ECONOMICS 616
EXAM 1
Spring 2002

Total Points: 100
Time: 1 hour, 15 minutes

NOTE: Answer all questions. Show your work. GOOD LUCK!

1. Consider the following regression function

$$Y_i = b_1 + b_2 X_{2i} + b_3 X_{3i} + u_i, i = 1,...,n \quad (1)$$

that satisfies all the standard assumptions of classical linear regression model.

Assume that the sample correlation between X_2 and X_3 is zero. Based on this information, someone suggested you to run the following regressions

$$Y_i = g_1 + g_2 X_{2i} + u_{1i}, i = 1,...,n \quad (2)$$

$$Y_i = l_1 + g_3 X_{31} + u_{2i}, i = 1,...,n \quad (3)$$

(a) **(8 points)** Is $\hat{g}_2 = \hat{b}_2$ and $\hat{g}_3 = \hat{b}_3$? Why?

Note: $\hat{b}_{2,} \hat{b}_3, \hat{g}_2, \hat{g}_3$ are the OLS estimators from (1), (2) and (3), respectively.

(b) **(8 points)** Is $\text{var}(\hat{b}_2) = \text{var}(\hat{g}_2)$ and $\text{var}(\hat{b}_3) = \text{var}(\hat{g}_3)$? Explain.

(c) **(12 points)** Is $R^2 = R_1^2 + R_2^2$ where R^2, R_1^2, and R_2^2 are coefficient of determination from the OLS regressions (1), (2) and (3), respectively? Explain.

(d) **(4 points)** What did you learn from this problem?

(e) **(8 points)** Is $\hat{g}_2 = \hat{b}_2$ and $\hat{g}_3 = \hat{b}_3$ if $b_1 = g_1 = l_1 = 0$ in (1)-(3)? Briefly explain.

2. A researcher carries out an OLS regression and obtains

$$b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \\ 2 \end{bmatrix}, \qquad s^2 (X'X)^{-1} = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix},$$

where s^2 is the OLS estimator of σ^2. Assume $n = 53$.

(a) **(10 points)** Using the above information, obtain the restricted least squares estimator b^* when $\boldsymbol{b}_1 + \boldsymbol{b}_2 + \boldsymbol{b}_3 = 0$.

(b) **(10 points)** Using the above information, test the hypothesis that $\boldsymbol{b}_1 + 2\boldsymbol{b}_2 + \boldsymbol{b}_3 = 0$ at the 5% level of significance.

(c) **(10 points)** Given $X_0' = [1\ 2\ 3]$ predict E(Y_0) and obtain a 95% confidence interval for Y_0.

(d) **(4 points)** Find the estimated var-cov matrix of (b_2, b_3).

(e) **(6 points)** Construct a 95% confidence interval for β_3.

(f) **(4 points)** Test the hypothesis that $\beta_2 = 3$ against the alternative that $\beta_2 < 3$.

(g) **(4+2+4 points)** Find (i) $\frac{ESS}{s^2}$, (ii) $\frac{RSS}{s^2}$, and (iii) R^2.

(h) **(6 points)** Test the hypothesis that there is no regression at the 5% level of significance.