ANS KEY

Fall 2006 Exam I ECON 466

a) $Y_i = \beta_0 + \beta_1 X_i + u_i$

i) $\hat{\beta}_1 = \dfrac{\sum(X_i Y_i) - n\bar{X}\bar{Y}}{\sum X_i^2 - n\bar{X}^2}$

$$= \frac{265 - 15(5)(4)}{430 - 15(5)^2} = \frac{-35}{55} = -0.636$$

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1\bar{X} = 4 + 0.636(5) = 7.18$$

ii) $Y_i = \beta_0 + \beta_1(X_i - \bar{X}) + u_i$

$$\hat{\beta}_1 = \frac{\sum(X_i - \bar{X})(Y_i - \bar{Y})}{\sum(X_i - \bar{X})^2} = \frac{\sum X_i Y_i - n\bar{X}\bar{Y}}{\sum X_i^2 - n\bar{X}^2}$$

$$= -0.636$$

$$\hat{\beta}_0 = \bar{Y} = 4$$

iii) $\hat{\beta}_1 = \dfrac{\sum(X_i - \bar{X})(Y_i - \bar{Y})}{\sum(X_i - \bar{X})^2} = \dfrac{\sum X_i Y_i - n\bar{X}\bar{Y}}{\sum X_i^2 - n\bar{X}^2} = -0.636$

$$\hat{\beta}_0 = -\hat{\beta}_1\bar{X} = 0.636(5) = 3.18$$

iv) $\hat{\beta}_1 = \dfrac{\sum(X_i - \bar{X})(y_i - \bar{Y})}{\sum(X_i - \bar{X})^2} = -0.636$

$$\hat{\beta}_0 = 0$$

Ques 1 cont'd

b) The slope coefficients are all identical because the covariance between X and Y haven't changed and the variance of X haven't changed.

i) $$\frac{\hat{\beta}_1^2 \Sigma(X_i - \bar{X})^2}{\Sigma Y_i^2 - n\bar{Y}^2} = \frac{(-0.636)^2 (430 - 15(5)^2)}{350 - 15(4)^2}$$
$$= \frac{\hat{\beta}_1^2 (\Sigma X_i^2 - n\bar{X}^2)}{\Sigma Y_i^2 - n\bar{Y}^2} = 0.2022$$

ii) $$R^2 = \frac{\hat{\beta}_1^2 \Sigma(X_i - \bar{X})^2}{\Sigma(Y_i - \bar{Y})^2} = \frac{\hat{\beta}_1^2 (\Sigma X_i^2 - n\bar{X}^2)}{\Sigma Y_i^2 - n\bar{Y}^2} = 0.2022$$

iii) $$R^2 = \frac{\hat{\beta}_1^2 \Sigma(X_i - \bar{X})^2}{\Sigma(Y_i - \bar{Y})^2} = \frac{\hat{\beta}_1^2 (\Sigma X_i^2 - n\bar{X}^2)}{\Sigma Y_i^2 - n\bar{Y}^2} = 0.2022$$

iv) $$R^2 = \frac{\hat{\beta}_1^2 \Sigma(X_i - \bar{X})^2}{\Sigma(Y_i - \bar{Y})^2} = \frac{\hat{\beta}_1 (\Sigma X_i^2 - n\bar{X}^2)}{\Sigma Y_i^2 - n\bar{Y}^2} = 0.2022$$

The R^2 in each case is identical because the SSE and SST are the same in each case.

Ques 1 cont'd

c) $H_0 : \beta_1 = -0.5$

$se(\hat{\beta}_1)' = \sqrt{(\hat{\sigma}^2 / \Sigma(x_i - \bar{x})^2)}$

$$t = \frac{\hat{\beta}_1 + 0.5}{se(\hat{\beta}_1)}$$

$\hat{\sigma}^2 = \Sigma \hat{u}_i^2 / (15-2) = (110 - 22.24728)/13 = 6.75$

$se(\hat{\beta}_1) = \sqrt{(6.75/55)} = 0.3503$

$$t = \frac{-0.636 + 0.5}{0.3503} = -0.3882$$

The t-values will remain the same because the standard errors of $\hat{\beta}_1$ will be the same in each case.

