

Fall 2002
Econ 466
Examination III
Total: 100 points
Time: 1 hour and 15 minutes

Answer all questions. Write clearly and legibly. Good Luck!!

1. a. What is heteroskedasticity? Why is it a problem? **(5+5 points)**
- b. Consider the model
$$Y_t = \beta X_t + u_t \text{ where } V(u_t | X_t) = \sigma^2 X_t^2, t = 1, \dots, n$$

How would you estimate β in this model? Show the steps and derive the WLS estimator of β . **(7 points)**
- c. Now consider the model
$$Y_t = \beta_0 + \beta_1 X_t + u_t, \text{ where } V(u_t | X_t) = \sigma^2 h(X_t).$$

(i) If $h(X_t)$ is known, how would you estimate β_0 , β_1 and σ^2 ?
Show all the steps. **(7 points)**
(ii) If $h(X_t)$ is not known, how would you test for heteroskedasticity?
Show the steps. **(5 points)**
(iii) If $\sigma^2 h(X_t) = \delta_0 + \delta_1 X_t$, how would you estimate β_0 and β_1 correcting for heteroskedasticity? Show all the steps in details. **(7 points)**
2. You are interested in estimating the model
$$Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + u_t, \quad t = 1, \dots, n \tag{1}$$

However, after checking the data you found that $X_2 = 3X_1$.

- a. You decided to substitute this relationship into (1). Show that the slope coefficient of the regression Y on X_1 is $(\beta_1 + 3\beta_2)$. **(5 points)**
- b. Your friend ran the regression Y on X_2 . Show that the slope coefficient of the regression is $(\beta_2 + \frac{1}{3}\beta_1)$. **(5 points)**
- c. Show that $R_{YX_1}^2 = R_{YX_2}^2$, that is both you and your friend will get the same R^2 .
Any comment? **(10 points)** (Hint: Note that $R_{YX_1}^2 = r_{YX_1}^2$ and $R_{YX_2}^2 = r_{YX_2}^2$).

3. Consider the following model of the demand for airline travel, estimated using annual data for the period 1947-1987. The number of observations is therefore 41.

$$\ln(Q) = \beta_1 + \beta_2 \ln(P) + \beta_3 \ln(Y) + B_4 \ln(ACCID) + B_5 FATAL + u$$

where

Q = Per-capita passenger miles traveled in a given year

P = Average price per mile

Y = Per-capita income

ACCID = Accident rate per passenger mile

FATAL = Number of fatalities from aircraft accidents

The model is double-log except for the fact that FATAL is not expressed in log form because the observation for some of the years is zero.

In 1979 the airlines were deregulated. Define the dummy variable D that takes the value 0 for 1947-1978 and 1 for 1979-1987. The following table presents the relevant values for two models. Model A is the basic model given above. Model B is the one derived by assuming that there has been a structural change of the entire relation

| Estimated Coeff | Variable | Model A Coeff (Std err) | Model B Coeff (Std err) |
|--------------------|----------------------|-------------------------------|-------------------------------|
| $\hat{\beta}_1$ | CONSTANT | 2.938 (1.050) | 2.635 (1.326) |
| $\hat{\beta}_2$ | $\ln(P)$ | -1.312 (0.315) | -1.029 (0.377) |
| $\hat{\beta}_3$ | $\ln(Y)$ | 0.716 (0.289) | -0.001 (0.433) |
| $\hat{\beta}_4$ | $\ln(ACCID)$ | -0.541 (0.100) | -0.821 (0.156) |
| $\hat{\beta}_5$ | fatal | 0.0004 (0.0003) | 0.0009 (0.0003) |
| $\hat{\beta}_6$ | D | | -1.688 (0.388) |
| $\hat{\beta}_7$ | $D \cdot \ln(P)$ | | 0.278 (0.796) |
| $\hat{\beta}_8$ | $D \cdot \ln(Y)$ | | 0.987 (0.558) |
| $\hat{\beta}_9$ | $D \cdot \ln(ACCID)$ | | 0.818 (0.252) |
| $\hat{\beta}_{10}$ | $D \cdot FATAL$ | | -0.001 (0.0006) |
| SSR | | 1.096191 | 0.700959 |
| R^2 | | 0.972 | 0.979 |

- a. Interpret the coefficients β_9 and β_{10} first and then comment on their estimated values. **(3+2 points)**
- b. Deregulation is supposed to reduce the accident rate per passenger mile. How would test such a hypothesis? Perform the test at the 5% level of significance. **(3 points)**
- c. Carry out a test for structural change after deregulation. To do this complete the following steps. **(6 points)**
 - (i) Write down the null hypothesis.
 - (ii) Compute the numerical value of the F statistic.
 - (iii) Carry out the test using a 5% level of significant.
- d. Compute the elasticity of Q with respect to P before and after deregulation. **(5 points)**
- e. Compute the elasticity of Q with respect to Y before and after deregulation. **(5 points)**
- f. Someone told you that the variable FATAL should not be included in the model. How would you test such a hypothesis in Model B? **(4 points)**

4. Consider the model

$$Y_t = \beta_0 + \beta_1 X_t + u_t \text{ where } u_t = \rho u_{t-1} + \varepsilon_t, |\rho| < 1 \text{ and } \varepsilon_t \sim (0, \sigma_\varepsilon^2).$$

- (i) Assume that ρ is **known**. How would you estimate β_0 and β_1 correcting for autocorrelation? Explain. **(8 points)**
- (ii) Assume that ρ is **unknown**. How would you estimate β_0 and β_1 correcting for autocorrelation? Explain. **(8 points)**