ANS KEY

Spring 2004 Exam I ECON 466

1) a) $\hat{\beta}_1 = \dfrac{\Sigma(X-\bar{X})(Y-\bar{Y})}{\Sigma(X-\bar{X})^2} = \dfrac{194000}{66000} = 2.94$

$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} = \frac{10700}{500} - 2.94\left(\frac{24000}{500}\right)$

$= 21.4 - 141.12 = -119.72$

b) $\hat{\beta}_1 = \dfrac{\Sigma(X-\bar{X})(Y-\bar{Y})}{\Sigma(X-\bar{X})^2} = \dfrac{\frac{1}{n}\Sigma(X-\bar{X})(Y-\bar{Y})}{\frac{1}{n}\Sigma(X-\bar{X})^2}$

$= \dfrac{Cov(X,Y)}{Var\,X}$

$= \dfrac{Cov(X,Y)\ Var\,Y}{Var\,X\ Var\,Y}$

$= \dfrac{Cov(X,Y)\sqrt{Var\,Y}\sqrt{Var\,Y}}{\sqrt{Var\,X}\sqrt{Var\,X}\sqrt{Var\,Y}\sqrt{Var\,Y}}$

$= \dfrac{Cov(X,Y)}{\sqrt{Var\,X}\sqrt{Var\,Y}} \cdot \dfrac{\sqrt{Var\,Y}}{\sqrt{Var\,X}}$

$= r\sqrt{\dfrac{Var\,Y}{Var\,X}}$

c) $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X$

$\bar{\hat{Y}} = \Sigma\dfrac{\hat{Y}}{n} = \dfrac{\Sigma\hat{\beta}_0}{n} + \hat{\beta}_1\dfrac{\Sigma X}{n}$

$\bar{\hat{Y}} = \hat{\beta}_0 + \hat{\beta}_1\bar{X}$

$$\bar{\hat{y}} = \bar{Y} - \hat{\beta}_1 \bar{X} + \hat{\beta}_1 \bar{X} = \bar{Y}$$

since $\bar{\hat{y}} = \bar{Y}$

then $\bar{Y} = \hat{\beta}_0 + \hat{\beta}_1 \bar{X}$

d) $\sum \hat{u} = \sum (Y - \hat{\beta}_0 - \hat{\beta}_1 X)$

$$= \sum Y - \sum \hat{\beta}_0 - \hat{\beta}_1 \sum X$$

$$= n\bar{Y} - n\hat{\beta}_0 - \hat{\beta}_1 n \bar{X}$$

$$= n(\bar{Y} - \hat{\beta}_0 - \hat{\beta}_1 X) = 0$$

e) $\sum \hat{u} X = 0$

$$\sum \hat{u} X = \sum (Y - \hat{\beta}_0 - \hat{\beta}_1 X) X$$

$$= \sum XY - \hat{\beta}_0 \sum X - \hat{\beta}_1 \sum X^2$$

$$= \sum XY - (\bar{Y} - \hat{\beta}_1 \bar{X}) n \bar{X} - \hat{\beta}_1 \sum X^2$$

$$= \sum XY - n\bar{X}\bar{Y} - \hat{\beta}_1 (\sum X^2 - n\bar{X}^2)$$

$$= \sum (X-\bar{X})(Y-\bar{Y}) - \hat{\beta}_1 \sum (X-\bar{X})^2$$

$$= \sum (X-\bar{X})(Y-\bar{Y}) - \frac{\sum (X-\bar{X})(Y-\bar{Y})}{\sum (X-\bar{X})^2} \sum (X-\bar{X})^2$$

$$= 0$$

f.) $\sum(X-\bar{X})(Y-\bar{Y}) = \sum(X-\bar{X})Y - \bar{Y}\sum(X-\bar{X})$

$= \sum(X-\bar{X})Y$

$\sum(X-\bar{X})(Y-\bar{Y}) = \sum X(Y-\bar{Y}) - \bar{X}\sum(Y-\bar{Y})$

$= \sum X(Y-\bar{Y})$

g) $Y = \beta_1 X + u$

$\sum \hat{u}^2 = \sum(Y - \hat{\beta}_1 X)^2$

$$\frac{\partial \sum \hat{u}^2}{\partial \hat{\beta}_1} = -2\sum(Y-\hat{\beta}_1 X)X = 0$$

$\Rightarrow \sum(Y-\hat{\beta}_1 X)X = 0$

$\sum XY - \hat{\beta}_1 \sum X^2 = 0$

$$\hat{\beta}_1 = \frac{\sum XY}{\sum X^2}$$

$\sum \hat{u} = \sum Y - \hat{\beta}_1 \sum X$

If $\sum \hat{u} = 0$, then $\sum Y = \hat{\beta}_1 \sum X$

$$\Rightarrow \hat{\beta}_1 = \frac{\sum Y}{\sum X}$$

but $\hat{\beta}_1 = \dfrac{\sum XY}{\sum X} \neq \dfrac{\sum Y}{\sum X}$

so $\sum \hat{u} \neq 0$

if $\sum \hat{u} X = 0$, then $\sum XY - \hat{\beta}_1 \sum X^2 = 0$

$$\hat{\beta}_1 = \frac{\sum XY}{\sum X^2}$$

this follows from the first order conditions.
So $\sum \hat{u} X = 0$

Ques 2

$\hat{\beta}_1$: Holding income, interest rate and unemployment rate fixed, if the new car price index goes up by one unit, then the predicted demand in number of new cars reduces by 0.07 units

