# Econ 466

Homework Assignment #2, Due: February 16.

1. Problem 2.8 from Introductory Econometrics by Jeffrey Wooldridge.

2. Let $\hat{\beta}_{YX}$ and $\hat{\beta}_{XY}$ represent the slopes in the regression of Y on X ($Y = \beta_0 + \beta_{YX} X + u$) and X on Y ($X = \alpha_0 + \beta_{XY} Y + v$), respectively. Show that $\hat{\beta}_{YX}\hat{\beta}_{XY} = r^2$ (see the formula for *r* in problem number 5 below).

3. Consider the following formulation of the two-variable regression:

$$\text{Model I:} \quad Y_i = \beta_1 + \beta_2 X_i + u_i$$

$$\text{Model II:} \quad Y_i = \alpha_1 + \alpha_2 \left(X_i - \overline{X}\right) + u_i$$

a) Find the estimators of $\beta_1$ and $\alpha_1$. Are they identical? Are there variances identical?

b) Find the estimators of $\beta_2$ and $\alpha_2$. Are they identical? Are there variances identical?

c) What is the advantage, if any, of model II over model I?

4. *Regression without any Regressor.* Suppose you are given the model: $Y_i = \beta_1 + u_i$. Use OLS to find the estimator of $\beta_1$. What is its variance and the RSS (residual sum of squares)? Does the estimated $\beta_1$ make intuitive sense? Now consider the standard two-variable model we have been discussing in class. Is it worth adding the X variable to the model? If not, why bother with regression analysis?

5. Let $r_1$ be the coefficient of correlation between n pairs of values $(Y_i, X_i)$ and $r_2$ be the coefficient of correlation between n pairs of values $(aY_i + b, cX_i + d)$ where a, b, c, and d are constants. Show that $r_1 = r_2$ and hence establish the principle that the coefficient of correlation is invariant with respect to the change of scale and the change of origin. **Hint:**

$$r = \frac{n\sum X_i Y_i - \left(\sum X_i\right)\left(\sum Y_i\right)}{\sqrt{\left[n\sum X_i^2 - \left(\sum X_i\right)^2\right]\left[n\sum Y_i^2 - \left(\sum Y_i\right)^2\right]}}$$

6. If r, the coefficient of correlation between n pairs of values $(X_i, Y_i)$, is positive, then determine whether each of the following statements is true of false:
   a. r between $(-X_i, -Y_i)$ is also positive.
   b. r between $(-X_i, Y_i)$ and that between $(X_i, -Y_i)$ can be either positive or negative.
   c. Both the slope coefficients $\beta_{XY}$ and $\beta_{YX}$ are positive where the definitions of the slope coefficients follow from 2. above.