Econometrics 616
Exam II
Spring 2004

Total Points: 100
Time: 1 hour 30 minutes

Note: Answer all questions and show work when necessary. **Good Luck!**

1. i. Explain what the term heteroskedasticity means in the context of classical linear regression model (CLRM) and why it is a problem for the OLS estimator in the CLRM. **(8 points)**

   ii. Consider the model $Y = \beta X + u$, $u \sim \left(0, \sigma^2 X^2\right)$. Compute the variances of the GLS and OLS estimators of β and show that $V\left(\hat{\beta}_{OLS}\right) > V\left(\hat{\beta}_{GLS}\right)$. **(8 points)**

   Hint: Cauchy-Schwartz Inequality states that $\sum w^2 \sum z^2 \geq \left(\sum wz\right)^2$

2. Use the following information to compute the LR, LM, and Wald tests. The model is

   $$Y = \beta X + u$$

   and the null hypothesis is $H_0 : \beta = 1, \quad H_1 : \beta \neq 1$. **(9+9+9 points)**
   Also show that the Wald test statistic is equivalent to the ordinary t-statistic squared. **(4 points)**

   $$\sum XY = 25, \quad \sum X^2 = 20, \quad \sum Y^2 = 40, \quad n = 50.$$

   You need to use the following formulas to perform the above tests.

   $$LR = n\left(\ln e_*' e_* - \ln e' e\right)$$
   $$LM = n\left(e_*' e_* - e' e\right) / e_*' e_*$$
   $$W = n\left(e_*' e_* - e' e\right) / e' e$$

3. Let the true model be:

   $$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + u, \qquad u \sim iid\left(0, \sigma_u^2\right) \qquad (1)$$

   but you estimate the model:

   $$Y = \beta_0 + \beta_1 X_1 + v, \qquad v \sim iid\left(0, \sigma_v^2\right) \qquad (2)$$

   i. Show that your OLS estimator of β_1 $\left(\text{say } \tilde{\beta}_1\right)$ from model (2) is biased and comment on the direction of the bias. **(8 points)**

ii. Show that $V(\hat{\beta}_1) = \sigma_u^2 \Big/ \left((1-r^2)\sum (X_1 - \bar{X}_1)^2\right)$ where $\hat{\beta}_1$ is the OLS estimator from the true model in (1), and r^2 is the squared correlation coefficient between X_1 and X_2. **(10 points)** **Hint:** For the K variable regression model, $V(b_k) = \sigma^2 / X_k' M_{-k} X_k$, k = 2, …, K, where b_k is OLS estimator of the coefficient of X_k.

iii. If you over specify the model as:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + w \qquad w \sim iid\left(0, \sigma_w^2\right) \qquad (3)$$

show that $V(\breve{\beta}_1) = \sigma_w^2 \Big/ \left((1-R_1^2)\sum (X_1 - \bar{X}_1)^2\right)$ where $\breve{\beta}_1$ is the OLS estimator from model (3) and R_1^2 is the R-squared from the regression of X_1 on a constant, X_2, and X_3. **(10 points) Use the hint in (ii) above.**

iv. Compare $V(\hat{\beta}_1)$ with $V(\breve{\beta}_1)$. Which one is smaller? **(6 points)**

4. Consider the model:

$$Y_t = \alpha + u_t, \quad u \sim \left(0, \sigma_u^2\right) \qquad (4)$$

a. If $u_t = \rho \cdot u_{t-1} + \varepsilon_t$ where $\varepsilon_t \sim iid\left(0, \sigma_\varepsilon^2\right)$, explain how you would estimate α and ρ. Show all of the necessary steps. **(9 points)**

b. If $u_t = \rho \cdot u_{t-1} + \varepsilon_t$ where $\varepsilon_t \sim iid\left(0, \sigma_\varepsilon^2 X_t^2\right)$, explain how you would estimate α and ρ. Show all of the necessary steps. **(10 points)**

Bonus Questions

c. In the model (4) if $u_t = \varepsilon_t + \phi \cdot \varepsilon_{t-1}$ where $\varepsilon_t \sim iid\left(0, \sigma_\varepsilon^2\right)$, explain how you would estimate α and ϕ. Show all of the necessary steps. **(8 points)**

d. Consider the model in (4). Using the Central Limit Theorem that states if a random variable $X_t \sim iid\left(0, \sigma^2\right)$, then $\sqrt{n}\bar{X} \overset{a}{\sim} N\left(0, \sigma^2\right)$, show that $\sqrt{n}(\hat{\alpha} - \alpha) \overset{a}{\sim} N\left(0, \sigma_u^2\right)$ where $\hat{\alpha}$ is the OLS estimator of α. **(7 points)**