$\hat{\beta}_2$: Holding price, interest rate and unemployment rate fixed, if income increases by 1 dollar, then the predicted demand in the number of new cars increases by 0.003 units

$\hat{\beta}_3$: Holding price, income, unemployment rate fixed, if interest rates goes up by 1% then the predicted demand in the number of new cars will reduce by 0.15 units

$\hat{\beta}_4$: Holding price, income and interest rate fixed, if unemployment rate goes up by 1 percent the the predicted demand in the number of new cars will fall by 0.07 units

b) $\bar{R}^2 = 1 - (1 - R^2)\dfrac{n-1}{n-k-1}$

$1 - R^2 = (1 - \bar{R}^2)\dfrac{n-k-1}{n-1}$

$1 - R_A^2 = (1 - 0.758)\dfrac{35}{39} = 0.2172$

$R_A^2 = 0.7828$

$1 - R_B^2 = (1 - 0.764)\left(\dfrac{36}{39}\right) = 0.2178$

$R_B^2 = 0.7822$

$1 - R_C^2 = (1 - 0.565)\left(\dfrac{37}{39}\right) = 0.4127$

$R_C^2 = 0.5873$

c) $H_0: \beta_2 = 0$

$H_1: \beta_2 \neq 0$

For model A

$t = \dfrac{\hat{\beta}_2}{se(\hat{\beta}_2)} = \dfrac{0.003159}{0.001763} = 1.792$

$t_c = 2.0315 > 1.792$

we fail to reject the null

For model B: $t = \dfrac{0.00356}{0.000627} = 5.678 > t_c = 2.0294$

we reject the null

For model B

d) $H_0: \beta_3 = 0$

$H_1: \beta_3 < 0$

$$t = \frac{\hat{\beta}_3}{se(\hat{\beta}_3)} = \frac{-0.146651}{0.039229} = -3.738 < -1.6892 = t_c$$

we reject the null hypothesis

For model C

$$t = \frac{-0.204769}{0.051442} = -3.98 < -1.6879 = t_c$$

we reject the null hypothesis

e) for part c)

A: $p\text{-value} = P(|T| > 2.0315) = 2P(T > 2.0315)$
$= 2(0.024926)$
$= 0.049852$

B: $p\text{-value} = P(|T| > 5.678) = 2P(T > 5.678)$
≈ 0

for part d)

B: $p\text{-value} = P(T < -3.738) = 0.00032$

C: $p\text{-value} = P(T < -3.98) = 0.00015$

$\hat{T}$ $H_0: \beta_2 = 0$, $\beta_4 = 0$
H_1: null is not true

$$F = \frac{(R^2_{UR} - R^2_R)/q}{(1 - R^2_{UR})/n-k-1} = \frac{(0.7828 - 0.5873)/2}{0.2172/35}$$

$$= \frac{0.1955/2}{0.2172/35} = \frac{0.09775}{0.00062}$$

$$= 157.66$$

$\Rightarrow$ we reject the null

g) model A
$H_0: \beta_1 = 0$; $\beta_2 = 0$, $\beta_3 = 0$, $\beta_4 = 0$
H_1: null hypothesis is not true

$$F = \frac{R^2_{UR}/q}{(1 - R^2_{UR})/n-k-1} = \frac{0.7828/4}{0.2172/35} = \frac{0.1957}{0.0062}$$

$$= 31.56$$

$F_c = F_{4,35} = 2.65 \Rightarrow$ we reject the null

model B
$H_0: \beta_1 = 0$, $\beta_2 = 0$, $\beta_3 = 0$
H_1: null hypothesis is not true

$$F = \frac{0.7822/3}{0.2178/36} = \frac{0.2607}{0.00605} = 43.09$$

$F_c = F_{3,36} = 2.872 \Rightarrow$ we reject the null

h) $\hat{\beta}_2$ - coefficient on income would change
it would become
in Model A : 3.159
In Model B : 3.56

i) $H_0: \beta_3 - 2\beta_4 = 0$
$H_1: \beta_3 - 2\beta_4 \neq 0$ the alternative hypothesis states that β_3 is not equal to 2 times β_4. In other words, it is saying that the effect of interest rate is not twice the effect of unemployment rate
- choose 5% significance level.
- a t-test can be used in this case

$$t = \frac{\hat{\beta}_3 - 2\hat{\beta}_4}{Se(\hat{\beta}_3 - 2\hat{\beta}_4)}$$

- the null hypothesis states that the effect of interest rate on the demand for new cars is twice as much as the effect of unemployment rate.