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BUYERS, SELLERS, AND MIDDLEMEN: VARIATIONS ON SEARCH-THEORETIC THEMES*

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We study exchange that is bilateral but indirect—it involves chains of intermediaries, or middlemen—in markets with frictions. These frictions include search and bargaining problems. We show how, and how many, intermediaries might get involved in a chain, and how bargaining with one depends on upcoming negotiations with those downstream. The roles of buyers, sellers, money, and prices are discussed, allowing us to clarify some neglected connections between different branches of search theory. Pursuing one such connection, with monetary economics, we show how bubbles can emerge in intermediation, even with fully rational agents and perfect foresight.

1. INTRODUCTION

"You sell your own works directly, Mr Nelson?" Siobhan asked.
"Dealers have got the market sewn up," Nelson spat. "Bloodsucking bastards that they are . . . "
Resurrection Men (1991) by Ian Rankin.

This article develops a model of exchange that is bilateral but indirect—it involves chains of intermediaries, or middlemen—in markets with frictions. The frictions include search and bargaining problems. We show how, and how many, intermediaries might get involved in a chain. Although there is much research on intermediation, in general, a neglected aspect that seems important to practitioners is that there are often *multiple middlemen* engaged in getting goods from the originator to the end user—say, from farmer to broker to distributor to retailer to consumer.[2] In such an intermediation chain, we analyze how bargaining over the terms of trade with one party depends on upcoming negotiations with those downstream. We also have something to say, in the context of the formal model, about the roles of buyers, sellers, money, and prices in bilateral exchange, allowing us to make some previously neglected connections between disparate branches of the search theory literature. Pursuing one such connection, with monetary economics, we demonstrate how *bubbles* can emerge in intermediation chains, even with fully rational agents and perfect foresight.

In terms of related papers, it was not so long ago that Rubinstein and Wolinsky (1987) motivated their work on middlemen as follows:

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[2] Similarly, trade on the Internet is described by Ellis (2009) as follows: "If a majority of the wholesale companies being advertised are not true wholesale companies, then what are they and where are they getting their products? They are likely just middleman operating within a chain of middleman. A middleman chain occurs when a business purchases its resale products from one wholesale company, who in turn purchases the products from another wholesale company, which may also purchase the products from yet another wholesale company, and so on."

> Despite the important role played by intermediation in most markets, it is largely ignored by the standard theoretical literature. This is because a study of intermediation requires a basic model that describes explicitly the trade frictions that give rise to the function of intermediation. But this is missing from the standard market models, where the actual process of trading is left unmodeled.

Since then, subsequent studies have attempted to rectify the situation by analyzing how middlemen affect the quantity and quality of trade, the time required to conduct transactions, the variety of goods on the market, bid–ask spreads, and so on.

Here is a review of the literature; it is not meant to be comprehensive, only to provide a brief catalog of related research. In Rubinstein and Wolinsky's (1987) model, the focus is exclusively on search frictions. Agents meet bilaterally and at random, and middlemen are agents with an advantage over the original suppliers in the rate at which they meet end users. Bose and Sengupta (2010) study intermediation in a search model where middlemen are always immediately available to buyers, and they can cater to repeat clientele. Tse (2009) presents a model where agents are dispersed over space and trading costs increase with distance. In spatial equilibrium, middlemen choose to cluster at central locations, which helps economic activity and improves welfare. In contrast to these papers, our goal is not to explain the existence of middlemen, but to focus on determining the terms of trade when intermediation chains are necessary and on determining when these chains are economically viable.

A different branch of the literature emphasizes the role of middlemen in validating quality. Biglaiser (1993) and Li (1998, 1999) have middlemen with expertise that allows them to distinguish high- from low-quality goods. Relatedly, Masters (2007, 2008) analyzes the emergence of middlemen as a consequence of heterogeneous production costs instead of heterogeneous information. Still another group of papers has middlemen helping buyers obtain their preferred goods by holding inventories of either more or more types of commodities, including Johri and Leach (2002), Shevichenko (2004), and Smith (2004), where buyers have idiosyncratic or match-specific preferences, and middlemen cater to diverse tastes by holding a variety of goods. A related model is presented in Watanabe (2010a,b), where middlemen also have technologies that allow them to hold inventories, but he uses directed search, and buyers seek out trading partners based on their capacity. In contrast to those papers, our middlemen do not have a comparative advantage in information or technology; they are simply a necessary part of the process of getting goods from their original suppliers to end users.[3]

Brokers and dealers are two types of middlemen: the former execute trades on behalf of others; the latter trade on their own behalf. In some models, middlemen set bid and ask prices at which they sell and buy as dealers, as in some financial-asset and used-car markets. See Yavas (1992), Gehrig (1993), Spulber (1996), van Raalte and Webers (1998), Rust and Hall (2003), Caillaud and Jullien (2003), and Loertscher (2007). Much of this literature studies the impact of market power, say monopoly or duopoly middlemen. Gehrig (1993) presents a static model in which buyers and sellers differ in valuations and costs that are private information, and can access intermediaries whose locations and prices are publicly observable. Spulber (1996) studies a dynamic model in which buyers, sellers, and intermediaries are heterogeneous, and the concerned is characterizing bid–ask spreads. Rust and Hall (2003) extend that model by adding a second type of intermediary that posts publicly observable prices. Other papers, including Yavas (1994, 1996), feature middlemen as brokers who get traders together but do not hold inventories themselves, such as real estate agents and employment agencies.[4]

[3] Dale Mortensen suggests one interpretation: in order to move goods from location A_1 to A_N, one has to transport them through $A_2, A_3, \ldots$, and those with property rights to the intermediate locations all want a cut of the profit (e.g., to get wheat from northern to southern Europe one has to ship it though Ghent). Although this is not the only interpretation, it is an interesting suggestion.

