Econometrics 616
2nd Midterm
Answer Key

1.

(i) Show that the OLS estimator of $\hat{\delta}_1$ can be expressed as the following:

$$\hat{\delta}_1 = \beta_1 + \beta_2 \frac{\sum x_1 x_2}{\sum x_1^2} + \frac{\sum x_1 u}{\sum x_1^2}$$

OLS on the misspecified model yields

$$\hat{\delta}_1 = \frac{\sum x_1 y}{\sum x_1^2} = \frac{\sum x_1(\beta_1 x_1 + \beta_2 x_2 + u)}{\sum x_1^2} = \beta_1 + \beta_2 \frac{\sum x_1 x_2}{\sum x_1^2} + \frac{\sum x_1 u}{\sum x_1^2}$$

(ii) Show that the above estimator is both biased and inconsistent.

$$E(\hat{\delta}_1) = \beta_1 + \beta_2 \frac{\sum x_1 x_2}{\sum x_1^2}, \text{ So Biased}$$

$$\text{plim}(\hat{\delta}_1) = p\lim(\beta_1 + \beta_2 \frac{\sum x_1 x_2}{\sum x_1^2} + \frac{\sum x_1 u}{\sum x_1^2}) = p\lim(\beta_1) + p\lim(\beta_2 \frac{\sum x_1 x_2}{\sum x_1^2}) +$$

$$p\lim(\frac{\sum x_1 u}{\sum x_1^2})) = \beta_1 + \beta_2 p\lim \frac{\frac{1}{n}\sum x_1 x_2}{\frac{1}{n}\sum x_1^2} + p\lim \frac{\frac{1}{n}\sum x_1 u}{\frac{1}{n}\sum x_1^2} = \beta_1 + \beta_2\left(\frac{Cov(x_1, x_2)}{Var(x_1)}\right).$$

$$\neq \beta_1$$

(iii) The nature of the bias and inconsistent depends on how correlated the two x variables are. If there is positive correlation then our estimate is biased up, and our estimate is biased down if the is negative correlation. Note also that the variance of the estimator will be larger than the estimate of the unbiased estimator, even if no correlation exists.

(iv) If the two regressors are uncorrelated then the asymptotic distribution of the estimator is: $\sqrt{n}(\hat{\delta}_1 - \beta_1) \sim N\left(0, \sigma^2 / \text{var}(x_1)\right)$

2.

(i) Derive the GLS estimator using the partitioned variables.

$$\hat{\beta}_{GLS} = \left(X'\Omega^{-1}X\right)^{-1} X'\Omega^{-1}Y = \left(\begin{bmatrix} X_1 & X_2 \end{bmatrix} \begin{bmatrix} \sigma_1^2 I & 0 \\ 0 & \sigma_2^2 I \end{bmatrix}^{-1} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \right)^{-1} X'\Omega^{-1}Y$$

$$= \left(\frac{1}{\sigma_1^2} X_1{}' X_1 + \frac{1}{\sigma_2^2} X_2' X_2 \right)^{-1} \left(\begin{bmatrix} X_1 & X_2 \end{bmatrix} \begin{bmatrix} \sigma_1^2 I & 0 \\ 0 & \sigma_2^2 I \end{bmatrix}^{-1} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \right)$$

$$= \left(\frac{1}{\sigma_1^2} X_1{}' X_1 + \frac{1}{\sigma_2^2} X_2' X_2 \right)^{-1} \left(\frac{1}{\sigma_1^2} X_1{}' Y_1 + \frac{1}{\sigma_2^2} X_2' Y_2 \right)$$

(ii) if the two variances are the same then we have the following:

$$\hat{\beta}_{GLS} = \left(\frac{1}{\sigma^2} X_1{}' X_1 + \frac{1}{\sigma^2} X_2' X_2 \right)^{-1} \left(\frac{1}{\sigma^2} X_1{}' Y_1 + \frac{1}{\sigma^2} X_2' Y_2 \right)$$

$$= \sigma^2 \left(X_1{}' X_1 + X_2' X_2 \right)^{-1} \frac{1}{\sigma^2} \left(X_1{}' Y_1 + X_2' Y_2 \right) = \left(X_1{}' X_1 + X_2' X_2 \right)^{-1} \left(X_1{}' Y_1 + X_2' Y_2 \right)$$

