EXAM 2 KEY

1.

	U	NU
F	$\beta_0+\beta_1+\beta_4+\beta_5$	$\beta_0+\beta_4$
M	$\beta_0+\beta_1$	β_0

(i)

(a) Note: Ref group = M, NU

$\therefore$ β_4 = differential for F

β_5 = " " F, U

(b)

	U	NU
F	$\delta_0+\delta_5$	$\delta_0+\delta_3$
M	δ_0	$\delta_0+\delta_4$

(2)

(i) Now, Ref group = M, U.

$\therefore$ δ_3 = differential for F, NU

δ_4 = " M, NU

δ_5 = " F, U

(ii) Algebraically:

$\delta_0 = \beta_0 + \beta_1$

$\delta_5 = \beta_4 + \beta_5$

$\delta_3 = \beta_4 - \beta_1$

$\delta_4 = -\beta_1$

Alternatively, you could show β's in terms of δ's ......

$E(\ln(wage) | x)$

	Male	Female
Union	$\underbrace{\delta_0 + \delta_1 exp + \delta_2 Edu}$ Call it aa	$aa + \delta_5$
Non-union	$aa + \delta_4$	$aa + \delta_3$

(c) (i) Mean log wage difference Male and Female (in the union group)

is $aa - (aa + \delta_5) = -\delta_5$

.... in the Non-union group $aa + \delta_4 - (aa + \delta_3)$

$= \delta_4 - \delta_3$

$\Rightarrow$ No difference if $\delta_5 = 0$ and $\delta_4 - \delta_3 = 0$.

F test.

- Run the model with the above restrictions, i.e.,

(restricted) $\ln wage = \delta_0 + \delta_1 Exp + \delta_2 Edu + \delta_3 Non\text{-}union + u$

(unrestricted) given in (2). Compute $F = \dfrac{(SSR_{rest} - SSR_{un}) / \# rest}{SSR_{un} / d.f.}$

$\# rest = 2$, $d.f. = n - 6$.

(ii) mean diff in log wage between Union and Non union

(for male) is $aa - (aa + \delta_4) = -\delta_4$

.... for female $= aa + \delta_3 - (aa + \delta_5) = \delta_3 - \delta_5$.

No diff. $\Rightarrow$ $\delta_4 = 0$ and $\delta_3 - \delta_5 = 0$

F test.

Restricted model $\ln wage = \delta_0 + \delta_1 Exp + \delta_2 Edu + \delta_3 Female + u$

F test as before — Same d.f. for both the numerator and denominator

(d) If there is no difference between U and NU wages the model will be the restricted model in (ii) above, i.e.,

$$\ln wage = \delta_0 + \delta_1 Exp + \delta_2 Edu + \delta_3 Female + u.$$

Note: if $\delta_3 = \delta_5$ then you can combine the two terms δ_3 Female * Non-union + δ_5 Female * Union

$$= \delta_3 \text{ Female } (\underbrace{\text{Non-union} + \text{union}}_{=1})$$

$$= \delta_3 \text{ Female}.$$

(2) (a) White test

1. Run (4). Get $\hat{u}^2$
2. Run $\hat{u}^2 = \alpha_0 + \alpha_1 X + \alpha_2 X^2 + e$.
3. F-test where $H_0: \alpha_1 = \alpha_2 = 0$.

(b) If ignored, then OLS estimates are not BLUE
To correct it, w/o knowing the form of $h(x)$,
use FGLS.

1. Assume $V(u|X) = \sigma^2 \underbrace{\left(e^{\delta_0 + \delta_1 X}\right)}_{h(x)}$

$$\Rightarrow u^2 = \sigma^2 e^{\delta_0 + \delta_1 X} \cdot V$$

2. Run $\log(\hat{u}^2) = \alpha_0 + \delta_1 X + e$ (Get $\hat{u}$'s from Y on X)
3. Get fitted values $\hat{g}$. Then $\hat{h} = e^{\hat{g}}$.
4. Divide everything in original equation by $\sqrt{\hat{h}}$. Run Y^* on X^*

(c)

Y	X	$Y^* = \frac{Y}{X}$	$X^* = \frac{1}{X}$
2	1	2	1
6	4	1.5	.25
15	8	1.875	.125
20	10	2	.1
30	16	1.875	.0625

Now run $Y = \beta_0 + \beta_1 X$ and $Y^* = b_0 + b_1 X^*$

$\hat{\beta}_0 = -.47$ $\qquad$ $\hat{b}_0 = 1.81$

$\hat{\beta}_1 = 1.73$ $\qquad$ $\hat{b}_1 = .13$

3 (a) $\hat{y} = 5.83 + 5(.869)\overbrace{\frac{x}{5}}^{x^*}$

$\Rightarrow \hat{y} = \underset{(1.23)}{5.83} + \underset{(.585)}{4.345}\, x^*$

(b) $\frac{\overbrace{\hat{y}}^{y^*}}{10} = \frac{5.83}{10} + \frac{.869}{10} x$

$\Rightarrow y^* = \underset{(.123)}{.583} + \underset{(.0117)}{.0869}\, x$

(c) Nothing. (How could we improve fit by changing scale?!)

(d) $\hat{b} = \left(\frac{s_x}{s_y}\right)\hat{\beta} = \frac{5}{6}(.869) = .724$

Beta coeff.