Econ616
Homework #4
Answer key to selected questions

<u>Question 2</u>

Model 1 log(Y)= γ +δlog(X)+ε
Model 2 log(X)= α+βlog(Y)+ε

Define variables x: log(X) in mean deviation form and y: log(Y) in mean deviation form.

(b) $d = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} y_i^2}$ and $b = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2}$ and thus $d/b = \frac{\sum_{i=1}^{n} x_i^2}{\sum_{i=1}^{n} y_i^2} \neq 1$

(c) The regressor is assumed not to be correlated with an error term.

(d) $R^2 = \frac{\sum_{i=1}^{n} \hat{y}_i^2}{\sum_{i=1}^{n} y_i^2} = \frac{d^2 \sum_{i=1}^{n} x_i^2}{\sum_{i=1}^{n} y_i^2} = \frac{d \sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} y_i^2} = d \cdot b$

(e) Define a variable X*=pX, where X denotes En and apply the above formulas. The rest follows…

<u>Question 3</u>

Model 1 En=α+βRGDP+u

Model 2 En*=βRGDP*+u*

Model 2 is Model 1 in mean deviation form.

Define Y=En and X=RGDP, small x and y denote respective variables in mean deviation form.

Solution

(a) **Model 1** OLS regression gives $b_1 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^{n} (Y_i - \bar{Y})^2} = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} y_i^2}$

Model 2 OLS through the origin gives $b_2 = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} y_i^2}$

Thus b_1=b_2.

(b) **Model 1** $R_1^2 = \frac{\sum_{i=1}^{n}(\hat{Y}_i - \bar{Y})^2}{\sum_{i=1}^{n}(Y_i - \bar{Y})^2} = \frac{\sum_{i=1}^{n}\hat{y}_i^2}{\sum_{i=1}^{n}y_i^2}$

Model 2 $R_2^2 = \frac{\sum_{i=1}^{n}\hat{y}_i^2}{\sum_{i=1}^{n}y_i^2}$

Since b_1=b_2 predicted y will be the same in both models thus $R_1^2 = R_2^2$.

<u>Question 4</u>

Model 1 En=α+βRGDP+u

Model 2 En*=βRGDP*+u*

Model 3 En=βRGDP+u

Define Y=En and X=RGDP, small x and y denote respective variables in mean deviation form.

(a) **Model 3** OLS through the origin gives $b_3 = \frac{\sum_{i=1}^{n} X_i Y_i}{\sum_{i=1}^{n} Y_i^2} \neq b_1 = b_2$

(b) **Model 3** $R_3^2 = \frac{\sum_{i=1}^{n}\hat{Y}_i^{\,2}}{\sum_{i=1}^{n}Y_i^2}$ First, note $R_1^2 = R_2^2 \neq R_3^3$. Second, note that $TSS_3 = \sum_{i=1}^{n} Y_i^2$ and $TSS_1 = TSS_2 = \sum_{i=1}^{n} y_i^{\,2}$ and thus $TSS_1 = TSS_2 \neq TSS_3$. The coefficients of determination are not comparable.

<u>Question 5</u>

Model ln(C)=β_1+$\beta_2$ln(P)+$\beta_3$ln(Y)+u

Solution

(b) β_2+β_3=-1 thus β_3=-(1+β_2). One may rewrite the regression equation as follows

ln(C)=β_1+$\beta_2$ln(P)+$\beta_3$ln(Y)+u

$\ln(C)=\beta_1+\beta_2\ln(P)+-(1+\beta_2)\ln(Y)+u$
$\ln(C)+\ln(Y)=\beta_1+\beta_2\ln(P)-\beta_2\ln(Y)+u$
$\ln(C\bullet Y)=\beta_1+\beta_2\ln(P/Y)+u$

Thus the restriction is built into the model.

(c) Built into the model one restriction at a time then reestimate the model and construct a F- test.

(i) $\beta_3=0$ and the model becomes
$\ln(C)=\beta_1+\beta_2\ln(P)+u$
(ii) see Question 5 (b)
(iii) $\beta_3=0$ and $\beta_2=-1$
$\ln(C)=\beta_1+\beta_2\ln(P)+\beta_3\ln(Y)+u$
$\ln(C)=\beta_1-\ln(P)+u$
$\ln(C\bullet P)=\beta_1+u$