Answer Key ECON 466 Midterm I Spring 05

Question 1

a) $$t\text{-value} = \frac{\hat{\beta}_j}{se(\hat{\beta}_j)}$$

$\Rightarrow t_{\hat{\beta}_0} = -5.204$, $t_{\hat{\beta}_1} = -7.292$, $t_{\hat{\beta}_2} = 4.548$

$t_{\hat{\beta}_3} = -5.577$

b) $\hat{\beta}_1$ – a 1% increase in car price results in a 1.75% decrease in predicted car sales, holding all other variables constant.

or elasticity of car sales with respect to price is -1.75

$\hat{\beta}_2$ – a 1% increase in average family income yields a 4.73% increase in predicted car sales, ceteris paribus.

$\hat{\beta}_3$ – a 1% increase in interest rate results in a 29% decrease in predicted car sales, ceteris paribus

R^2 – 74.8% of the variation in y is explained by the regressors.

c) It is not possible to use the table to find $\frac{\partial \hat{y}}{\partial X_1}$, $\frac{\partial \hat{y}}{\partial X_2}$, $\frac{\partial \hat{y}}{\partial X_3}$. Why?

$$\ln Y = \beta_0 + \beta_1 \ln X_1 + \beta_2 \ln X_2 + \beta_3 X_3 + u$$

$$\Rightarrow \hat{\beta}_1 = \frac{\partial \ln \hat{y}}{\partial \ln X_1} = \frac{\partial \hat{y} / \hat{y}}{\partial X_1 / X_1} = \frac{\partial \hat{y}}{\partial X_1} \cdot \frac{X_1}{\hat{y}}$$

$$\Rightarrow \frac{\partial \hat{y}}{\partial X_1} = \hat{\beta}_1 \cdot \frac{\hat{y}}{X_1}$$

Similarly,

$$\hat{\beta}_2 = \frac{\partial \ln \hat{y}}{\partial \ln X_2} \Rightarrow \frac{\partial \hat{y}}{\partial X_2} = \hat{\beta}_2 \cdot \frac{\hat{y}}{X_1}$$

$$\hat{\beta}_3 = \frac{\partial \ln \hat{y}}{\partial X_3} \Rightarrow \frac{\partial \hat{y}}{\partial X_3} = \hat{\beta}_3 \cdot \hat{y}$$

Thus to calculate these marginal effects we need the predicted values for each y, $\hat{y}$, and we also need the actual observations on X_1, X_2 X_3.

d) (i) $H_0: \beta_1 = -1$

$H_1: \beta_1 \neq -1$

$$t_{\hat{\beta}_1} = \frac{\hat{\beta}_1 - \beta_1}{se(\hat{\beta}_1)} = \frac{-1.75-(-1)}{0.24} = -3.125$$

$t_{5\%}(29) = 2.045 = C$

Since $|t_{\hat{\beta}}| > C \Rightarrow$ reject H_0

(ii) $H_0: \beta_2 = 4$

$H_1: \beta_2 \neq 4$

$$t_{\hat{\beta}_2} = \frac{\hat{\beta}_2 - \beta_2}{se(\hat{\beta}_2)} = 0.702$$

$0.702 < 2.045 \Rightarrow$ accept H_0.

(iii) $H_0: \beta_1 = \beta_2 = \beta_3 = 0$

H_1: not H_0

unrestricted model : $\ln Y = \beta_0 + \beta_1 \ln X_1 + \beta_2 \ln X_2 + \beta_3 X_3 + u$

Restricted model : $\ln Y = \beta_0 + u \Rightarrow R^2_R = 0$

$$F = \frac{R^2_{uR} - R^2_R / q}{(1 - R^2_{uR}) / n-k-1} = \frac{0.748 / 3}{(1-0.748)/29} = 28.69$$

$F(3, 29)$ at $5\% = 2.934$ = Critical value

$F > F_{5\%}(3,29) \Rightarrow$ reject H_0.

e) $H_0: \beta_1 = 3\beta_3 \Rightarrow H_0: \beta_1 - 3\beta_3 = 0$

H_1: not H_0

let $\theta_1 = \beta_1 - 3\beta_3$

$\Rightarrow \beta_1 = \theta_1 + 3\beta_3$

$\ln y = \beta_0 + \beta_1 \ln X_1 + \beta_2 \ln X_2 + \beta_3 X_3 + u$

$\ln y = \beta_0 + (\theta_1 + 3\beta_3) \ln X_1 + \beta_2 \ln X_2 + \beta_3 X_3 + u$

$= \beta_0 + \theta_1 \ln X_1 + 3\beta_3 \ln X_1 + \beta_2 \ln X_2 + \beta_3 X_3 + u$

(2) $\ln y = \beta_0 + \theta_1 \ln X_1 + \beta_2 \ln X_2 + \beta_3 (3 \ln X_1 + X_3) + u$

Run model (2) in excel, creating a new regressor $z = 3\ln X_1 + X_3$.