[4] Additional papers that focus on intermediation in financial markets include Duffie et al. (2005, 2007), Miao (2006), Weill (2007), Lagos and Rocheteau (2009), and Trejos and Wright (2012). Some of this work is related to the New Monetarist literature recently surveyed by Williamson and Wright (2010) and Nosal and Rocheteau (2011), where one can find more discussion of and citations to work on financial intermediaries. We discuss in detail the connection between intermediation and money below, but for now, note that middlemen emerge endogenously in the original

As in many of these papers, we use search theory, but our environment is quite different. We are also interested in comparing search-based models of middlemen, in general, to other branches of search theory. This allows us to bridge some gaps between these literatures and to show how ostensibly different models are logically related. It also allows us to discuss several issues concerning bilateral trade at an abstract level, including: Who is a buyer and who is a seller? How should one define price? Moreover, we develop a particular bargaining solution. Although we do not consider this as a major contribution to bargaining theory, per se, we argue that it is very useful in search models with nonlinear utility, especially when one is interested in nonstationary equilibria. Finally, in terms of contributions, when we consider nonstationary equilibria, we show that with nonlinear utility there exist bubbles in intermediation chains—dynamic equilibrium paths where the terms of trade differ from their fundamental values, and can vary over time, even though fundamentals are constant, based purely on self-fulfilling prophecies.

Although this project mainly concerns theory, we mention a natural application to the recently popular activity of *flipping*. This is defined by Wikipedia (as good a source as any) as "purchasing a revenue-generating asset and quickly reselling ... it for profit." Although one can flip any asset, in principle, the label is usually applied to real estate or sometimes IPOs. As in our intermediation chains, "Under the multiple investor flip, one investor purchases a property at below-market value, assigns or sells it quickly to a second investor, who subsequently sells it to the final consumer, closer to market value." Of course, "Profits from flipping real estate come from either buying low and selling high (often in a rapidly rising market), or buying a house that needs repair and fixing it up before reselling." It seems people believe that this has something to do with the generation of housing and other price bubbles, where prices differ from their fundamental value, and hence the reference to a "rapidly rising market." This is consistent with our theory.

The rest of the article is summarized as follows: Section 2 introduces the basic framework and presents the bargaining solution. Here, we prove existence and generic uniqueness of equilibrium for any finite number N of agents. Section 3 studies case of $N = 3$ agents. We characterize the regions of parameter space where middlemen either are or are not active, and show how outcomes are not necessarily efficient. Section 4 allows any finite N, studies intermediation chains, shows how to construct maximal chains, and discusses their qualitative properties. In particular, even though the market can exhibit the appearance of trading frenzies interspersed by lulls, plus increases in the rate of change in the terms of trade, we argue that this should not be interpreted as a bubble. Section 5 digresses to discuss some broader issues of interpretation. Section 6 takes up the possibility of genuine intermediation bubbles. We show that this requires $N = \infty$ as well as nonlinear utility. We also discuss how, once we have nonlinear utility, the results depend on which bargaining solution one uses. Section 7 contains some comments comparing our setup to some ostensibly very different models, including overlapping generations (OLG) models. Section 8 concludes.

2. THE BASIC MODEL

Time is continuous and unbounded. The set of agents is $\mathcal{A} = \{A_1, A_2, \ldots, A_N\}$, where $N < \infty$ for now. Agents are spatially separated in the following fashion: A_n can meet, and hence trade, with A_{n-1} and A_{n+1} but no one else. Therefore, trade between A_{n-1} and A_{n+1} must go through A_n, and we cannot ask why the market does not "cut out the middleman," or disintermediate (this is relaxed in the working paper, Wright and Wong, 2011).[5] Given these assumptions, the

search-based model of money in Kiyotaki and Wright (1989), where certain agents choose to acquire and retrade costly-to-store commodities. See also Wright (1995), Camera (2001), Corbae et al. (2003), and Howitt (2005). Other work on intermediation includes Kurz and Wilson (1974), Townsend (1978), Kalai et al. (1978), Williamson (1987), Glosten and Milgrom (1985), Seward (1990), and Admati and Pfleiderer (1990).

[5] On disintermediation, in general, practitioners say this: "Why doesn't every wholesaler just buy from the manufacture and get the deepest discount? The answer is simple—not all wholesalers (or companies claiming to be wholesalers)

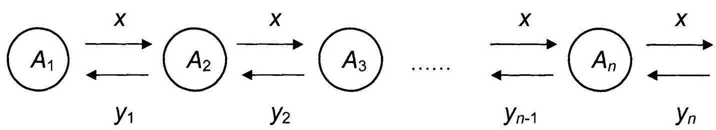

FIGURE 1

POPULATION GRAPH

population can be represented as a *network*, or a graph with the set of nodes $\mathcal{A}$ connected as in Figure 1. There are search frictions: It can take time and other resources for A_n to meet A_{n+1}. There is an indivisible object x in fixed supply and a divisible object y that anyone can produce at unit cost up to some arbitrarily big upper bound $\bar{y}$. Only A_1 is endowed with x. He can either try to trade it to A_2 in exchange for some amount of y, say y_1, or consume it himself for utility ν_1. Hence, ν_1 is A_1's opportunity cost of trading. One can add production costs, say A_1 may need to pay k_1 to originate x, or A_n may need to pay k_n to maintain or improve it, as in flipping real estate; this omitted here to reduce notation.

