ECONOMICS 616
EXAM 2
Spring 2002

Total Points: 100
Time: 1 hour and 15 minutes

NOTE: Answer all questions. Show your work fully. GOOD LUCK!!

1. You have a sample of 15 observations that are grouped into 2 categories. The first group (identified by the dummy variable D_1=1) has 5 observations and the second group (identified by the dummy variable D_2=1) has 10 observations. (Note that $D_1 + D_2 = 1$). You have values of the dependent variable Y and an independent variable X for all 15 observations. You run several regressions and the results are reported as follows:

$$Y = -.07 + .52X, \quad RSS = 6.56 \qquad (1)$$
$$Y = -.46D_1 + .55D_2 + .49X, \quad RSS = 3.49 \qquad (2)$$
$$Y = -.06D_1 + .4D_2 + .44D_1 * X + .51D_2 * X, \quad RSS = 3.16 \qquad (3)$$
$$Y = .23 + .4D_1 * X + .52D_2 * X, \quad RSS = 3.33 \qquad (4)$$

Part A: (Hint: All you need here is to identify the restricted and unrestricted regressions in each case).

i. How would you test for structural change (both intercept and slope) at the 5% level of significance? (8 points)
ii. How would you test for equality of slopes (assuming that intercepts are the same) at the 5% level of significance? (8 points)
iii. How would you test for equality of slopes (assuming that intercepts are different) at the 5% level of significance? (8 points)
iv. How would you test for equality of intercepts (assuming that slopes are the same) at the 5% level of significance? (8 points)
v. How would you test for equality of intercepts (assuming that slopes are different) at the 5% level of significance? (8 points)

Part B: Instead of running the regressions in (1)-(4) your friend estimated the following

$$Y = c_0 + c_2D_2 + c_3X + e_2, \quad RSS = 3.49 \qquad (2a)$$
$$Y = b_0 + b_1D_2 + b_2X + b_3D_2 * X + e_1, \quad RSS = 3.16 \qquad (3a)$$
$$Y = f_0 + f_1X + f_2D_2 * X + e_3, \quad RSS = 3.33 \qquad (4a)$$

i. Use your results from (2) to calculate c_0, c_1, c_2 for her in (2a). **(5 points)**
ii. Use your results from (3) to calculate b_0, b_1, b_2 for her in (3a). **(5 points)**
iii. Use your results from (4) to calculate f_0, f_1, f_2 for her in (4a). **(5 points)**

2. Let the true regression (in deviation form) be $y = \beta_2 x_2 + \beta_3 x_3 + u$ and the OLS estimates of the coefficients are b_2 and b_3, respectively. However, one mistakenly excludes X_3 from the regression and estimates the model (in deviation form) $y = \alpha_2 x_2 + v$. Let the OLS estimate of α_2 from this misspecified model be a_2.

(a) Show that $a_2 = b_2 + b_3\, c_{32}$, where c_{32} is the slope coefficient of the regression X_3 on X_2. (Hint: You may start from the relationship $y = b_2 x_2 + b_3 x_3 + e$ where e is the OLS residual). Can you generalize this result to the k regressors model? **(8 points)**

(b) Use the above result to show that $E(a_2) = \beta_2 + \beta_3 c_{32}$. **(3 points)**

(c) What happens to the bias when X_3 and X_2 are uncorrelated? How about hypothesis testing in this special case? **(4 points)**

3. Consider the following regression function

$$Y_i = \beta X_i + u_i,\ i = 1, \ldots, n \quad (1)$$

where $u_i \sim (0, \sigma^2 X_i^2)$. Note that X is a scalar.

(i) Derive the GLS estimator of β and call it b_G. Show that b_G is consistent. (Note: state any assumptions you need to make). **(12 points)**

(ii) Consider two other estimators of β, viz.,

a. $\hat{b} = \overline{Y} / \overline{X}$, and

b. $\tilde{b} = \sum x_i y_i / \sum x_i^2$

where $\overline{Y}$ and $\overline{X}$ are sample means of Y and X; y and x are Y and X in deviation forms.

Show that $\hat{b}$ and $\tilde{b}$ are consistent. (Hint: use convergence in quadratic mean to prove your result. State any assumptions you need to make.) **(18 points)**