Economics 466

Introduction to Econometrics

Spring 2007, Midterm Exam I

Total points - 100, Time - 1 hr. 15 min.

Answer all questions on the blue book. There are three questions in this exam.

1. (25 points, 5 each) Consider the following two-variable regression model: $Y = \beta_0 + \beta_1 X + u_.$ Prove the following algebraic properties.

   (a) $\sum_{i=1}^{n} \hat{u}_i = 0.$

   (b) $\sum_{i=1}^{n} X_i \hat{u}_i = 0.$

   (c) $\sum_{i=1}^{n} \hat{Y}_i \hat{u}_i = 0.$

   (d) $\overline{\hat{Y}} = \overline{Y}.$

   (e) $\sum_{i=1}^{n} \hat{y}_i \hat{u}_i = 0$, whereas y is the deviation of Y, i.e., $y = Y - \overline{Y}$

2. (28 points) We have the following summary of 10 observations on X and Y to estimate the simple regression $Y = \beta_0 + \beta_1 X + u_.$

   $\sum X = 40$, $\sum Y = 8$, $\sum X^2 = 200$, $\sum Y^2 = 26$, $\sum XY = 20$

   Use the above information to answer the following questions.

   (a) (10 points) Estimate the model using OLS method. Calculate the regression coefficients and their standard errors.

   (b) (5 points) What is the total variation in Y and how much of the variation in Y is explained by X?

   (c) (5 points) Calculate a 95% confidence interval for β_1.

   (d) (4 points) Test the hypothesis that β_1=1, using 5% level of significance.

   (e) (4 points) Test the hypothesis that there is no regression using 5% level of significance.

3. (47 points) Some Economists who study crime believe that locking up criminals deters crime. Suppose a politician who is running for office has employed you to help him determine if locking up criminals reduces crime. You gathered data on all 50 states and the District of Columbia on crime rates (Y) and prison populations per capita (X) to guide you in your analysis, which resulted in the following table. Answer each of the following questions. Standard errors are reported in parentheses beneath each estimate.

   (a) (5 points) Model I estimates $\ln(Y) = \beta_0 + \beta_1 \ln(X) + u$. Interpret $\hat{\beta}_1$.

   (b) (6 points) Describe a formal test that allows you to assess if prison population have a crime deterring effect. Be sure to state your null and alternative correctly.

   (c) (6 points) Perform the test you described in (b) at the 5% level of significance and construct a 95% confidence interval for β_1 as well.

   One might argue that it is not the prison population alone but things like per capita income might also influence crime rates. In view of this, you change the model and estimate the following model.

   Model II estimates

$$\ln(Y) = \beta_0 + \beta_1 \ln(X) + \beta_2 \ln PI + u$$

   where PI is per capital income.

   (d) (6 points) Interpret $\hat{\beta}_1$ and $\hat{\beta}_2$ in the above regression. What sign do you expect on the estimated β_2?

   (e) (6 points) State and test the hypothesis, at the 1% level, that prison population and per capita income has no effect on crime rates.

   (f) (5 points) If you measure PI in thousands of dollars what would happen to the estimated coefficient on $\ln PI$? Explain why it will/will not change.

   (g) (5 points) Explain in details how you would construct the 95% confidence interval of either the mean or the point predictor of $\ln(Y)$, given $ln(X) = -5$ and $\ln(PI) = 3$.

Dependent Variable: $\ln(Y)$		
Regressors	$(Model\ I)$	$(Model\ II)$
intercept	0.5462	0.5440
	(0.1215)	(0.1227)
$\ln(X)$	1.0693	1.0893
	(0.1093)	(0.1206)
$\ln(PI)$		−0.4132
		(1.0177)
Observations	51	51
R^2	0.6615	0.6627

Critical values you may find useful for Question 3:

1% Critical Values for the F-Distribution

$F(1,51) = 7.1595$, $F(2,51) = 5.0472$, $F(3,51) = 4.1906$, $F(2,50) = 5.0566$, $F(2,49) = 5.0665$, $F(2,48) = 5.0767$, $F(3,50) = 4.1994$, $F(3,49) = 4.2085$, $F(3,48) = 4.2180$

5% Critical Values for the t-Distribution **(2-sided)**

$t(50) = 2.0086$, $t(49) = 2.0096$, $t(48) = 2.0106$, $t(47) = 2.0117$

5% Critical Values for the t-Distribution **(1-sided)**

$t(50) = 1.6759$, $t(49) = 1.6766$, $t(48) = 1.6772$, $t(47) = 1.6779$