Generally, if A_n acquires x from A_{n-1}, he can either consume it for payoff ν_n or try to trade it to A_{n+1} for y_n, which generates a payoff $U_n(y_n)$. We often use $U_n(y_n) = y_n$, but in Section 6 it becomes important to consider general utility, with $U_n' > 0$ and $U_n'' < 0$. If A_1 trades x to A_2 and A_2 trades it to A_3 ... before some A_N eventually consumes x, we say that trade is intermediated and call $A_2, \dots, A_{N-1}$ intermediaries or middlemen (we do not allow A_n to trade x back to A_{n-1}, but typically this would not be desirable anyway). All parameters describing utilities, costs, etc., are common knowledge. Usually, we assume agent A_n exits the market after consuming or trading away x. If one wanted to keep the economy going forever, one can "recycle" agents by allowing them to continue instead of exit after trading, replenishing A_1's endowment each time, or one can replace every A_n with a "clone" of himself after he leaves the market (see Nosal et al., 2013). Instead, after studying the finite model, we keep things going by letting $N = \infty$.

We now discuss the trading protocol when A_{n-1} with x meets A_n. It is possible that there are no gains from trade, in which case A_{n-1} consumes x. But if there are gains from trade, they enter into the following game:

Initial-offer stage: A_{n-1} proposes to A_n "give me $y_{n-1} \in [0, \bar{y}]$ for x."

- If A_n accepts they trade and the game ends;
- If A_n rejects they go to the next stage.

Final-offer stage: Nature moves—a coin toss—such that:

- With probability θ_{n-1}, A_{n-1} makes A_n a take-it-or-leave-it offer;
- With probability $1 - \theta_{n-1}$, A_n makes A_{n-1} a take-it-or-leave-it offer.

Figure 2 shows the game tree, with W_n denoting the payoff for A_n from acquiring x, as defined below.

One can think of the an interval of time Δ elapsing between the initial- and final-offer stages of the game, and then take $\Delta \to 0$ so we can ignore discounting at this point; one could, however, easily allow discounting between stages. For a general utility function $U_{n-1}(y)$, there are gains from trade if and only if $U_{n-1}(W_n) > \nu_{n-1}$ where W_n is A_n's payoff from acquiring x, since $y = W_n$ is the most he would produce to get it. If there are no gains from trade A_{n-1} consumes x (although in this case A_{n-1} and A_n would never meet in equilibrium). Suppose there are gains from trade. At the final-offer stage of bargaining, if A_{n-1} wins the coin toss he can extract A_n's entire payoff by asking him to produce $y_n = W_n$, while if A_n wins the coin toss he has

can afford to purchase the minimum bulk-order requirements that a manufacture requires. Secondly, many manufactures only do business with companies that are established" (Ellis, 2009). More work would be welcome on this issue, to be sure, but that is not the point of our analysis.

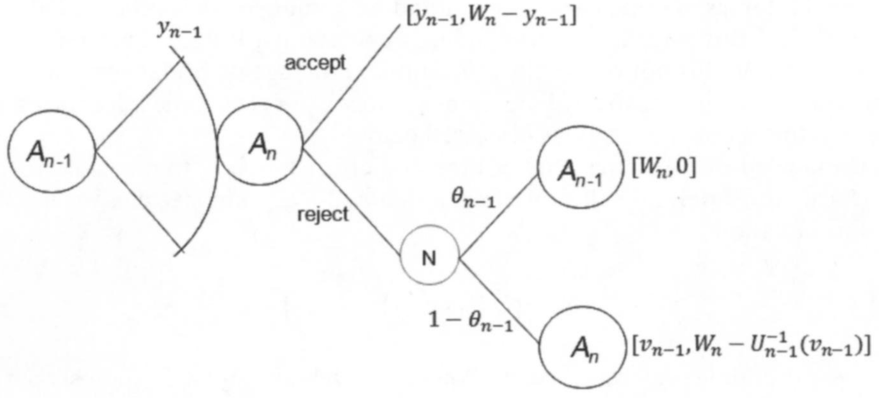

FIGURE 2

GAME TREE

to compensate A_{n-1} for his opportunity cost by producing y_{n-1} such that $U_{n-1}(y_{n-1}) = \nu_{n-1}$, leaving A_n a surplus $W_n - U_{n-1}^{-1}(\nu_{n-1})$.

A subgame-perfect equilibrium (SPE) is the following: At the initial-offer stage A_{n-1} makes A_n his reservation offer—i.e., the one that makes him indifferent between accepting and moving to the random final-offer stage—and A_n accepts. It is easy to see that A_n's reservation offer is

$$y_{n-1} = \theta_{n-1} W_n + (1 - \theta_{n-1}) U_{n-1}^{-1}(\nu_{n-1}), \tag{1}$$

and this generates surpluses $S_{n-1} = U(y_{n-1}) - \nu_{n-1}$ and $S_n = W_n - y_{n-1}$. Of course, the agents are not compelled to trade, but it is easy to check that $U_{n-1}(W_n) \geq \nu_{n-1}$ implies $S_{n-1} \geq 0$ and $S_n \geq 0$, so exchange is voluntary. Note that with risk neutrality one might worry that an initial offer gets rejected, leading to the coin toss, but one may rule this out by assuming a probability $\varepsilon > 0$ of an exogenous breakdown or a discount factor $\delta < 1$ between stages. Given this, the SPE where the initial offer is accepted is unique.

