ECONOMICS 616
EXAM 2
Spring 2003

Total Points: 100
Time: 1 hour, 15 minutes

NOTE: Answer all questions. Show your work. GOOD LUCK!

1. Let the true model be $y = \beta_1 x_1 + \beta_2 x_2 + u$ which satisfies all the assumptions of the classical linear regression model. Note that the Y and X variables are in deviation forms. You estimated the model $y = \delta_1 x_1 + \varepsilon$.

   (i) Show that the OLS estimator of $\hat{\delta}_1$ can be expressed as

   $$\hat{\delta}_1 = \beta_1 + \beta_2 \frac{\Sigma x_1 x_2}{\Sigma x_1^2} + \frac{\Sigma x_1 u}{\Sigma x_1^2}.$$ **(5 points)**

   (ii) Use the above result to show that $\hat{\delta}_1$ is both biased and inconsistent. **(3+5 points)**

   (iii) Comment on the nature of bias and inconsistency (sign). **(3 points)**

   (iv) Derive the asymptotic distribution of $\hat{\delta}_1$ assuming that X_1 and X_2 are uncorrelated. (State the results – no proof of normality is necessary). **(7 points)**

2. Consider the model $Y = X\beta + u$ where Y and X are portioned as $Y = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$, $X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$ and $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$. The variance-covariance matrix for u is $\Omega = \begin{bmatrix} \sigma_1^2 I_1 & 0 \\ 0 & \sigma_2^2 I_2 \end{bmatrix}$ where I_1 and I_2 are identity matrices of order n_1 and n_2 ($n_1 + n_2 = n$). Assume that σ_1^2 and σ_2^2 are **known**.

(i) Derive the GLS estimator of β using the partitioned variables (X_1, X_2, Y_1, Y_2, etc.) **(6 points)**

(ii) If σ_1^2 and σ_2^2 are the same ($=\sigma^2$), show that the OLS and GLS estimators are the same. **(4 points)**

(iii) Now assume that the relationship is $Y_1 = X_1\beta_1 + u_1$ and $Y_2 = X_2\beta_2 + u_2$ where subscripts 1 and 2 refer to two sub-periods with n_1 and n_2 observations. The variance-covariance matrix for u is $\Omega = \begin{bmatrix} \sigma_1^2 I_1 & 0 \\ 0 & \sigma_2^2 I_2 \end{bmatrix}$. Assuming that σ_1^2 and σ_2^2 are **known**, derive the GLS estimators of β_1 and β_2. Show that these are the same as the OLS estimators from the above two equations. How would you test the hypothesis that $\beta_1 = \beta_2$? You may use any one of the asymptotic tests. State all the steps in details. **(4+4+6 points)**

3. Consider the following model of the demand for airline travel, estimated using annual data for the period 1947-1987 (41 observations)

$$\ln(Q) = \beta_1 + \beta_2 \ln(P) + \beta_3 \ln(Y) + \beta_4 \ln(ACCID) + \beta_5 FATAL + u \text{ ,}$$

where
Q = Per-capita passenger miles traveled in a given year, P = Average price per mile, Y = Per-capita income, ACCID = Accident rate per passenger mile, and FATAL = Number of fatalities from aircraft accidents.

In 1979 the airlines were deregulated. Define the dummy variable *D* that takes the value 0 for 1947-1978 and 1 for 1979-1987. Results from three models are presented below. Model A is the basic model given above, Model B is the one derived by assuming that there has been a structural change of the entire relation, and Model C is derived after omitting a number of variables from Model B.

	Variable	**Model A** **Coeff** **(Stderr)**	**Model B** **Coeff** **(Stderr)**	**Model C** **Coeff** **(Stderr**
$\hat{\beta}_1$	CONSTANT	2.938 (1.050)	2.635 (1.326)	2.476 (1.128)
$\hat{\beta}_2$	1n(P)	-1.312 (0.315)	-1.029 (0.377)	-0.991 (0.273)
$\hat{\beta}_3$	1n(Y)	0.716 (0.289)	-0.001 (0.433)	
$\hat{\beta}_4$	1n(ACCID)	-0.541 (0.100)	-0.821 (0.156)	-0.817 (0.035)
$\hat{\beta}_5$	FATAL	0.0004 (0.0003)	0.0009 (0.0003)	0.0009 (0.0003)
$\hat{\beta}_6$	*D*		-1.688 (0.388)	
$\hat{\beta}_7$	*D* x 1n(P)		0.278 (0.796)	
$\hat{\beta}_8$	*D* x 1n(Y)		0.987 (0.558)	0.883 (0.187)
$\hat{\beta}_9$	*D* x 1n(ACCID)		0.818 (0.252)	0.849 (0.185)
$\hat{\beta}_{10}$	*D* x FATAL		-0.001 (0.0006)	-0.001 (0.0005)
RSS		1.096191	0.700959	0.711423

(i) Write down the estimated model before and after deregulation using the estimated parameters in the above table and interpret the results. **(8 points)**

(ii) Carry out a test (Chow test as well as the asymptotic tests LR, LM and W) for the null hypothesis that the structure is the same before and after deregulation. Do you get similar answer? Why or why not? **(16 points)**

(iii) In Model C, perform the Wald test to test the null hypothesis that *during the period 1979-19*87 the elasticity for ACCID is equal to 1. **(4 points)**

(iv) In Model C, you want to test the null hypothesis that *during the period 1979-19*87 the elasticity for ACCID is equal to 1. Describe in step-by-step how to perform the LM test. You may use algebraic notations instead of the estimated parameter values in the table above. **(6 points)**

4. Consider the model $Y = X\beta + u$ which satisfies all the assumptions of the classical linear regression model. Furthermore assume that *u* is normally distributed. Based on 29 observations a researcher obtains the following $X'X$ matrix

$$X'X = \begin{bmatrix} 29 & 0 & 0 \\ 0 & 50 & 10 \\ 0 & 10 & 80 \end{bmatrix}, \qquad X'Y = \begin{bmatrix} 116 \\ 29 \\ 76 \end{bmatrix} \text{ and TSS } (y'y) = 600.$$

(a) **(8 points)** Use $X'Xb = X'Y$, i.e., $\begin{bmatrix} 29 & 0 & 0 \\ 0 & 50 & 10 \\ 0 & 10 & 80 \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 116 \\ 29 \\ 76 \end{bmatrix}$ to show that

$$b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0.4 \\ 0.9 \end{bmatrix} \text{ and RSS } (=e'e) = 520.$$

(b) **(7 points)** You are asked to test the hypothesis $\beta_2 + \beta_3 = 1$. Use the formula $e_*'e_* = e'e + (Rb - r)'\left[R(X'X)^{-1}R'\right]^{-1}(Rb - r)$ to calculate RSS for the restricted model ($e_*'e_*$).

(c) **(4 points)** Compute the value of the LR statistic and test the above hypothesis.