d) $Y_i^* = 10 Y_i$

then $Y_i = \frac{Y_i^*}{10}$

$$\bar{Y} = \frac{\Sigma Y_i}{n} = \frac{\Sigma Y_i^*/10}{n} = 4$$

$$\Rightarrow \frac{\Sigma Y_i^*}{10n} = 4 \Rightarrow \frac{\Sigma Y_i^*}{n} = 4(10)$$

$$\Rightarrow \bar{Y}^* = 4(10)$$

Ques 1 d) cont'd

$$Y_i^{*2} = (10\,Y_i)^2$$

$$\sum Y_i^{*2} = 100 \sum Y_i^2 = 100(350)$$

$$\sum X_i Y_i^* = \sum X_i (10\,Y_i) = 10 \sum X_i Y_i = 10(265)$$

Let slope coefficient of the regression Y^* on X be $\tilde{\beta}$!

$$\tilde{\beta} = \frac{\sum (X_i - \bar{X})(Y_i^* - \bar{Y}^*)}{\sum (X_i - \bar{X})^2}$$

$$= \frac{\sum X_i Y_i^* - n\bar{X}^* \bar{Y}^*}{\sum X_i^2 - n\bar{X}^2}$$

$$= \frac{265(10) - 15(5)(10)(4)}{430 - 15(5)^2}$$

$$= \frac{10[265 - 15(5)(4)]}{55} = \frac{-35}{55}(10) = -6.36$$

$$= 10(\hat{\beta}_1)$$

Ques 2

a) $\hat{\beta}_{NOX^2}$: Holding CRIME and PTRATIO fixed, a unit increase in the level of nitric oxides will decrease the predicted median home values by 96 percent

$\hat{\beta}_{CRIME}$: Holding PTRATIO and NOX^2 fixed, a unit increase in the per capita crime rate will decrease the predicted median home values by 1 percent

$\hat{\beta}_{PTRATIO}$: Holding CRIME and NOX^2 fixed, a unit increase in the ratio of students to teachers in the community will increase predicted median home values by 7 percent.

b) $H_0 : \beta_{NOX^2} = 0$ $\qquad t_c = t_{(502)} = \pm 1.9648$

$H_1 : \beta_{NOX^2} \neq 0$

$$t = \frac{\hat{\beta}_{NOX^2}}{se(\hat{\beta}_{NOX^2})} = \frac{-0.96}{0.10} = -9.6 < -1.9648$$

we reject the null hypothesis

$H_0 : \beta_{CRIME} = 0$

$H_1 : \beta_{CRIME} \neq 0$

$$t = \frac{\hat{\beta}_{CRIME}}{se(\hat{\beta}_{CRIME})} = \frac{-0.01}{0.002} = -5 < -1.9648$$

we reject the null hypothesis

$H_0: \beta_{PTRATIO} = 0$
$H_1: \beta_{PTRATIO} \neq 0$

$$t = \frac{\hat{\beta}_{PTRATIO}}{se(\hat{\beta}_{PTRATIO})} = \frac{0.07}{0.006} = 11.667 > 1.9648$$

we reject the null hypothesis

c) $H_0: \beta_{CRIME} = 0 ; \beta_{PTRATIO} = 0$
$H_1:$ null is not true
This is a joint hypothesis test. Use F test

$$F = \frac{(R^2_{UR} - R^2_r)/q}{(1 - R^2_{UR})/n-(k+1)} = \frac{(0.50 - 0.25)/2}{(1-0.5)/502}$$
$$= \frac{0.125 * 502}{0.5} = 125.5$$

$F_c = F_{2,502} = 3.0138$; $125.5 > 3.0138$
we reject the null hypothesis

d) For each intercept, add log 10 (natural log)

e) $H_0: \beta_{CRIME} = \beta_{NOX^2}$
using t-test. $t = \dfrac{\hat{\beta}_{CRIME} - \hat{\beta}_{NOX^2}}{se(\hat{\beta}_{CRIME} - \hat{\beta}_{NOX^2})}$

Ques 2 cont'd

f) using F test

unrestricted regression ; $\widehat{lmdev} = \hat{\beta}_0 + \hat{\beta}_{NOX^2} NOX^2$

$\widehat{lmdev} = \hat{\beta}_0 + \hat{\beta}_{NOX^2} NOX^2 + \hat{\beta}_{CRIME} CRIME + \hat{\beta}_{PTRATIO} PTRATIO$

restricted regression $\beta_{CRIME} = \beta_{NOX^2} = \tilde{\beta}$

$\widehat{lmdev} = \hat{\beta}_0 + \tilde{\beta}(NOX^2 + CRIME) + \hat{\beta}_{PTRATIO} PTRATIO$

$$F = \frac{(R^2_{UR} - R^2_R)/q}{(1 - R^2_{UR})/n-(k+1)}$$

so get R^2_{UR} from unrestricted regression
and get R^2_R from restricted regression
$q = 1$ and $df = n-(k+1) = 502$