With risk neutrality, $U_{n-1}(y) = y$, (1) simplifies to

$$y_{n-1} = \theta_{n-1} W_n + (1 - \theta_{n-1}) \nu_{n-1}, \tag{2}$$

and the surpluses reduce to $S_{n-1} = \theta_{n-1}(W_n - \nu_{n-1})$ and $S_n = (1 - \theta_{n-1})(W_n - \nu_{n-1})$. In this linear case, the outcome is the same as some other common bargaining models. Consider generalized Nash (1950), with θ_{n-1} denoting the bargaining power of A_{n-1} and threat points given by outside options:

$$y_{n-1} = \arg\max_y (y - \nu_{n-1})^{\theta_{n-1}} (W_n - y)^{1-\theta_{n-1}}.$$

The solution is (2), so our game implements Nash. One can also consider Kalai's (1977) proportional bargaining solution, giving A_{n-1} a fraction θ_{n-1} of the total surplus. In fact, Kalai and Nash are the same in this linear example, even though they are not the same in general, as discussed in Section 6.[6]

For now, with linear utility it does not matter which bargaining solution one uses; later it will. In fact, with linear utility, one could skip the first round and just use a coin toss to determine who

[6] Kalai bargaining has recently become popular in search theory with nonlinear utility, especially in models with liquidity constraints, because it is more tractable and has several other advantages relative to Nash (see Aruoba et al., 2007; Lester et al., 2011, and references therein).

makes the final offer, as in some earlier search-and-bargaining models (Gale, 1990; Mortensen and Wright, 2002). But since a coin toss induces uncertainty, if agents are risk averse, then skipping the first round is not bilaterally efficient, and the agents prefer our game. Although there is a certain arbitrariness to any bargaining protocol, ours has some advantages, especially in nonlinear, nonstationary models, as discussed below.[7]

Search is modeled by assuming it takes time and effort for A_{n-1} to meet A_n. If the former sets out to locate the latter, his value of search is denoted V_{n-1}. This satisfies the usual dynamic programming equation

$$rV_{n-1} = \alpha_{n-1}\left[U_{n-1}(y_{n-1}) - V_{n-1}\right] - c_{n-1},$$

where r is the rate of time preference, α_n is a Poisson arrival rate, and c_n is a flow search cost. One can alternatively think of $1/\alpha_n$ as A_n's expected transportation time and c_n his transportation (or inventory holding or maintenance) cost. Rearranging, we have

$$V_n = \frac{\alpha_n U_{n-1}(y_n) - c_n}{r + \alpha_n}, \tag{3}$$

and the value mentioned above of acquiring x is $W_n = \max\{V_n, \nu_n\}$. For search by A_n to be *viable*, the opportunity cost cannot be too high: $\nu_n \le V_n$.

We now analyze the full model using backward induction. We make a choice here to impose $U_n(y) = y$ for all n, even though Proposition 1 holds for a general $U_n(y)$, for two reasons: It eases the presentation, and in this case the results are the same for all the bargaining solutions discussed above. Since the method should be clear after seeing the linear case, we leave the general case as an exercise.

To begin the argument, at the last potential link in the chain, where A_{N-1} with x meets A_N, obviously $W_N = \nu_N$. Hence, $\nu_N < \nu_{N-1}$ (no gains from trade) implies A_{N-1} consumes x. But $\nu_N > \nu_{N-1}$ (gains from trade) implies A_N consumes it after transferring y_{N-1} to A_{N-1}. Hence, from (3)

$$V_{N-1} = \frac{\alpha_{N-1}\left[\theta_{N-1}\nu_N + (1-\theta_{N-1})\nu_{N-1}\right] - c_{N-1}}{r + \alpha_{N-1}},$$

after inserting y_{N-1}. Therefore, $\nu_{N-1} \le V_{N-1}$, and search by A_{N-1} is viable if and only if

$$\nu_{N-1} \le \frac{\alpha_{N-1}\theta_{N-1}\nu_N - c_{N-1}}{r + \alpha_{N-1}\theta_{N-1}} \equiv \nu^*_{N-1}. \tag{4}$$

Notice $\nu^*_{N-1} < \nu_N$, so there is a *wedge* between ν_{N-1} and ν_N, for several reasons: First, unless we take $r \to 0$, impatience makes A_{N-1} prefer immediate gratification to the delayed payoff from search. Second, unless we take $c_{N-1} \to 0$, search is costly. Or, we can take $\alpha_{N-1} \to \infty$ and

[7] We are not sure of the original use of the extensive form shown in Figure 2, but one can find versions of it in the literature (e.g., Marchesiani and Nosal, 2012), and it is obviously related to much previous work (see any textbook on bargaining, e.g., Osborne and Rubinstein, 1990). We first saw the particular specification, with just two rounds of bargaining and a coin toss to determine who makes the final offer, in early versions of Cahuc et al. (2006), but they ultimately switched to a more standard game that gives the same results in their model, which is linear. Those authors and others we asked did not know more about the origins, although some said that it was basically a homework exercise once one knows standard bargaining theory. We like the fact that it is not special, but we think that those who say it is merely a homework might not appreciate its convenience in models with nonlinear utility, especially when one considers nonstationary equilibria, as discussed in Section 6.

eliminate both of these frictions. But given r and the search parameters (c_{N-1}, α_{N-1}), there is also a bargaining friction, if we may call it that, which can be seen by rearranging (4) as

$$\theta_{N-1} \geq \frac{r\nu_{N-1} + c_{N-1}}{\alpha_{N-1}(\nu_N - \nu_{N-1})} \equiv \theta^*_{N-1}. \tag{5}$$

