Key Test #3

Heteroscedasticity is a problem arising from the error term having a nonconstant value, that is a functional form of one or more of the specified independent variables, for the conditional variance. The existence of heteroscedasticity makes the estimator of $var(\beta_j)$ biased and thus t-values, F. values are incorrect.

$$Y_t = \beta X_t + U_t \quad \text{where} \quad V(U_t \mid X_t) = \sigma^2 x_t^2$$

How to estimate β in this model

① Take $Y_t = \beta X_t + U_t$ and divide the T observations by $\sqrt{X_t}$

so $$y_t/\sqrt{x_t} = \beta(X_t/\sqrt{X_t}) + U_t/\sqrt{X_t}$$

$$Y_t/\sqrt{X_t} = \beta\sqrt{X_t} + U_t/\sqrt{X_t}$$

$$(*) \quad y_t^* = \beta x_t^* + U_t^*$$

② Run OLS on *

$$\beta^{WLS} = \frac{\sum_{t=1}^{T} x_t^* y_t^*}{\sum_{t=1}^{T} (x_t^*)^2}$$

$Y_t = \beta_0 + \beta_1 X_t + U_t$ where $V(U_t \mid X_t) = \sigma^2 h(X_t)$

(i) if $h(X_t)$ is known do same as before to get β_0 & β_1.
To find σ^2

(ii) If $h(X_t)$ is not known can do white's test or B-P test

(iii) if $\sigma^2 h(x_t) = \delta_0 + \delta_1 X_t$ how would you correct for heteroscedasticity
(i) run OLS, retain residuals, square them
(ii) run OLS of $\hat{U}^2$ on X_t or $\log(\hat{U}^2)$ on X_t
(iii) from the OLS find fitted weights or obtain fitted values $\hat{g}$
(iv) use WLS use weights $\hat{h}_i = e^{\hat{g}_i}$

2. $Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + u_t \qquad t = 1, 2, \ldots, n$

$X_{2t} = 3X_{1t}$

a) $Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 3X_{1t} + u_t$

$Y_t = \beta_0 + \gamma_1 X_{1t} + u_t \qquad \gamma_1 = \beta_1 + 3\beta_2$

b) $Y_t = \beta_0 + \beta_1 \frac{X_{2t}}{3} + \beta_2 X_{2t} + u_2$

$Y_t = \beta_0 + \gamma_2 X_{2t} + u_t \qquad \gamma_2 = \frac{1}{3}\beta_1 + \beta_2$

c) $R^2_{yx_1} = R^2_{yx_2}$ or $R^2_{yx_1} = r_{yx_1} = r_{yx_2} = R^2_{yx_2}$

$R^2_{yx_1} = \frac{ESS_{x_1}}{TSS_y} = \frac{ESS_{x_2}}{TSS_y} = R^2_{yx_2}$

$= \frac{cov(y, x_1)}{\sqrt{var(y)\, var(x_1)}} = \frac{cov(y, x_2)}{\sqrt{var(y)\, var(x_2)}}$

show $ESS_{x_1} = ESS_{x}$

$\frac{cov(y, x_1)}{\sqrt{var(y)\, var(x_1)}} = \frac{cov(y, 3x_1)}{\sqrt{var(y)\, var(3x_1)}}$

$= \frac{3\, cov(y, x_1)}{\sqrt{var(y)\, 9\, var(x_1)}}$

$= \frac{3\, cov(y, x_1)}{3\sqrt{var(y)\, var(x_1)}}$

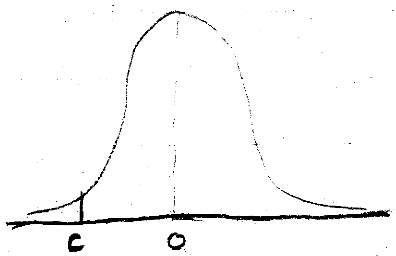
$r_{yx_1} = r_{yx_2}$

3. b) $H_0: \beta_9 = 0$ $\quad t = \frac{.818}{.252} = 3.246 \quad n-k-1 = 31$

$H_1: \beta_9 < 0$

test @ 5% level $\quad c = -1.697 \quad$ fail to reject

$t = 3.246$



c) i) $H_0: \beta_6 = \beta_7 = \beta_8 = \beta_9 = \beta_{10} = 0$ $\quad$ 5 restrictions

$H_1: H_0$ is false

ii) $F = \frac{(R^2_u - R^2_R)/q}{(1-R^2_u)/n-k-1} = \frac{(.979 - .972)/5}{(1-.979)/41-10} = \frac{.0014}{.000677} = 2.0\overline{66}$

$F < c$ $\quad$ reject @ 5% level $\quad$ no structural change after deregulation in 1979.

d) Elasticity of Q wrt P before & after deregulation

Before: -1.029

After: $-.751$

e) Elasticity of Q wrt Y before & after deregulation

Before: $-.001$

After: $.986$

f) F-test

$H_0: \beta_5 = \beta_{10} = 0$

$H_1: H_0$ is false

(4) $Y_t = \beta_0 + \beta_1 X_t + U_t \qquad U_t = \rho U_{t-1} + \varepsilon_t \qquad |\rho|<1 \qquad \varepsilon_t \sim (0, \sigma_\varepsilon^2)$

(i) assume ρ is known

$$Y_t = \beta_1 + \beta_2 X_t + U_t \qquad (1)$$
$$Y_{t-1} = \beta_1 + \beta_2 X_{t-1} + U_{t-1} \qquad (2)$$

a) multiply (2) by ρ on both sides

$$\rho Y_{t-1} = \rho \beta_1 + \rho \beta_2 X_{t-1} + \rho U_{t-1} \qquad (3)$$

b) subtract (3) from (1)

$$Y_t - \rho Y_{t-1} = \beta_1 (1-\rho) + \beta_2 (X_t - \rho X_{t-1}) + (U_t - \rho U_{t-1})$$
$$= \beta_1 (1-\rho) + \beta_2 (X_t - \rho X_{t-1}) + \varepsilon_t$$

$$Y_t^* = \beta_1^* + \beta_2^* X_t^* + \varepsilon_t$$

c) Since ε_t satisfies OLS assumptions we can run OLS to get unbiased estimators

Note that first observation is excluded

reason $Y_1^* = Y_1 - Y_0$ but we dont have Y_0 information

$$\hat{\beta}_1^* = \overline{Y^*} - \beta^* \bar{X}^*$$

$$\hat{\beta}_2^* = \frac{\sum_{t=2}^{n} (x_t - \rho x_{t-1})(y_t - \rho y_{t-1})}{\sum_{t=2}^{n} (x_t - \rho x_{t-1})^2}$$

(ii) assume that ρ is unknown

a) run OLS on $Y_t = \beta_1 + \beta_2 X_t + u_t$
and obtain $\hat{u}_t$

b) run regression $\hat{u}_t = \rho \hat{u}_{t-1} + v_t$ and obtain $\hat{\rho}$.

c) use $\hat{\rho}$ as you would in part (i)