beta_1(\beta_1 - \alpha_1)} z_2 - \frac{\alpha_1}{\beta_1} \frac{u_2}{\beta_1 - \alpha_1} + \frac{u_1}{\beta_1 - \alpha_1} - \frac{\beta_2}{\beta_1} z_2 - \frac{u_2}{\beta_1}$$

$$P = \frac{\alpha_2}{\beta_1 - \alpha_1} z_1 - z_2\left(\frac{\alpha_1 \beta_2}{\beta_1(\beta_1 - \alpha_1)} + \frac{\beta_2}{\beta_1}\right) + \frac{u_1}{\beta_1 - \alpha_1} - u_2\left(\frac{\alpha_1}{\beta_1(\beta_1 - \alpha_1)} + \frac{1}{\beta_1}\right)$$

$$\frac{\alpha_1 \beta_2}{\beta_1(\beta_1-\alpha_1)} + \frac{\beta_2}{\beta_1} = \frac{\alpha_1 \beta_2 + \beta_2(\beta_1-\alpha_1)}{\beta_1(\beta_1-\alpha_1)} = \frac{\beta_2}{\beta_1-\alpha_1}$$

$$\frac{\alpha_1}{\beta_1(\beta_1-\alpha_1)} + \frac{1}{\beta_1} = \frac{\alpha_1+(\beta_1-\alpha_1)}{\beta_1(\beta_1-\alpha_1)} = \frac{1}{\beta_1-\alpha_1}$$

$$P = \frac{\alpha_2}{\beta_1-\alpha_1} z_1 - \frac{\beta_2}{\beta_1-\alpha_1} z_2 + \frac{v_1}{\beta_1-\alpha_1} - \frac{v_2}{\beta_1-\alpha_1} = \pi_{21} z_1 + \pi_{22} z_2 + v_2$$

C) $\hat{\pi}_{11} = -1 \quad \hat{\pi}_{12} = -.6 \quad \hat{\pi}_{21} = -.5 \quad \hat{\pi}_{22} = 1.2$

$$(\alpha_1-\beta_1)\hat{\pi}_{21} = -\alpha_2 \qquad (\alpha_1-\beta_1)\hat{\pi}_{22} = \beta_2$$

$$\hat{\pi}_{12} = \frac{\beta_2 \alpha_1}{\alpha_1-\beta_1} = \frac{(\alpha_1-\beta_1)\hat{\pi}_{22}\alpha_1}{\alpha_1-\beta_1} = \hat{\pi}_{22}\alpha_1 \qquad \hat{\alpha}_1 = \hat{\pi}_{12}/\hat{\pi}_{22} = \frac{-.6}{1.2} = -.5$$

$$\hat{\pi}_{11} = \frac{-\beta_1 \alpha_2}{\alpha_1-\beta_1} = \frac{\beta_1(\alpha_1-\beta_1)\hat{\pi}_{21}}{\alpha_1-\beta_1} = \hat{\pi}_{21}\beta_1 \qquad \hat{\beta}_1 = \hat{\pi}_{11}/\hat{\pi}_{21} = \frac{-1}{-.5} = 2$$

$$\hat{\beta}_2 = (\hat{\alpha}_1-\hat{\beta}_1)\hat{\pi}_{22} = (-.5-2)\,1.2 = -3$$
$$\hat{\alpha}_2 = -(\hat{\alpha}_1-\hat{\beta}_1)\hat{\pi}_{21} = -(-.5-2)(-.5) = -1.25$$

$\hat{\alpha}_1 = -.5 \quad \hat{\alpha}_2 = -1.25$
$\hat{\beta}_1 = 2 \quad \hat{\beta}_2 = -3$