To reiterate, if $\nu_N \leq \nu_{N-1}$ there are no gains from trade if A_{N-1} were to meet A_N, so A_{N-1} does not search, but even if $\nu_N > \nu_{N-1}$ the binding constraint for search by the penultimate agent in the network is (4) or, equivalently, (5). The next (backwardly inductive) step is to ask what would happen if the antepenultimate A_{N-2} had x. Inserting the relevant value of W_{N-1}, depending on whether A_{N-1} searches or consumes x, (2) generates y_{N-2}, and (3) generates V_{N-2}. It is then routine to check if search by A_{N-2} if viable. For instance, in the case where A_{N-1} searches, we have $W_{N-1} = V_{N-1}$ and

$$\begin{aligned} y_{N-2} &= \theta_{N-2}V_{N-1} + (1-\theta_{N-2})\,\nu_{N-2} \\ &= \theta_{N-2}\frac{\alpha_{N-1}\left[\theta_{N-1}\nu_N + (1-\theta_{N-1})\nu_{N-1}\right] - c_{N-1}}{r + \alpha_{N-1}} + (1-\theta_{N-2})\,\nu_{N-2}. \end{aligned}$$

In this case, $V_{N-2} \geq \nu_{N-2}$ if and only if

$$\begin{aligned} (r+\alpha_{N-2})\,\nu_{N-2} \leq{} & -c_{N-2} + \alpha_{N-2}(1-\theta_{N-2})\nu_{N-2} \\ & + \alpha_{N-2}\theta_{N-2}\frac{\alpha_{N-1}\left[(1-\theta_{N-1})\nu_{N-1} + \theta_{N-1}\nu_N\right] - c_{N-1}}{r+\alpha_{N-1}}. \end{aligned}$$

Continuing in this way, one can check the viability of search by each agent in the network. This generates an equilibrium trading pattern. As usual, it is generically unique: The only possible issue is that A_n may be indifferent between consuming x and searching, $\nu_n = V_n$, for some values of n. In this case, we can perturb parameters to make everyone who was indifferent strictly prefer one or the other.[8] We can now state the following result, the proof of which follows from the above discussion:

PROPOSITION 1. *If $N < \infty$, then there exists an SPE and it is generically unique.*

We repeat that Proposition 1 does not use $U_n(y) = y$, except to reduce notation. So we have existence and generic uniqueness for any preferences as long as $N < \infty$; what happens when $N = \infty$ will be analyzed later. The equilibrium trading pattern depends on parameters, of course, and in principle we can have anything from the market shutting down, with A_1 consuming x, to the opposite extreme where A_1 passes x to A_2 who passes it to A_3 ... until A_N ultimately consumes it. The remainder of the article involves studying these possibilities.

3. A SINGLE INTERMEDIARY

To facilitate comparison with the literature, consider the case $N = 3$. This means that, as in many other models, there are producers, potential middlemen, and potential end users. There are exactly three possible equilibrium trading patterns: A_1 consumes x; A_1 trades x to A_2 who consumes it; and A_1 trades x to A_2 who trades it to A_3 who consumes it. In the third case, A_2

[8] It is always possible to perturb parameters so as to break all possible cases of indifference. Although there are generally many ways to this, here is one: Let n be the highest n such that A_n is indifferent between searching and not searching: $\nu_n = V_n$. Then, change c_n by ε to make him strictly prefer one. This does not affect downstream negotiations (those with $n' > n$). If ε is small, it does not affect outcomes with $n'' < n$, either, unless $A_{n''}$ was also indifferent, in which case it might. But then perturb $c_{n''}$. Continuing in this way, we can make sure that no one is indifferent.

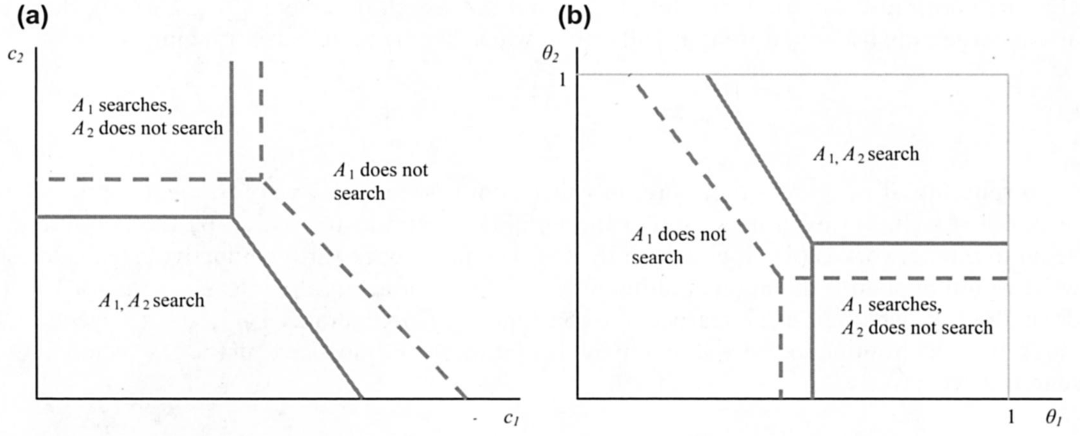

FIGURE 3

(A) EQUILIBRIUM TRADING PATTERN IN (c_1, c_2), $N = 3$; (B) EQUILIBRIUM TRADING PATTERN IN (θ_1, θ_2), $N = 3$

acts as a middleman. We want to determine when this happens as a function of parameters. We also want to analyze the efficiency of equilibrium and discuss some of its other qualitative properties. For this exercise, we use $U_n(y) = y$, but again this is not important for anything except reducing notation.