Test the statistical significance of θ_1

Question 2

a) $Y_i = \gamma_0 + \gamma_1 (X_i - \bar{X}) + u_i$

$$\Rightarrow \hat{\gamma}_0 = \bar{Y} = \frac{\sum Y_i}{n} = \frac{10,700}{500} = 21.4$$

$$\hat{\gamma}_1 = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sum (X_i - \bar{X})^2} = \frac{194,000}{66,000} = 2.939$$

Fitted line is $\hat{Y} = \hat{\gamma}_0 + \hat{\gamma}_1 (X_i - \bar{X})$

$$\Rightarrow \hat{Y} = 21.4 + 2.934 (X_i - \bar{X})$$

If fitted line passes through $(\bar{X}, \bar{Y})$

$$\Rightarrow \bar{Y} = 21.4 + 2.934 (\bar{X} - \bar{X})$$

$$\Rightarrow \bar{Y} = 21.4$$

So fitted line passes through $(\bar{X}, \bar{Y})$.

d) $Y_i = \beta_0 + \beta_1 X_i + u_i$

$$\Rightarrow \hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} \quad , \quad \hat{\beta}_1 = \frac{\sum (\bar{X}_i - X)(Y_i - \bar{Y})}{\sum (X_i - \bar{X})^2}$$

$$\Rightarrow \hat{\beta}_1 = 2.939 \quad , \quad \hat{\beta}_0 = 21.4 - 2.939(48) = -119.672.$$

$\hat{\gamma}_1 = \hat{\beta}_1$ because subtracting a constant term from each observation on X does not change the slope coefficient of the regression.

b) $SSE = \hat{\gamma}_1^2 \sum(x_i - \bar{x})^2 = (2.939)^2 (66,000) = 570089.59$

$SSR = SST - SSE = 1,398,000 - 570089.59$
$= 827,910.41$

$R^2 = \frac{SSE}{SST} = 0.408$

c) $\hat{Var}(\hat{\gamma}_1) = \frac{\hat{\sigma}^2}{\sum(x_i - \bar{x})^2}$, $\hat{\sigma}^2 = \frac{SSR}{n-k} = \frac{SSR}{498} = 1662.47$

$\Rightarrow \hat{Var}(\hat{\gamma}_1) = \frac{1662.47}{66,000} = 0.0252$

$se(\hat{\gamma}_1) = \sqrt{\hat{Var}(\hat{\gamma}_1)} = 0.1587$

e) $\hat{u} = y - \hat{\beta}_0 - \hat{\beta}_1 x$

Numerically,

$\sum \hat{u} = \sum y_i - n\hat{\beta}_0 - \hat{\beta}_1 \sum x_i$

$= 10,700 - 500(-119.672) - (2.939)(24,000) = 0$

Algebraically,

$\sum \hat{u} = \sum y - n\hat{\beta}_0 - \hat{\beta}_1 \sum x_i$ but $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$

$\Rightarrow \sum y - n\bar{y} + \hat{\beta}_1 n\bar{x} - \hat{\beta}_1 \sum x_i = \sum y - \sum y + \hat{\beta}_1 \sum x - \hat{\beta}_1 \sum x = 0$

$\sum x\hat{u} = 0$, Algebraically

Various way of showing this!!

