Fall 2003
Econ 466
Final Examination
Total: 100 points
Time: 2 hours

Answer all questions. Write clearly and legibly. Good Luck!!

1. The linear regression model, as you have been exposed to it in this class, consists of the following equations and assumptions:

$$Y_t = \beta_0 + \beta_1 X_t + u_t,\ t = 1,...,n \qquad (1)$$

with E(u_t) = 0, and V(u_t) = σ^2. The u_t are independent across observations.

(a) Under the above assumptions show that the OLS estimator of β_0 and β_1 are unbiased and consistent. (**8 points**)

(b) If E(u_t) = 0 but V(u_t) = σ^2 is replaced by $V(u_t \mid X_t) = \sigma^2 X_t^2, \quad t = 1,...,n$, what are the properties of the OLS estimator of β_0 and β_1? How would you estimate β_0 and β_1 in this model taking the above heteroskedasticity problem into account? Show all the steps. (**10 points)**

(c) You suspect that the model is heteroskedastic (i.e., $V(u_t \mid X_t) = \sigma_t^2$).
   (i) How would you test for heteroskedasticity? Explain a test of your choice and explain step-by-step how you would perform the test. (**5 points)**
   (ii) If you find evidence for heteroskedasticity, how would you obtain heteroskedasticity corrected standard errors of the OLS of estimators β_0 and β_1? Explain the steps. (**5 points)**

(d) Consider the model in (1). Now assume that E(u_t) = 0 but Y is a dummy variable. Show that Y_t is heteroskedastic. How would you estimate such a model taking the heteroskedasticity problem into account? Explain the steps. (**10 points)**

(e) Consider the model in (1). If E(u_t) = 0 but the assumption that u_t's are independent over observation is replaced by $u_t = \rho u_{t-1} + \varepsilon_t, |\rho| < 1$ and $\varepsilon_t \sim$i.i.d.$(0, \sigma_\varepsilon^2)$. Are the OLS estimator of β_0 and β_1 unbiased? How would you estimate β_0 and β_1 correcting for autocorrelation if (i) ρ is known, and (ii) ρ is unknown? Show all the steps in each case. (**12 points)**

(f) If the X variable in model (1) is measured with error (i.e., $X_t = X_t^* - u_t$) and the true model is $Y_t = \beta_0 + \beta_1 X_t^*$ can the OLS estimator of β_1 from (1) be unbiased/consistent. State your assumptions clearly. (**8 points)**

2. Consider the following relationship between the amount of money spent by a state on welfare programs (Y) and the state's revenue (X): (**8 points)**

$$Y = \alpha_1 + \alpha_2 D1 + \alpha_3 X + \alpha_4 (D1 * X) + u$$

where D1 is a dummy variable that takes the value 1 if the state legislature is controlled by Democrats and 0 otherwise, and * means multiplication. Your friend decided to define another dummy variable, D2, which takes the value 1 if the legislature is controlled by non-Democrats and 0 otherwise, and estimate the following model instead.

$$Y = \beta_1 + \beta_2 D2 + \beta_3 X + \beta_4 (D2 * X) + u$$

Describe step by step how you can obtain estimates of the α's from those of the β's, *without running another regression*. That is, you should derive expressions for the α's in terms of the βs.

3. The table below has the estimated coefficients and associated statistics (standard errors in parentheses) for the demand for cigarettes in turkey. The data are annual for the years 1960-1988 (29 observations). The definitions of the variables are as follows:

LQ = Logarithm of cigarette consumption per adult (dependent variable)
LY = Logarithm of per-=capita real GNP in 1968 prices (in Turkish liras)
LP = Logarithm of real price of cigarettes in Turkish liras per kg.

D82 = 1 for 1982 onward, 0 prior to that

D86 = 1 for 1986 onward, 0 prior to that

LYD1 = LY multiplied by D82

LYD2 = LY multiplied by D86

LPD1 = LP multiplied by D82

LPD1 = LP multiplied by D86

	Variable	Model A	Model B	Model C	Model D
$\hat{\beta}_1$	CONSTANT	-4.997 (0.511)	-4.800 (0.677)	-4.186 (0.535)	-4.997 (0.509)
$\hat{\beta}_2$	D82	23.364 (5.547)	-0.108 (0.207)	-0.103 (0.026)	21.793 (5.254)
$\hat{\beta}_3$	D86	-36.259 (12.859)	-0.406 (0.236)	-0.103 (0.037)	-28.291 (9.433)
$\hat{\beta}_4$	LY	0.732 (0.069)	0.705 (0.091)	0.621 (0.071)	0.732 (0.068)
$\hat{\beta}_5$	LYD1	-2.798 (0.661)			-2.602 (0.623)
$\hat{\beta}_6$	LYD2	4.251 (1.516)			3.298 (1.098)
$\hat{\beta}_7$	LP	-0.371 (0.097)	-0.337 (0.129)	-0.201 (0.090)	-0.371 (0.097)
$\hat{\beta}_8$	LPD1	0.405 (0.206)	0.016 (0.246)		0.288 (1.787)
$\hat{\beta}_9$	LPD2	-0.236 (0.258)	0.288 (0.250)		
$\bar{R}^2$	---	0.921	0.859	0.852	0.921
R^2	---				
SSR	---	0.0186	0.0364	0.04195	0.0194

Note: Values in parentheses are the corresponding standard errors.

a. *In Model D*, write down the numerical values of the income and price elasticities of demand for each of the three periods (enter them in the flowing table). **(12 points)**

Period	**Income Elasticity**	**Price Elasticity**
1960-1981		
1982-1985		
1986-1988		

b. Compute R^2 values for Models A- D. (**8 points)**

c. Use the information in the above table to test for the null hypothesis that:

   i. The price and income elasticities for 1960-1981, 1982-1985, and 1986-1988 are the same. (**7 points)**
   ii. The income elasticities for 1960-1981, 1982-1985, and 1986-1988 are the same but the price elasticities are NOT the same. (**7 points)**

Hint: To do the above complete the following for each part: identify the restricted and unrestricted models ; write down the null hypothesis in terms of the β s; compute the numerical values of the test statistics; state the distribution and degrees of freedom of the test statistic; and perform the test at the 1% level of significance.