Econ 616
Midterm Exam 2
Spring 2006
Total Points: 100
Time: 1 hr. and 20 min.

Answer all parts. Note that each part has different weight. Good Luck!!

1. The linear regression model consists of the following equation (C is consumption expenditure, and D is defense expenditure):

$$C_i = \beta_1 + \beta_2 GDP_i + \beta_3 D_i + u_i, i = 1, \cdots, n \tag{3}$$

(a) Suppose that $E(u_i) = 0$ but you suspect that the model is heteroskedastic (i.e., $E(u_i|GDP_i, D_i) = \sigma_i^2$).

(i) (**8 points**) How would you test for the presence of heteroskedasticity? Suggest a test of your choice and explain step-by-step how you would perform the test.

(ii) (**9 points**) If you find evidence for heteroskedasticity, how would you obtain heteroskedasticity corrected standard errors of the OLS estimators of β_1, β_2 and β_3? Explain the steps.

(b) (**8 points**) If $E(u_i) = 0$ and $E(u_i^2) = \sigma^2 GDP_i$, how would you estimate β_1, β_2 and β_3 in this model taking the above heteroskedasticity problem into account? Show all the steps.

2. Consider the following two models

$$Y = \beta_1 + \beta_2 D_2 + \beta_3 D_3 + \beta_4 D_4 + u \tag{1}$$

$$Y = \alpha_1 D_1 + \alpha_2 D_2 + \alpha_3 D_3 + \alpha_4 D_4 + u \tag{2}$$

where $D_1, \cdots, D_4$ are quarterly dummies. Assume that there are equal (T) observations in each quarter.

(a) (**10 points**) Write the normal equations for the above models.

(b) (**10 points**) Derive the OLS estimators of the α and β coefficients.

(c) (**5 points**) Show the relationship between the OLS estimates of the α and β coefficients.

3. Consider the following simple regression model

$$Y_{ig} = \beta X_{ig} + \epsilon_{ig}, \tag{1}$$

where $E(\epsilon_{ig}) = 0$, and $Var(\epsilon_{ig}) = \sigma^2$. The subscript i refers to ith individual in the gth group. The data consists of 5 group means which are to be used to estimate

$$\bar{Y}_g = \beta \bar{X}_g + \bar{\epsilon}_g, \; g = 1, \ldots, 5. \tag{2}$$

Group	1	2	3	4	5	*All*
n_g	10	20	15	30	25	100
$\overline{Y}_g$	2	4	3	1	5	$--$
$\overline{X}_g$	1	4	2	1	6	$--$

(a) (**10 points**) Estimate β using both OLS and GLS procedures.

(b) (**10 points**) Compute the variances of OLS and GLS estimators of β assuming $\sigma^2 = 1$. Show that the GLS estimator is more efficient.

4. Data from two sub-periods with 50 observations each produced the following moment matrices (note that X contains a column of ones and another variable)

$$\text{Sample 1} \quad X'X = \begin{pmatrix} 50 & 300 \\ 300 & 2100 \end{pmatrix}, Y'X = \begin{pmatrix} 300 & 2000 \end{pmatrix}, Y'Y = 2100.$$

$$\text{Sample 2} \quad X'X = \begin{pmatrix} 50 & 300 \\ 300 & 2100 \end{pmatrix}, Y'X = \begin{pmatrix} 300 & 2200 \end{pmatrix}, Y'Y = 2500.$$

(a) (**15 points**) Test the hypothesis that the coefficients (intercept and slope) in both sub-periods are the same ($\beta^1 = \beta^2$), assuming that variances are the same for both sub-periods ($\sigma_1^2 = \sigma_2^2$).

(b) (**15 points**) Assume that the coefficients (intercept and slope) in both sub-periods are the same, but the variances for both sub-periods are not the same, i.e., $\sigma_1^2 \neq \sigma_2^2$. How would you estimate such a model when (i) σ_1^2 and σ_2^2 are **known**, (ii) σ_1^2 and σ_2^2 are **unknown**?