Econ 616 Assignment # 2

1.) Consider the multiple regression of y on K variables, x, and an additional variable, z. Prove that the true variance of the least squares estimator of the coefficients of x can be larger when z is included in the regression than when it is not.

2.) Consider the least squares regression of y on x. Consider an alternative set of regressors, $z = xP$, where P is a nonsingular matrix. Thus, each column of z is a mixture of some of the columns of x. Prove that the redidual vectors in the regressions of y on x and y on z are identical. What relevance does this have to the question of changing the fit of a regression by changing the units of measurement of the independent variables?

3.) Let the regression equation be partitioned as

$$y = x_1\beta_1 + x_2\beta_2 + \epsilon.$$

Let b_1 and b_2 be the usual least squares estimators. Suppose that $E(\epsilon) = x_1\gamma$, that is, the mean vector of the disturbances is a linear combination of some of the regressors. Prove that b_1 is biased but b_2 is not.

4.) Consider an alternative setup for the regression equation where the right-hand side regressors are expressed in deviation form, that is, $X = [i\ X_*]$, where X_* is the $n \times (k-1)$ matrix of deviations. Show that when Y is regressed on X the LS vector is

$$b = \begin{bmatrix} \bar{Y} \\ b_2 \end{bmatrix}$$

where b_2 is the $(k-1)$ element vector,

$$b_2 = (X_*'X_*)^{-1}X_*'y = (X_*'X_*)^{-1}X_*'y_*$$

5. Consider the model $Y = X\beta + u$ which satisfies all the assumptions of the classical linear regression model.

(a) Let X be partitioned as $X = [X_1 \;\; X_2]$ and

$$X'X = \begin{bmatrix} X_1'X_1 & 0 \\ 0 & X_2'X_2 \end{bmatrix}.$$

i) Find b_1, b_2 — the OLS estimators of β_1 and β_2.

ii) Regress Y on X_1 only. Let the OLS estimator of β_1 be $\tilde{b}_1$ and OLS residuals be $\tilde{e}$. Now regress $\tilde{e}$ on X_2 to find estimator of β_2, $\tilde{b}_2$.

Show that $\tilde{b}_1 = b_1$ and $\tilde{b}_2 = b_2$. Comment on this result.

(b) Consider the following method of selecting the appropriate X variables in your regression. You are regressing Y on each of the independent variables <u>separately</u>.

[illegible] to reject

(c) When can you expect the parameter estimates of these separate regressions to be the same as that of the multiple regression (i.e., Y on $X_1, \ldots, X_K$)?

(d) Let b_n denotes the OLS estimator of β based on n observations and b_{n+1} is the one with $(n+1)$ observations. Show that $V(b_{n+1}) - V(b_n)$ is NSD.

[Hint: If $A - B$ is psd then $A^{-1} - B^{-1}$ is NSD].