$$= (X'X)^{-1} X'Y = \hat{\beta}_{OLS}$$

$$\begin{bmatrix} \hat{\beta}_1^{GLS} \\ \hat{\beta}_2^{GLS} \end{bmatrix} = \left(\begin{bmatrix} X_1' & 0 \\ 0 & X_2{}' \end{bmatrix} \begin{bmatrix} \frac{1}{\sigma_1^2} I & 0 \\ 0 & \frac{1}{\sigma_2^2} I \end{bmatrix}^{-1} \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} \right)^{-1} X'\Omega^{-1}Y$$

(iii)

$$= \begin{bmatrix} \frac{1}{\sigma_1^2} X_1{}' X_1 & 0 \\ 0 & \frac{1}{\sigma_2^2} X_2{}' X_2 \end{bmatrix}^{-1} \left(\begin{bmatrix} X_1' & 0 \\ 0 & X_2{}' \end{bmatrix} \begin{bmatrix} \frac{1}{\sigma_1^2} I & 0 \\ 0 & \frac{1}{\sigma_2^2} I \end{bmatrix}^{-1} \begin{bmatrix} Y_1 & 0 \\ 0 & Y_2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} \frac{1}{\sigma_1^2} X_1{}' X_1 & 0 \\ 0 & \frac{1}{\sigma_2^2} X_2{}' X_2 \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{\sigma_1^2} X_1{}' Y_1 \\ \frac{1}{\sigma_2^2} X_2{}' Y_2 \end{bmatrix} = \begin{bmatrix} \hat{\beta}_1^{OLS} \\ \hat{\beta}_2^{OLS} \end{bmatrix}$$

To test the hypothesis of equal coefficients we will use the model in (i) as our restricted model and the model from (iii) as our unrestricted model. We then use the likelihood ratio test which is the following:

$LR = n\left[\ln\left({e_*{}' e_*}/{e'e} \right) \right] \sim \chi_k^2$ where k is the # of restrictions and the e* are the residuals from the restricted model.

3.

(i) Before deregulation:

$$\ln(Q) = 2.635 - 1.029\ln(P) - .001\ln(Y) - .821\ln(ACCID) + .0009FATAL$$

After deregulation:

$$\ln(Q) = .947 - .751\ln(P) + .986\ln(Y) - .003\ln(ACCID) - .0001FATAL \text{ n:}$$

What we see is that both the price elasticity and the accident elasticity became more inelastic while the income elasticity became positive. This signified that airplane tickets became more of a normal good and the fact that the price elasticity became relatively more inelastic means that people were more indifferent about the price, that is riding on airplanes became more of a necessity than a luxury.

(ii)

Chow Test:

$$\frac{(RSS_R - RSS_u)}{RSS_U} * \frac{n-k}{j} = \frac{(1.096191 - .700959)}{.700959} * \frac{41-5}{5} = 4.05968$$

Wald Test: $n\left(\frac{RSS_R}{RSS_u} - 1\right) = 41\left(\frac{1.096191}{.700959} - 1\right) = 23.1176$

LM Test: $n\left(1 - \frac{RSS_u}{RSS_R}\right) = 41\left(1 - \frac{.700959}{1.096191}\right) = 14.7826$

LR Test: $n\ln\left(RSS_r / RSS_U\right) = 41\ln\left(\frac{1.096191}{.700959}\right) = 18.333$

We also note the following: Wald > LR > LM

(iii) To test the null that after deregulation the elasticity for ACCID is equal to 1 we want the two coefficients to sum to 1. We will impose this in the model and run the restricted regression. From this regression we can use the R-squared to compute the value of the LM Test.

4.

(i) See the answer to Exam #1.

$\beta_2 + \beta_3 = 1$

$r = 1, R = [0 \quad 1 \quad 1]$

(ii)

$$RSS_R = RSS_U + .3\left[R[X'X]^{-1}R'\right]^{-1}.3 = 520 + .3\left[\frac{11}{390}\right]^{-1}.3$$

$$RSS_R = 520 + 3.19091 = 523.19091$$

(iii) LR Test: $29\ln\left(523.19091 / 520\right) = .177411$