Solution : Econ 616 , Exam 2 , SP 2002

1. Part A: Given the information you have to use F test

$$\frac{(RSS_R - RSS_U)/\#\text{ rest}}{RSS_U/df} \sim F_{\#\text{ rest},\, d.f}$$

(i) use equations ① and ③

$$\frac{(6.56-3.16)/2}{3.16/(15-4)} \sim F_{2,11}$$

(ii) Use equations (1) and (4)

$$\frac{(6.56-3.33)/1}{3.33/(15-3)} \sim F_{1,12}$$

(iii) Use equations (2) and (3)

$$\frac{(3.49-3.16)/1}{3.16/(15-4)} \sim F_{1,11}$$

(iv) Use equations (1) and (2)

$$\frac{(6.56-3.49)/1}{3.49/(15-3)} \sim F_{1,12}$$

(v) use equation (3) and (4)

$$\frac{(3.33-3.16)/1}{3.16/(15-4)} \sim F_{1,11}$$

Part B:

(i) Write (2a) as $Y = C_0(D_1+D_2) + C_2 D_2 + C_3 X + e$

$$= C_0 D_1 + (C_0 + C_2) D_2 + C_3 X + e$$

and compare it with (2) to get

$$C_0 = -.46,\ C_0 + C_2 = .55 \Rightarrow C_2 = .55 + .46 = 1.01$$

and $C_3 = .49$

(ii) Write (3a) as $Y = b_0 (D_1+D_2) + b_1 D_2 + b_2 X (D_1+D_2) + b_3 D_2 X + e$

$= b_0 D_1 + (b_0+b_1) D_2 + b_2 D_1 X + (b_2+b_3) D_2 X + e$

and then compare it with equation (3) to get

$b_0 = -.06$, $b_0+b_1 = .4 \Rightarrow b_1 = .46$, $b_2 = .44$, $b_2+b_3 = .51$

$\Rightarrow b_3 = .51 - .44 = .07$

(iii) Write (4a) as $Y = f_0 + f_1 X (D_1+D_2) + f_2 D_2 X + e_4$

$= f_0 + f_1 D_1 X + (f_1+f_2) D_2 X + e_4$

and equate it to equation (4) to get

$f_0 = .23$, $f_1 = .4$, $f_1+f_2 = .52 \Rightarrow f_2 = .52 - .4 = .12$

2. (a) Start from the hint: $y = b_2 x_2 + b_3 x_3 + e$. Then multiply both sides by x_2 and take summation. This gives

$$\Sigma y x_2 = b_2 \Sigma x_2^2 + b_3 \Sigma x_2 x_3 + \underbrace{\Sigma x_2 e}_{=0} \quad \text{since } x'e = 0.$$

$$\Rightarrow \frac{\Sigma y x_2}{\Sigma x_2^2} = b_2 + b_3 \frac{\Sigma x_2 x_3}{\Sigma x_2^2}$$

$$\Rightarrow a_2 = b_2 + b_3 C_{32}$$

In general, with K regressors and X_2 omitted

$$a_2 = b_2 + b_3 C_{32} + b_4 C_{42} + \dots + b_K C_{42}$$

(b) $E(a_2) = E(b_2) + E(b_3) \cdot C_{32} = \beta_2 + \beta_3 C_{32}$

(Note: OLS estimators of the true model are unbiased. And C_{32} is a constant since X_2 and X_2 are non-random).

(c) If X_2 and X_3 are uncorrelated, $E(b_2) = \beta_2$

but $E(s^2) > \sigma^2$ (biased upward)

$\Rightarrow$ t values are biased downward.

3. (i) If you rewrite (1) as

$\frac{Y}{X} = \beta + \frac{u}{X}$ then all the assumptions of CRM will be satisfied. (Note ~~this~~ ~~this~~ is equivalent to writing the model as $P^{-1}Y = P^{-1}X\beta + P^{-1}u$ with $P^{-1} = \text{diag}(1/x_i)$).

$\Rightarrow b_a = \frac{1}{n}\sum\left(\frac{Y}{X}\right)$ [This is the OLS estimator of the intercept only model.]

Use convergence in quadratic mean to prove consistency.

$$E(b_a) = \frac{1}{n}\sum_i \frac{1}{X_i} E(Y_i) = \frac{1}{n}\sum_i \frac{1}{\cancel{X_i}}(\beta \cancel{X_i}) = \frac{1}{n}\cdot n\beta = \beta.$$

$$V(b_a) = \frac{1}{n^2}\sum \frac{1}{X_i^2} V(Y_i) = \frac{1}{n^2}\sum \frac{1}{\cancel{X_i^2}}\sigma^2 \cancel{X_i^2} = \frac{\sigma^2}{n}.$$

Thus $\left.\begin{array}{l} E(b_a) = \beta \\ V(b_a) \to 0 \end{array}\right\}$ as $n \to \infty$

$\Rightarrow$ plim $b_a = \beta$.

(ii) a. $\hat{b} = \frac{\bar{Y}}{\bar{X}} \Rightarrow E(\hat{b}) = \frac{1}{\bar{X}} E(\bar{Y}) = \frac{\beta\bar{X}}{\bar{X}} = \beta.$

$$V(\hat{b}) = \frac{1}{(\bar{X})^2} V(\bar{Y}) = \frac{1}{\bar{X}^2} V(\bar{u}) = \frac{1}{\bar{X}^2}\left(\frac{1}{n^2}\sum_i \sigma^2 X_i^2\right)$$

$$= \underbrace{\frac{\sigma^2}{n}}_{\downarrow\, 0} \cdot \underbrace{\frac{\sum X_i^2}{n}}_{\downarrow\,\text{constant (second raw moment)}} \cdot \underbrace{\frac{1}{\bar{X}^2}} \to 0 \text{ as } n \to \infty.$$

$\text{plim}(\bar{X})^2 = (\text{plim}\,\bar{X})^2 = (\text{constant})^2$

(Assume that plim $\bar{X}$ = population mean of $\bar{X}$ = constant)

$\Rightarrow \hat{b}$ is consistent estimator of β.

b. $$\tilde{b} = \frac{\Sigma x_i y_i}{\Sigma x_i^2} = \frac{\Sigma x_i (\beta x_i + u_i - \bar{u})}{\Sigma x_i^2} = \beta + \frac{\Sigma x_i u_i - \bar{u}\,\overset{\to 0}{\Sigma x_i}}{\Sigma x_i^2}$$

$$= \beta + \frac{\Sigma x_i u_i}{\Sigma x_i^2}.$$

$$E(\tilde{b}) = \beta + \frac{\Sigma x_i E(u_i)}{\Sigma x_i^2} = \beta.$$

$$V(\tilde{b}) = \frac{V(\Sigma x_i u_i)}{(\Sigma x_i^2)^2} = \frac{\Sigma x_i^2 \cdot \sigma^2 x_i^2}{(\Sigma x_i^2)^2} = \frac{\frac{\sigma^2}{n}\frac{1}{n}\Sigma x_i^4}{\left(\frac{1}{n}\Sigma x_i^2\right)^2} \to 0$$

Assume $\frac{1}{n}\Sigma x_i^4 \longrightarrow$ fourth moment (constant) $\Big\}$ population
$\frac{1}{n}\Sigma x_i^2 \to$ second moment (const)

Thus $\tilde{b}$ is a consistent estimator of β.