Econometrics 616
Midterm Exam II
Spring 2005

Total Points: 100
Time: 1 hour and 30 minutes

Note: Answer all questions. Be neat and show work when necessary. **Good Luck!**

1. Suppose you collect data on wages, education, experience, gender and marijuana use among the adults below 30 years of age. Your objective is to estimate the effects of marijuana use on wages, controlling for other factors. The regression you want to estimate is

$$\ln(wage) = \beta_0 + \beta_1\, usage + \beta_2 Edu + \beta_3 Exp + \beta_4 Female + \beta_5 Female * usage + u \qquad (1)$$

where Female is the female dummy and usage is a dummy variable for marijuana usage.

(a) How would you measure the effect of marijuana use on wages? Is it the same for males and females? **(6 points)**

(b) Instead of (1) your friend is estimating the following model

$$\ln(wage) = \alpha_0 + \alpha_1\, Exp + \alpha_2 Edu + \alpha_3 D_2 + \alpha_4 D_3 + \alpha_5 D_4 + u \qquad (2)$$

where $D_2 = 1$ for female users, $D_3 = 1$ for male users, and $D_4 = 1$ for female nonusers. Show the relationship between the α and the β parameters. **(8 points)**

(c) Another friend of yours is not comfortable in using interaction of dummy variables. To examine the differential effect of usage on males and females, she ran the following regression separately, first for male and then for female, instead of (1)

$$\ln(wage) = \beta_0 + \beta_1\, usage + \beta_2 Edu + \beta_3 Exp + u \qquad (3)$$

She then took the difference in the estimates of β_1 for males and females. What is difference between her approach and the one you used in (1)? Which model is more general? Can you test the hypothesis that the model in (3) and (1) is the same? Explain. **(12 points)**

2. Consider two alternative regressions

$$Y = \alpha_1 D_1 + \alpha_2 D_2 + \alpha_3 D_3 + \alpha_4 D_4 + u \qquad (1)$$

$$Y = \beta_1 + \beta_2 D_2 + \beta_3 D_3 + \beta_4 D_4 + u \qquad (2)$$

where D_1, D_2, D_3 and D_4 are quarterly dummies. Obtain the OLS estimates of the coefficients in each model in terms of quarterly means of Y. **(7+8 points)**

3. If the true model is

$$Y = \alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + u \qquad (1)$$

but the model you estimate is

$$Y = \alpha_0 + \alpha_1 X_1 + v \qquad (2)$$

(a) Show that the OLS estimator of α_1 from (2) is biased and inconsistent. **(10 points)**

(b) Show that $b_1 + b_2 . b_{X1X2} = b_{YX1}$ where b_1 and b_2 are OLS estimates α_1 and α_2 from (1), b_{X1X2} is the OLS coefficient of the simple regression X_1 on X_2, and finally b_{YX1} is the OLS coefficient of the simple regression Y on X_1. **(10 points)**

(c) If $X_2 = X_1^2$ what can you say about the direction of bias in the OLS estimate of α_1 from (2)? Carefully present your arguments. **(8 points)**

4. Suppose that the regression model is

$$Y_i = \beta + u_i, \quad i = 1,...,n \qquad (1)$$

where $E(u_i \mid X_i) = 0$ but $V(u_i \mid X_i) = \sigma^2 X_i^2$, for ($i$ = 1, …, n). Note that X is a scalar variable, and Ω -- the variance-covariance matrix is diagonal. Y**ou may answer the following questions either algebraically or using the following data on**

Y and X, viz., $Y = \begin{bmatrix} 3 \\ 5 \\ 10 \\ 19 \end{bmatrix}, \quad X = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}.$

(a) Given a sample of n (=4) observations on Y and X, find the P^{-1} matrix and the transformed model so that the OLS can be used. What is the OLS estimator of β in the transformed model? What is its variance? **(8 points)**

Now consider the same model in (1) and assume that $E(u_i) = 0$ but $V(u_i) = \sigma_1^2$ for $i = 1, \ldots, n_1$ and $V(u_i) = \sigma_2^2$ for $i = n_1+1, \ldots, n_1 + n_2$ with $n_1 + n_2 = n$. **If you are using the above data assume that $n_1 = n_2 = 2$.**

(b) Write down the Ω matrix and derive the GLS estimator of β and its variance. **(6 points)**

(c) Assume that u_i is $N(0, \sigma_1^2)$ for $i = 1, \ldots, n_1$ and u_i is $N(0, \sigma_2^2)$ for $i = n_1+1, \ldots, n_1 + n_2$ with u_i's being independent. What is the ML estimator of β, σ_1^2, and σ_2^2? **(9 points)**

(d) Derive the LR test for testing $H_0 : \sigma_1^2 = \sigma_2^2$ in part (c). Describe the steps if you cannot perform the test. **(8 points)**