$\sum x(y - \hat{\beta}_0 - \hat{\beta}_1 x)$

$= \sum x_i y_i - \hat{\beta}_0 \sum x_i - \hat{\beta}_1 \sum x_i^2 \quad (*)$

$= \sum x_i y_i - (\bar{y} - \hat{\beta}_1 \bar{x}) \sum x_i - \hat{\beta}_1 \sum x_i^2$

$= \sum x_i y_i - \bar{y} \sum x_i + \hat{\beta}_1 \bar{x} \sum x_i - \hat{\beta}_1 \sum x_i^2$

$= \sum x_i y_i - n\bar{x}\bar{y} - \hat{\beta}_1 (\sum x_i^2 - n\bar{x}^2)$

we've shown this a million times!!

but $\hat{\beta}_1 = \dfrac{\sum x_i y_i - n\bar{x}\bar{y}}{\sum x_i^2 - n\bar{x}^2} = \dfrac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$

$\therefore \sum x_i \hat{u}_i = 0$ as required

Numerically, $\sum x_i y_i = \sum (x_i - \bar{x})(y_i - \bar{y}) + n\bar{x}\bar{y} = 707{,}600$

$\sum x_i^2 = \sum (x_i - \bar{x})^2 + n\bar{x}^2 = 1{,}218{,}080$

put these number into $(*)$, eqn should be approximately zero (due to rounding off errors)

(f) $y_i = \alpha_0 + u_i \Rightarrow \hat{\alpha}_0 = \bar{y} = 21.4$

$Var(\hat{\alpha}_0) = \dfrac{\hat{\sigma}^2}{n} = ?$

$\hat{\sigma}^2 = \dfrac{SSR}{n-K}$, what is SSR for this model?

$SSR = SST - SSE = SST$ b/c $SSE = 0$; $R^2 = 0$

$\Rightarrow Var\ \hat{\alpha}_0 = \dfrac{1{,}398{,}080}{499} \times \dfrac{1}{500} = 5.603$

Yes $\hat{\gamma}_0 = \hat{\alpha}_0$.

g) Using Model (2)

$Y = \beta_0 + \beta_1 X + u$ old model

let X^* – new X , X "old" X

$\Rightarrow X^* = X + 10 \Rightarrow X = X^* - 10$

new model $Y = \beta_0^* + \beta_1^*(X^* - 10) + u$

$Y = \beta_0^* - 10\beta_1^* + \beta_1^* X^* + u$

$$\hat{\beta}_1^* = \frac{\Sigma(X^* - \bar{X}^*)(Y - \bar{Y})}{\Sigma(X^* - \bar{X}^*)^2} = \frac{\Sigma(X - \bar{X})(Y - \bar{Y})}{\Sigma(X - \bar{X})^2} = \hat{\beta}_1$$

$\Rightarrow$ no change in slope

$$\hat{\beta}_0^* = \bar{Y} - \hat{\beta}_1^* \bar{X}^* = \bar{Y} - \hat{\beta}_1(\bar{X} + 10) = \underbrace{\bar{Y} - \hat{\beta}_1 \bar{X}} - 10\hat{\beta}_1$$
$$= \hat{\beta}_0 - 10\hat{\beta}_1$$

intercept change by $\underline{\underline{-10\hat{\beta}_1}}$

h) let $X^* = X/10 \Rightarrow X = 10X^*$

$Y = \beta_0 + \beta_1(10X^*) + u$

$\Rightarrow Y = \beta_0 + 10\beta_1 X^* + u$

set $Y = \beta_0^* + \beta_1^* X^* + u$

$$\hat{\beta}_1^* = \frac{\Sigma(X^* - \bar{X}^*)(Y - \bar{Y})}{\Sigma(X^* - \bar{X}^*)^2}$$

$$= \frac{1/10\ \Sigma(X - \bar{X})(Y - \bar{Y})}{1/100\ \Sigma(X - \bar{X})^2}$$

$\Rightarrow \hat{\beta}_1^* = 10\hat{\beta}_1$

and $\hat{\beta}_0^* = \hat{\beta}_0$ no change in intercept but change in slope

(i) let $y^* = y/10$

old model $y = \beta_0 + \beta_1 x + u$

new model $\frac{y}{10} = \beta_0/10 + \beta_1/10\, x + \frac{u}{10}$

$\Rightarrow y^* = \beta_0^* + \beta_1^* x + u^*$

$$\hat{\beta}_1^* = \frac{\Sigma(x-\bar{x})(y^* - \bar{y}^*)}{\Sigma(x-\bar{x})^2} = 1/10 \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$$

$$= 1/10\ \hat{\beta}_1$$

$$\hat{\beta}_0^* = \bar{y}^* - \hat{\beta}_1^* \bar{x} = \frac{\bar{y}}{10} - \frac{\hat{\beta}_1}{10}\bar{x}$$

$$= \frac{1}{10}(\bar{y} - \hat{\beta}_1 \bar{x}) = \frac{1}{10}\hat{\beta}_0 .$$

both intercept and slope decrease by a factor 10.