Answer Key Midterm 2 Spring 05

Question 1

$$C_t = \beta_0 + \beta_1 GNP_t + \beta_2 D_t + u_t \qquad \text{Mode 1}$$

a) $$V(u_t \mid GNP_t, D_t) = \sigma_t^2$$

One possible test is ~~the~~ White test

Step 1: estimate model 1, save ~~the~~ residuals, square them $\hat{u}_t^2$.

Step 2: run ~~the~~ regression below and write down its R^2

$$\hat{u}_t^2 = \delta_0 + \delta_1 GNP + \delta_2 D_t + \delta_3 GNP_t^2 + \delta_4 D_t^2 + \delta_5 GNP_t \cdot D_t + \text{error}$$

Test ~~the~~ hypothesis

$H_0: \delta_1 = \cdots = \delta_5 = 0 \Rightarrow$ homoskedasticity

H_1: not H_0

$$F = \frac{R^2_{\hat{u}^2}/5}{1 - R^2_{\hat{u}^2}/44}$$

Critical value $F_{5\%}(5, 44)$

$\Rightarrow$ reject if $F > F_{5\%}(5, 44)$

(ii) $$HSRE(\hat{\beta}_j) = \sqrt{\hat{Var}(\hat{\beta}_j)} = \sqrt{\frac{\sum_{i=1}^{n} \hat{r}_{ij}^2 \hat{u}_i^2}{SSR_j^2}}$$

where

$\hat{r}_{ij}^2$ - is the i^{th} residual from regressing x_j on all other independent variables

SSR_j^2 - sum of squared residual from the regression x_j on all other independent variables

$\hat{u}_i$ - ols residuals from model ①.

For e.g. to get $HRSE(\hat{\beta}_1)$

- Run model ① and save the residuals, square them $\hat{u}_i^2$
- Run the regression

  $GNP_t = \alpha_0 + \alpha_1 D_t$ + error

  save the residuals from this regression as $\hat{r}_{i1}$, square them $\hat{r}_{i1}^2$. Also write down the SSR as SSR_1.

  Then use the formula above to compute $HRSE(\hat{\beta}_1)$.

Repeat the steps for the other coefficients.

(b) $Var(u_t | GNP_t, D_t) = \sigma^2 GNP_t$ $E(u) = 0$

Properties of the OLS estimators — unbiased, consistent but inefficient.
$\Rightarrow$ no longer BLUE.

Taking ~~the~~ above heteroskedasticity into account, we would transform model ① so that the variance of the error term in the transformed model is equal to σ^2 (homoskedasticity).

$$\frac{C_t}{\sqrt{GNP_t}} = \frac{\beta_0}{\sqrt{GNP_t}} + \beta_1 \frac{GNP_t}{\sqrt{GNP_t}} + \beta_2 \frac{D_t}{\sqrt{GNP_t}} + \underbrace{\frac{u}{\sqrt{GNP_t}}}_{\to u^*}$$

$$E(u^*) = E\left(\frac{u}{\sqrt{GNP}}\right) = 0$$

$$Var(u^*) = Var\left(\frac{u}{\sqrt{GNP}}\right) = \frac{1}{GNP} \cdot Var(u)$$

$$= \frac{1}{\cancel{GNP}} \cdot \sigma^2 \cancel{GNP} = \sigma^2.$$

Run ~~the~~ transformed model, the estimators will be unbiased, consistent and efficient.

c) (i) Reason - model is heteroskedastic
Assumption - $Var(u_t | GNP_t, D_t) - \sigma^2 GNP_t^2$

Transforming ~~the~~ regression did not change the estimates by much $\Rightarrow$ heteroskedasticity was not a problem.

(ii) Can't compare R^2 of the two regression because the dependent variables are different.

d) $\hat{C} = E(\hat{C}) = 83.79$

$se(E(\hat{C})) = 4.77$

$se(\hat{C}) = \sqrt{4.77^2 + \hat{\sigma}^2} = \sqrt{4.77^2 + \left(\frac{2.97}{47}\right)} =$

Question 2

$$\ln(wage) = \beta_0 + \beta_1 \, usage + \beta_2 \, Edu + \beta_3 \, Exp + \beta_4 \, Female + \beta_5 \, Female * usage + u$$

a) β_4 - differential intercept

β_5 - differential slope coefficient

b) $$\ln(wage) = \alpha_0 + \alpha_1 Exp + \alpha_2 Edu + \alpha_3 Female * usage + \alpha_4 Female * nonuse + \alpha_5 Male * usage + u$$

Model 4

	Usage	Nonusage
Female	$\beta_0+\beta_1+\beta_4+\beta_5$	$\beta_0+\beta_4$
Male	$\beta_0+\beta_1$	β_0

Model 5

	Usage	Nonusage
Female	$\alpha_0+\alpha_3$	$\alpha_0+\alpha_4$
Male	$\alpha_0+\alpha_5$	α_0

Using ~~the~~ fitted values transform model (4) to get

$$\frac{\ln(wage)_i}{\sqrt{\hat{h}_i}} = \frac{\beta_0}{\sqrt{\hat{h}_i}} + \beta_1 \frac{usage_i}{\sqrt{\hat{h}_i}} + \beta_2 \frac{educ_i}{\sqrt{\hat{h}_i}} + \beta_3 \frac{exp_i}{\sqrt{\hat{h}_i}}$$

$$+ \beta_4 \frac{female_i}{\sqrt{\hat{h}_i}} + \beta_5 \frac{(female * usage)_i}{\sqrt{\hat{h}_i}} + \frac{u_i}{\sqrt{\hat{h}_i}}$$

This transformation correct for the heteroskedasticity. Estimate this equation. (FGLS)

e) This is equivalent to

$$\ln(wage) = \alpha_0 + \alpha_1 usage + \alpha_2 Educ + \alpha_3 Exp + \alpha_4 female + \alpha_5 female*usage + \alpha_6 female * educ + \alpha_7 female * exp$$

Your friend allows female to interact with <u>all</u> ~~the~~ other regressors.

Model (6) is more general.

Testing hypothesis

H_0: $\alpha_6 = \alpha_7 = 0$

H_1: not H_0

do a F-test.