PROPOSITION 2. *If $N = 3$, A_2 acts as a middleman in the generically unique equilibrium if and only if*

$$c_2 \leq \alpha_2\theta_2\nu_3 - (r + \alpha_2\theta_2)\,\nu_2$$
$$(r + \alpha_2)\,c_1 + \alpha_1\theta_1 c_2 \leq \alpha_1\theta_1\alpha_2\theta_2\nu_3 + \alpha_1\theta_1\alpha_2(1 - \theta_2)\nu_2 - (r + \alpha_1\theta_1)\,(r + \alpha_2)\nu_1.$$

The proof is obvious as a special case of the analysis in Section 2. The first condition is simply the viability of search by A_2 for A_3, rewritten to isolate search costs on the left-hand side. Given it holds, the second condition is the viability of search by A_1 for A_2, rewritten the same way. To complete the characterization of equilibrium, we now ask what else might happen. If the first condition in Proposition 2 fails, then A_2 would rather consume x than search for A_3. Consider

$$c_1 \leq \alpha_1\theta_1\nu_2 - (r + \alpha_1\theta_1)\,\nu_1.$$

If this holds, then A_1 searches for A_2 and A_2 consumes x; if it fails, then A_1 consumes x.

Figure 3 shows where the different outcomes obtain in (c_{n-1}, c_n) space and (θ_{n-1}, θ_n) space. The regions enclosed by the solid and dashed lines correspond to higher and lower values of the discount rate r, respectively. Naturally, for middlemen to be active, it is necessary that c_{n-1} and c_n are low, and as r falls we can support this outcome with larger c_{n-1} and c_n. Similarly, it is necessary that θ_{n-1} and θ_n are big, and as r falls we can support this outcome with smaller θ_{n-1} and θ_n. One can draw a similar picture in (α_{n-1}, α_n) space, as emphasized in Rubinstein and Wolinsky (1987). Also, although here $U_n(y) = y$, it is not hard to work out the case with $U_n'' < 0$, and the figures look similar.

As discussed above in the context of the general model, having ν_3 exceed ν_1 and ν_2 is obviously not enough for the market to deliver x to A_3. To develop some intuition, let $\nu_2 = 0$ so that A_2 is a pure middleman with no desire to consume x himself. If A_2 has x, for him to search for A_3 we need the payoff to exceed the cost, $\alpha_2\theta_2\nu_3 \geq c_2$. If this condition does not hold, the market shuts down; if it holds, then A_2 would search for A_3 , and then A_1 would search for A_2 if and only if

$$(r + \alpha_2)\, c_1 + \alpha_1\theta_1 c_2 \leq \alpha_1\theta_1\alpha_2\theta_2\nu_3 - (r + \alpha_1\theta_1)\,(r + \alpha_2)\nu_1.$$

The right-hand side is A_1's expected share of A_2's expected share of the end user's payoff, net of opportunity cost; the left-hand side is A_1's search cost plus the amount he has to compensate A_2 for his search cost, all appropriately capitalized.

Suppose for the sake of illustration that, in addition to $\nu_2 = 0$, we let $r \to 0$ and $c_n \to 0$. Then the market can get x from A_1 to A_3 if and only if $\theta_2\nu_3 \geq \nu_1$. So, even when r, c_n, and ν_2 vanish, although there appear to be gains from trade whenever $\nu_3 > \nu_1$, we need $\theta_2\nu_3 \geq \nu_1$ for the market to function. This is due to a standard *holdup problem*. Heuristically, potential middleman A_2 knows that A_3 is willing to give anything up to ν_3 to get x, and A_1 would be willing to let it go for as little as ν_1. But when he eventually meets A_3, the intermediary only gets $y_2 = \theta_2\nu_3$, which may not cover his cost $y_1 = (1 - \theta_1)\,\nu_1 + \theta_1\theta_2\nu_3$. This is because that cost is sunk when he is negotiating with the end user. Hence, A_2 will not intermediate the transaction unless $y_2 \geq y_1$, which reduces to $\theta_2\nu_3 \geq \nu_1$. This is a market failure, and it is caused by a lack of commitment.

If A_3 and A_2 could sign a binding contract ex ante, before search begins, A_3 would agree to pay the middleman at least his cost, but that is proscribed in our environment by the assumption that you cannot contract with someone before you contact someone. Alternatively, Rubinstein and Wolinsky (1987) describe a consignment arrangement, where (in our notation) A_2 only pays A_1 after he trades x to A_3. This can ameliorate the above-mentioned problem, but it is also proscribed by the assumption that after A_1 passes x to A_2 they cannot meet again. Actually, it is enough to assume A_2 cannot *commit* to meet and settle the obligation, ruling out the consignment contract for opportunistic reasons. Note that we are not claiming that it is always impossible to contract away holdup problems in the real world; we are only claiming that one can imagine situations where it is impossible. More generally, Figure 3 shows the equilibria without assuming r, c_n, and ν_2 vanish, where the results are similar but richer—e.g., there is an additional aspect of holdup because the cost c_2 is also sunk when A_2 meets A_3. Notice the asymmetry: A_3 does not compensate A_2 for his search cost, but A_1 does compensate A_2 for his, for the obvious reasons.

In Wright and Wong (2011) and Nosal et al. (2013), we consider an alternative setup that is more like the original Rubinstein–Wolinsky model. There are many agents, each of which can be one of three types (again, originators, middlemen, and end users). Also, we assume $\nu_1 = \nu_2 = 0 < \nu_3$, as in Rubinstein and Wolinsky, and add production costs. Also, agents continue in rather than exit from the market after trading. The main difference, however, is that in the alternative setup anyone can meet anyone else. Now when A_1 meets A_2, he can either trade x to him or hold out for a direct trade with an end user. Again middlemen may or may not be active, depending on parameters. This generalized Rubinstein–Wolinsky model (generalized to allow heterogeneous bargaining power, search costs, and production costs, not just heterogeneous arrival rates) yields results that are similar to our baseline model, where A_1 and A_3 must trade through A_2. In particular, the equilibrium set in that model looks similar to Figure 3. Hence, for the rest of this article, we use this baseline model.

