Econ 466, Assignment #2

Note Title 9/12/2006

1. Use the definitions $\bar{X} = \frac{1}{n}\sum X_i$ and $s_X^2 = \frac{1}{n}\sum(X_i - \bar{X})^2$ to show that if $Y_i = a + bX_i$ then

(i) $\bar{Y} = a + b\bar{X}$ and $s_Y^2 = \frac{1}{n}\sum(Y_i - \bar{Y})^2 = b^2 s_X^2$.

(ii) If $Z = \dfrac{X_i - \bar{X}}{s_X}$ then $\bar{Z} = 0$ and $s_Z^2 = 1$.

2. Show that $\sum(X_i - \bar{X})(Y_i - \bar{Y}) =$

(i) $\sum X_i Y_i - n\bar{X}\bar{Y}$

(ii) $\sum(X_i - \bar{X})Y_i$

(iii) $\sum X_i(Y_i - \bar{Y})$

3. If $k_i = \dfrac{X_i - \bar{X}}{\sum(X_i - \bar{X})^2}$ then

(i) $\sum k_i = 0$,

(ii) $\sum k_i^2 = \dfrac{1}{\sum(X_i - \bar{X})^2}$

(iii) $\sum k_i(X_i - \bar{X}) = \sum k_i X_i = 1$

4. Use $\hat{\beta}_1 = \dfrac{\sum(X_i - \bar{X})(Y_i - \bar{Y})}{\sum(X_i - \bar{X})^2}$ to show that

(i) $\hat{\beta}_1 = \sum(X_i - \bar{X})Y_i \,/\, \sum(X_i - \bar{X})^2$

(ii) $R^2 = \dfrac{SSE}{SST} = \hat{\beta}_1^2 \sum(X_i - \bar{X})^2 \,/\, \sum(Y_i - \bar{Y})^2$

(iii) $R^2 = r_{YX}^2 = \{cov(X,Y)\}^2 / var(X) \cdot var(Y)$

5. Consider the following regression

$$Y_i = \beta X_i + u_i$$

Show that the OLS estimator of β is

(i) $\hat{\beta} = \sum X_i Y_i / \sum X_i^2$

(ii) $\dfrac{\sum \hat{u}_i}{n} = \frac{1}{n} \sum (Y_i - \hat{Y}_i) \neq 0$

(iii) $\sum \hat{u}_i X_i \neq 0$