Economics 466
Exam I, Spring 2004

Total Points: 100, Time 1.5 hrs

Answer all questions. Note that each question has different weight. **Good Luck!**

1. Consider the model: $Y = \beta_0 + \beta_1 X + u$ and that you are given the following information:

$$n = 500, \quad \sum X = 24{,}000, \quad \sum Y = 10{,}700$$
$$\sum (X - \bar{X})^2 = 66{,}000, \quad \sum (Y - \bar{Y})^2 = 1{,}398{,}000$$
$$\sum (X - \bar{X})(Y - \bar{Y}) = 194{,}000$$

   a) Find the estimates of the slope and intercept from the model. **(6 points)**

   b) Show that $\hat{\beta}_1 = \dfrac{Cov(X,Y)}{Var(X)} = r\sqrt{\dfrac{Var(Y)}{Var(X)}}$ where r is the correlation coefficient. **(5 points)**

   c) Show that the fitted line $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X$ passes through the mean of the data (i.e. $\bar{X}, \bar{Y}$ is a point on the fitted line). **(4 points)**

   You may answer the questions below (parts (d)- (g)) either algebraically or using the above data.

   d) Prove that $\sum \hat{u} = 0,$ where $\hat{u} = Y - \hat{\beta}_0 - \hat{\beta}_1 X$. **(5 points)**

   e) Prove that $\sum \hat{u} X = 0$. **(5 points)**

   f) Show that $\sum (X - \bar{X})(Y - \bar{Y}) = \sum (X - \bar{X})Y = \sum (Y - \bar{Y})X$. **(6 points)**

   g) If the model is $Y = \beta_1 X + u$ compute $\hat{\beta}_1$. Is $\sum \hat{u} = 0$? Is $\sum \hat{u} X = 0$? Note that $\hat{u} = Y - \hat{\beta}_1 X$. **(4+4 points)**

2. Using **quarterly** data for 10 years, the following model of demand for new cars was estimated.

$$NUMCARS = \beta_1 + \beta_2 PRICE + \beta_3 INCOME + \beta_4 INTRATE + \beta_5 UNEMP + u$$

where
NUMCARS = Number of new car sales per thousand population
PRICE = New car price index
INCOME = Per-capita real disposable income (in actual dollars)
INTRATE = Interest rate
UNEMP = Unemployment rate

The following table has the estimates of the coefficients for three alternative models. Values in parentheses are standard errors (if an entry is blank, it indicates that the particular variable has been excluded from the model).

Variables	Model A	Model B	Model C
CONSTANT	-7.453352	-10.554074	15.238094
	(13.578200)	(4.621104)	(1.167467)
PRICE	-0.071391	-0.079392	-0.024883
	(0.0347300)	(0.0110220)	(0.0073660)
INCOME	0.003159	0.00356	
	(0.001763)	(0.000627)	
INTRATE	-0.153699	-0.146651	-0.204769
	(0.049190)	(0.039229)	(0.051442)
UNEMP	-0.072547		
	(0.298195)		
SSR	23.510464	23.550222	44.65914
$\overline{R}^2$	0.758	0.764	0.565
R^2			

a) Interpret the slope coefficients in Model A. **(8 points)**
b) Compute R^2 values for Models A-C using the formula $\overline{R}^2 = 1 - R^2 \cdot ((n-1)/(n-k-1))$. **(6 points)**
c) Test the hypothesis that $\beta_2 = 0$ (in Models A and B) against a **two-sided** alternative at the 5% level, being careful of how you set up the test and state the null. **(7 points)**
d) Test the hypothesis that $\beta_3 = 0$ (in Models B and C) against a **one-sided** alternative at the 5% level, being careful of how you set up the test and state the null. **(7 points)**
e) Find the p-values of the tests from parts b) and c) to within reasonable accuracy from your table. **(8 points)**
f) Test the joint hypothesis that $\beta_2 = 0, \beta_4 = 0$ at the 5% level. Be careful of how you state the alternative hypothesis. **(6 points)**
g) Perform the test of **no regression** for Models A and B at the 5% level of significance. Be sure to state the null and alternative hypotheses. **(8 points)**
h) If income were measured in thousands of dollars which coefficient(s) would change? What would its (their) new estimates be? **(5 points)**
i) For the model above, how would you test $H_0 : \beta_3 - 2\beta_4 = 0$? Give a detailed description of the alternative hypothesis, your significance level of choice, each step you would perform if you had access to excel and also the intuition behind the null hypothesis. **(6 points)**