4. MULTIPLE MIDDLEMEN

In this section, we restrict attention to $U_n(y) = y$ (again one can consider general utility, and we return to this below, but for now it eases the presentation to begin by considering the linear case). Equilibrium can be represented as a simple dynamical system

$$y_{n-1} = (1 - \theta_{n-1})\,\nu_{n-1} + \theta_{n-1}W_n,$$

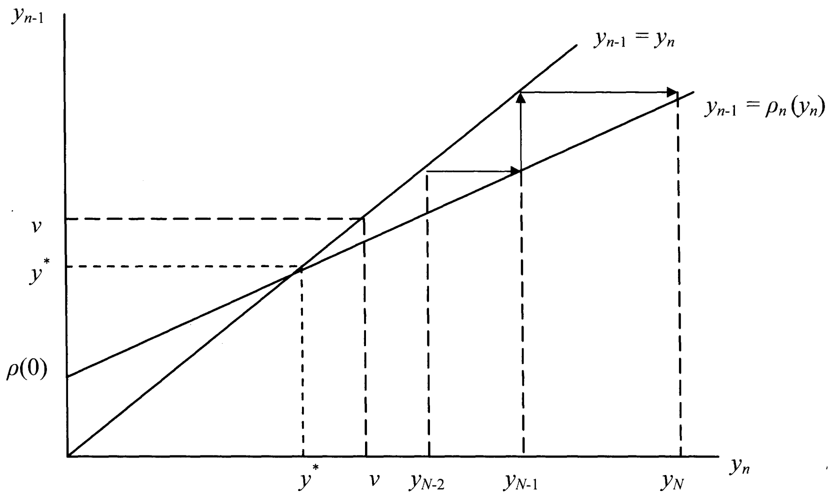

FIGURE 4

PATH OF y_{n-1}

where $W_n = \max\{v_n, V_n\}$ and V_n satisfies (3). Collapsing this into one equation,

$$y_{n-1} = (1-\theta_{n-1})\, v_{n-1} + \theta_{n-1} \max\left\{v_n, \frac{\alpha_n y_n - c_n}{r+\alpha_n}\right\} \equiv \rho_n(y_n). \tag{6}$$

We interpret $y_{n-1} = \rho_n(y_n)$ as a best response condition, giving the initial-offer strategy of A_{n-1} as a function of others' strategies, as summarized by y_n, since that is the only endogenous variable one needs to know to choose y_{n-1}. Proposition 1 tells us that for any finite N there is generically a unique solution to this system.

One application of the framework is to consider the special case where there is a natural end user A_N and ask if the equilibrium delivers x to him—i.e., we ask how long an intermediation chain can be. To this end, consider a quasi-stationary environment where α_n, c_n, and θ_n are the same for all n, while $v_n = v$ for $n < N$ and $v_N = \hat{v} > v$. Then, A_N is the natural end user. If A_N is to get x, then the terms of trade along the chain are given by $y_{n-1} = \rho(y_n)$ for $n \le N$, where now $\rho(y) = (1-\theta)v + \theta(\alpha y - c)/(r+\alpha)$ is stationary (does not depend on n) as well as linear. Figure 4 plots $y_{n-1} = \rho(y_n)$ and shows the unique fixed point $y^* = \rho(y^*)$, where

$$y^* = \frac{(1-\theta)v(r+\alpha) - \theta c}{r+\alpha(1-\theta)}. \tag{7}$$

Figure 4 is drawn assuming $y^* > 0$, but the following discussion does not depend on this. A more relevant observation (see below) is

$$y^* - v = \frac{-\theta(c+rv)}{r+\alpha(1-\theta)} < 0, \tag{8}$$

which implies $y^* < v$.

To determine the maximal intermediation chain, start at the end with the terminal condition $y_{N-1} = (1-\theta)\, v + \theta\hat{v}$. Then compute $y_{N-2} = \rho(y_{N-1})$, $y_{N-3} = \rho(y_{N-2})$, and so on. It is clear that $y_{N-n} \to y^*$ and $V_{N-n} \to V^* = (\alpha y^* - c)/(r+\alpha)$ as $n \to \infty$, and the convergence is monotone. A quick calculation implies $V^* < v$. Hence, as n gets big, eventually A_{N-n} strictly prefers to consume x instead of search. Summarizing this discussion:

PROPOSITION 3. *There exists a unique $\bar{n}$ such that $n = \bar{n}$ is the first n such that $V_{N-n} < \nu$. Then $\bar{n} - 1$ is the maximal length of a viable intermediation chain.*

If $N \geq \bar{n}$, then we cannot get x from the originator to A_N, and trade never gets off the ground: x is consumed by A_1. If $N < \bar{n}$, then we can get x to A_N. In the limiting case $\nu = c = 0$, which means there is neither a search cost nor an opportunity cost to trade, $y^* = 0$, and we can sustain arbitrarily long chains; for $\nu > 0$ or $c > 0$ there is a maximal chain. Also note that as we move forward in time, with n getting smaller as we approach the end user, y_{N-n} increases with every trade. Indeed, from Figure 4 it is clear that y_{N-n} increases at an increasing rate as we move forward in time. Indeed, since $\rho(y)$ is linear we can solve explicitly for the terms of trade along the equilibrium path,

$$y_{N-n} = \left[(1-\theta)\,\nu - \frac{\theta c}{r+\alpha}\right]\sum_{j=0}^{n-2}\left(\frac{\theta\alpha}{r+\alpha}\right)^j + \left(\frac{\theta\alpha}{r+\alpha}\right)^{n-1}\hat{y}.$$

