Econometrics 616
Final Exam
Spring 2004

Total Points: 100
Time: 2 hours

Note: Answer all questions and show work when necessary. **Good Luck!**

1. (a) What is multicollinearity? Are the OLS estimators biased by the presence of it? Is there any easy way to figure out whether serious multicollinearity exists in a particular data set, after you have run a regression? **(8 points)**
   (b) Give an example of perfect multicollinearity. Is there a way to fix the problem in your example? **(6 points)**
   (c) It can be shown that the variance of the OLS estimator of β_k is

   $$V(b_k) = \frac{\sigma^2}{\sum x_k^2 (1 - R_k^2)} \equiv \frac{\sigma^2}{\sum x_k^2} VIF, k = 2, ..., K$$

   where VIF is called the variance inflation factor. (The other notations are self-explanatory.) What is the minimum and maximum value of VIF? How is VIF related to the severity of the multicollinearity problem? **(6 points)**

2. Suppose household demand for gasoline (G) is thought to be a linear function of household income (Y) but that the intercept depends on the region where the driver resides, namely Binghamton (B), Endicott (E), Johnson City (JC), and Vestal (V). Mobil regresses G on an intercept, Y, and the dummy variables B, E, and JC, while Chevy regresses G on Y and all four of the dummies.
   a) How would you estimate the difference between the intercepts for Vestal and Binghamton using (i) Mobil's results and (ii) Chevy's results? Which estimated difference would you expect to be larger and why? Interpret this difference. **(6 points)**
   b) How would you test the hypothesis that the intercepts for Vestal and Binghamton are the same using (i) Mobil's results and (ii) Chevy's results? Be explicit about the hypothesis specification as well as the test statistic. Assume that the number of observations is 500. **(6 points)**
   c) Suppose that Mobil believes that the slope for Vestal is identical to that of Binghamton, but differs from Endicott and Johnson City. Setup a model that could be estimated to incorporate this belief and explain the test procedure that Mobil would use to confirm his belief. **(6 points)**
   d) Suppose that Chevy believes that each region has a unique slope and a unique intercept. Setup Chevy's new model(s) and explain in detail how Chevy would test this hypothesis. **(6 points)**
   e) Compare Mobil's model in c) with Chevy's model in d). Specifically, comment on the functional form, ease of estimation (in case you had a computer in front of you), and the interpretation of the results. **(5 points)**

3. Consider the model

$$Y = \beta X + u, \quad u \sim i.i.d.\, N(0,\sigma^2)$$

    a) Show that the ML estimator of β is MVUE. **Hint: For simplicity in computation assume that σ^2 is known. (7 points)**

    b) Assume that X is a random variable and is correlated with u. Show that the OLS estimator of β is inconsistent. **(5 points)**

    c) Now assume that Z can be used as an instrument for X. Derive the IV estimator of β and show that it is consistent. **(7 points)**

    d) Let the true model be $Y = \beta X^* + u$ but X^* is measured with error (i.e., $X = X^* + v$ where v is measurement error). If you run the regression Y on X (without an intercept) then the OLS estimator of β will be inconsistent. Show the degree of inconsistency. (Assume that v is independent of X^* and u). **(7 points)**

    e) If the model is $Y_t = \beta_0 + \beta_1 Y_{t-1} + u_t$, $|\beta_1| < 1$, show that the OLS estimator of β_1 is consistent if u_t is i.i.d. $(0,\sigma^2)$ but inconsistent if u_t follows an AR (1) process. How would you estimate the model if u_t follows an AR (1) process? **(5+8+5 points)**

4. Why is identification an issue in a simultaneous equation system? Consider the following system of demand and supply equations:

Demand: $\quad Q = \alpha_1 P + \alpha_2 Z_1 + u_1$

Supply: $\quad\quad Q = \beta_1 P + \beta_2 Z_2 + u_2$

where Q and P are endogenous variables.

    (a) Show (using both rank and order conditions) that the above system is identified. **(5 points)**

    (b) Derive the reduced form equations and show (give an intuitive explanation) that the parameters in the reduced form are consistently estimated. **(10 points)**

    (c) You want to estimate the demand function only. Show that the OLS estimators of α_1 and α_2 from the demand function are inconsistent (give an intuitive explanation). Derive the IV estimators of α_1 and α_2 using Z_1 and Z_2 as instruments. **(12 points)**