Econometrics 616
Exam I
Spring 2004

Total Points: 100
Time: 1 hour and 30 minutes

Note: Answer all questions and show work when necessary. **Good Luck!**

You are given the following formulas:

$$\textbf{Model}:\ Y = \beta_1 + \beta_2 X_2 + \beta_3 X_3 + u \qquad (1)$$

The OLS estimators of β_2 and β_3 (denoted by b_2, b_3) are

$$b_2 = \frac{\sum x_3^2 \sum x_2 y - \sum x_2 x_3 \sum x_3 y}{\sum x_2^2 \sum x_3^2 - \left(\sum x_2 x_3\right)^2}$$

$$b_3 = \frac{\sum x_2^2 \sum x_3 y - \sum x_2 x_3 \sum x_2 y}{\sum x_2^2 \sum x_3^2 - \left(\sum x_2 x_3\right)^2}$$

where the lower case variables are mean deviation of the corresponding uppercase variable.

For the same model the variances of the estimators b_2 and b_3 are

$$V(b_2) = \frac{\sigma^2 \sum x_3^2}{\sum x_2^2 \sum x_3^2 - \left(\sum x_2 x_3\right)^2}$$

$$V(b_3) = \frac{\sigma^2 \sum x_2^2}{\sum x_3^2 \sum x_2^2 - \left(\sum x_2 x_3\right)^2}$$

1. You are given the following data matrix in deviation form $x'x = \begin{bmatrix} 50 & 60 \\ 60 & 100 \end{bmatrix}$ and $x'y = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, $y'y = 20.75$ and $n = 20$.

   a) Compute $b_2, b_3,$ and the estimated variances of b_2, b_3. **(6+8 points)**

   b) Compute the **t-value** for testing the hypothesis $H_0 : \beta_2 + \beta_3 = 0$. **(4 points)**

c) Denote $\delta = \beta_2 + \beta_3$ and rewrite the model in (1) as

$$Y = \beta_1 + \beta_2 \tilde{X}_2 + \delta X_3 + u \qquad (2)$$

where $\tilde{X}_2 = X_2 - X_3$. Compute the **t-value** for testing the hypothesis $H_0 : \delta = 0$ in (2). Do you get the same result as in 1(b)? **(8 points)**

d) Show that model in (1) can be written in a standardized form, viz.,

$$Y^* = \beta_1^* + \beta_2^* X_2^* + \beta_3^* X_3^* + u/S_Y \qquad (3)$$

where the variables in asterisks are standardized. **(3 points)**

(i) Find the OLS estimators of β_2^* and β_3^*. Also show that the OLS estimator of $\beta_1^* = 0$. **(6 points)**

(ii) Compute the ***estimated*** variances of the OLS estimators of β_2^* and β_3^* as well as their t-values. **(8 points)**

2. Consider the simple regression model

$$Y = \beta_1 + \beta_2 X + u \qquad (4)$$

which is estimated using the following data.

$$\bar{Y} = 10, \quad \bar{X} = 20, \quad \sum XY = 6{,}000$$
$$\sum X^2 = 12{,}000, \quad \sum Y^2 = 4{,}000, \quad n = 25$$

a) Predict $E(Y_0)$ when $X_0 = 25$. Label it as $E(\hat{Y}_0)$. **(3 points)**

b) Compute the estimated variance of $[E(\hat{Y}_0)]$ using the formula $V[E(\hat{Y}_0)] = s^2 \left[X_0'(X'X)^{-1}X_0\right]$, where $X_0' = [1 \;\; 25]$ and $(X'X)^{-1} = \frac{1}{n\left[\sum X^2 - n\bar{X}^2\right]}\begin{bmatrix} \sum X^2 & -n\bar{X} \\ -n\bar{X} & n \end{bmatrix}$. **(8 points)**

c) Alternative method: Write $\theta_0 = E(Y_0) = \beta_1 + \beta_2 X_0$ and rewrite the model in (4) as

$$Y = \theta_0 + \beta_2 \tilde{X} + u \qquad (5)$$

where $\tilde{X} = X - X_0$. Show that the OLS estimator of θ_0, $\hat{\theta}_0$, is the same as $E(\hat{Y}_0)$ in 2(a). **(4 points)**

d) Compute the variance of $\hat{\theta}_0$ and show that it is the same as $V\left[\hat{E}(Y_0)\right]$ in 2(b). **(6 points)**

e) Given your results in 2(c) and 2(d) what can you say about the prediction procedure for the K variable regression model? **(6 points)**

3. Dummy Variables: You are given the following information on wages (Y) for male and female candidates

	Number	Mean wage
Male	n_1	$\bar{Y}_1$
Female	n_2	$\bar{Y}_2$
Total	n	$\bar{Y}$

Note: $\bar{Y} = \left(n_1\bar{Y}_1 + n_2\bar{Y}_2\right)/n$

Model A: $Y = \beta_1 + \beta_2 D_F + u$ (6)

where D_F is the female dummy variable.

Model B: $Y = \gamma_1 D_M + \gamma_2 D_F + v$ (7)

where D_M is the male dummy. Note that there is no intercept in (7).

a) Find the OLS estimators of β_1 and β_2 using the information in the above table. Give an interpretation of the coefficients. *Be careful about what you say*. **(10 points)**

b) Find the OLS estimators of γ_1 and γ_2 using the information in the above table. Give an interpretation of the coefficients. *Be careful about what you say*. **(10 points)**

c) Are the two models different? Explain in either case. **(5points)**

d) Do you think the interpretation of β_2 will change if I were to add some explanatory variables (X) to Model A? Explain in either case. **(5 points)**