Assignment #1
Due: Feb 18??
Econometrics 616

1. The file tbrate.txt contains data for the period 1950:1 to 1996:4 for three economics series: r_t, the interest rate on 90-day treasury bills, π_t, the rate of inflation, and y_t, the logarithm of real GDP. Using this data estimate the model:
$$\Delta r_t = \beta_1 + \beta_2 \pi_{t-1} + \beta_3 \Delta y_{t-1} + \beta_4 \Delta r_{t-1} + \beta_5 \Delta r_{t-2} + u_t$$
Where Δ is the first difference operator defined as $\Delta x_t = x_t - x_{t-1}$. Record both the estimates of all model parameters as well as determining the fitted residuals of the model, $\hat{u}_t$. This will be regression 1. Next regress Δr_t on a constant, Δy_{t-1}, Δr_{t-1}, and Δr_{t-2}. Save the fitted residuals and call them $\hat{e}_t$. When this is done regress π_{t-1} on a constant, Δy_{t-1}, Δr_{t-1}, and Δr_{t-2}. Save the fitted residuals from this regression and call them $\hat{v}_t$. Now run the regression $\hat{e}_t = \alpha \hat{v}_t + w_t$. What do you notice about the parameter estimate from this regression? What do you notice about the fitted residuals, $\hat{w}_t$, from this regression? The niceness of your results are called the Frisch-Waugh-Lovell theorem. Curious readers may consult Frisch and Waugh, *Econometrica* 1933, Lovell, *Journal of the American Statistical Association*, 1963 or Davidson and MacKinnon, *Estimation and Inference in Econometrics*, 1993, particularly chapter 1.

2. *Simple versus multiple regression coefficients*. Consider the multiple regression $Y_i = \alpha + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$ along with the following auxiliary regressions
$X_{2i} = \hat{a} + \hat{b} X_{3i} + \hat{v}_{2i}$
$X_{3i} = \hat{c} + \hat{d} X_{2i} + \hat{v}_{3i}$
In class it was shown that $\hat{\beta}_2$ could be interpreted directly from the regression of Y on the OLS residuals $\hat{v}_2$. A similar interpretation can be given to $\hat{\beta}_3$. Kennedy (1981, pg.416, it is not necessary to read Kennedy's paper) claims that $\hat{\beta}_2$ is not necessarily the same as $\hat{\delta}_2$, the OLS estimate of obtained from the regression $Y_i = \gamma + \delta_2 \hat{v}_{2i} + \delta_3 \hat{v}_{3i} + w_i$. Prove this claim by finding a relationship between both $\hat{\beta}_2$ and $\hat{\beta}_3$ and $\hat{\delta}_2$ and $\hat{\delta}_3$.

3. For the simple regression $Y_i = \alpha + \beta_2 X_{2i} + u_i$ show that
   a) $\hat{\beta}_2 = \sum xy \big/ \sum x^2$ can be obtained using the residual interpretation by regressing X on a constant first, obtaining the residuals $\hat{v}$, and then regressing Y on $\hat{v}$.

b) $\hat{\alpha} = \overline{Y} - \hat{\beta}_2 \overline{X}_2$ can be obtained using residuals as well by regressing a column of ones on X_2, obtaining the residuals $\hat{w}$, and then regressing Y on $\hat{w}$.

c) Check the $Var(\hat{\alpha})$ and $Var(\hat{\beta})$ in parts (a) and (b) with those obtained from the residualing interpretation.

4. *Effect of additional regressors on* R^2. Suppose that the multiple regression has K_1 regressors in it. Denote the OLS **residual** sum of squares by RSS_1. Now add K_2 additional regressors so that the total number of regressors in the model is $K = K_1 + K_2$. Denote the corresponding OLS residual sum of squares by RSS_2. Show that $RSS_2 \leq RSS_1$ and conclude that the corresponding R-squares from the models satisfy $R_2^2 \geq R_1^2$.

5. Suppose that we have estimated the parameters of the multiple regression model $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$ by OLS. Denote the residuals as e_i and the fitted values by $\hat{Y}_i$.

   a) What is the R^2 of the regression e on a constant, X_2, and X_3?
   b) If we regress Y on a constant and $\hat{Y}$, what are the estimated intercept and slope coefficients? What is the relationship between the R^2 of this regression and the R^2 of the original regression?
   c) If we regress Y on a constant and e, what are the estimated intercept and slope coefficients? What is the relationship between the R^2 of this regression and the R^2 of the original regression?
   d) Suppose that we add a new explanatory variable X_4 to the model and re-estimate the parameters using OLS. Show that the estimated coefficient of X_4 and its estimated standard error will be the same as in the OLS regression of e on a constant, X_2, X_3, and X_4.

<u>References</u>

Kennedy, P.E.(1981), "The Balentine: A Graphical Aid for Econometrics," *Australian Economic Papers*, 20:414-416.