In the nonlinear case, one cannot solve for y_n explicitly, of course, but the qualitative properties are similar.[9]

To describe some additional properties of equilibrium, let T_n be the random date when A_n trades x to A_{n+1}. Since the underlying arrival times are Poisson, as is standard, the interarrival times $T_n - T_{n-1}$ are distributed exponentially—see any textbook on stochastic processes, e.g., Çinlar (1975). This entails a high probability of a short interarrival time and a low probability of a long interarrival time. Hence, typical realizations of the process have trades *clustered*, with many exchanges occurring in relatively rapid succession, separated by relatively long periods of inactivity. Combined with the feature described in the previous paragraph—i.e., y_{N-n} increases at an increasing rate with every trade—one might conclude that there is a bubble in this market, because one sees trading lulls interspersed by frenzies with y_n not only increasing but accelerating. This conclusion would be a mistake. The fact that y_n is accelerating with every trade is simply due to x getting closer and closer to the end user, and since Poisson arrivals are memoryless there are no frenzies or lulls in any meaningful economic sense.[10]

Since there is a unique SPE, with the stochastic process for y_n pinned down by fundamentals (preferences, search costs, etc.), we assert that there is no bubble here, despite appearances. The moral of the story is that one has to be careful how one interprets "bubbly" observations—just like they are not so easy to construct in theory, bubbles are not so easy to identify in data. Having said this, in Section 7 we show there are bubbles in an extension of the model, after a digression on some other issues of interpretation.

5. BUYERS, SELLERS, MONEY, AND PRICES

Since the Introduction, we have tried to refrain from using the words *buyer*, *seller*, and *price*, because here we want to raise certain issues associated with such usage. First, in the analog to our model found in the literature, the authors say (in our notation) that x is a good, y is the price, the agent who trades x for y is the seller, and the one who trades y for x is the buyer. The price y is typically described as measured in money, even though these are nonmonetary models.[11]

[9] In the linear case, one can also solve for V_{N-n} and derive a formula for $\bar{n}$ in terms of fundamentals, but there is not much more information in that than there is in Proposition 3.

[10] As Çinlar (1975, pp. 79–80), says, "this [exponential density of interarrival times] is monotone decreasing. As a result, an interarrival time is more likely to have a length in $[0, s]$ than in a length in $[t, t+s]$ for any t. Thus, a Poisson process has more short intervals than long ones. Therefore, a plot of the time series of arrivals on a line looks, to the naive eye, as if the arrivals occur in clusters." Yet with Poisson processes, the memorylessness property means "knowing that an interarrival time has already lasted t units does not alter the probability of its lasting another s units."

[11] From middlemen papers, consider this: Rubinstein and Wolinsky (1987, p. 582) say payoffs are given "in monetary terms." Biglaiser (1993, p. 213) says "Each buyer is endowed with money." Yavas (1992) says "The sellers and the

There is no money in these models in any serious sense; more reasonably, one might say that they have *transferable utility*. Identifying money with transferable utility is all too common in economics. Consider Binmore (1992):

> Sometimes it is assumed that contracts can be written that specify that some utils are to be transferred from one player to another.... Alert readers will be suspicious about such transfers.... Utils are not real objects and so cannot really be transferred; only physical commodities can actually be exchanged. Transferable utility therefore only makes proper sense in special cases. The leading case is that in which both players are risk-neutral and their von Neumann and Morgenstern utility scales have been chosen so that their utility from a sum of money x is simply $U(x) = x$. Transferring one util from one player to another is then just the same as transferring one dollar.

We disagree. In monetary theory, transferring dollars is not the same as transferring utils, because people tend to run out of dollars: for almost all inflation rates (except the Friedman rule) they carry less cash than the amount required for unconstrained trade. Moreover, in most monetary models payoffs are not linear in dollars, with exceptions, like Lagos and Wright (2005), but even there payoffs are only locally linear, and agents are typically constrained by their money holdings, which are insufficient to get all that they want. Examples of search-based models, chosen because their environments are similar to the one in this article, but with money included explicitly, include Shi (1995), Trejos and Wright (1995), Kocherlakota (1998), and Wallace (2001), just to mention a few. All these models, like the one in Lagos and Wright, would look *very different* if they had transferable utility.

Interestingly, those models take a diametric position to the above-mentioned applications outside of monetary economics: They call y a consumption good and x money. Which makes more sense? Under the first interpretation, from nonmonetary search theory, the object called money (i.e., y) is divisible, whereas the consumption good (i.e., x) is indivisible. Under the second interpretation, in the monetary papers, money is indivisible and goods are divisible. Superficially this favors the first position, since divisibility is a property commonly associated with money. On reflection, however, we do not think this should be given much weight.[12] Better discriminating criteria stem from the functional definitions of money: It is a unit of account, a store of value, and a medium of exchange. The unit of account function—American prices tend to be quoted in dollars, European prices in euros—is relatively uninteresting, as for most issues measuring prices in dollars or euros matters no more than measuring distance in feet or meters. Therefore, we concentrate on the store of value and a medium of exchange functions.

As regards the store of value function, in the model, it is actually x and not y that plays this role: x is a durable object that when acquired enables A_n to enjoy payoff y_n at some future date. The natural interpretation of y is that it is a perishable good, or a service, that is not carried across time but produced for immediate consumption. It is certainly not an asset. Moreover, x satisfies the standard definition of a medium of exchange: an object that is accepted in trade not to be consumed, or used in production, but to be traded again later. Clearly, y is not a medium of exchange in these models. One can also say that x solves a standard double-coincidence problem of the sort that generally makes money useful: When A_n wants y from A_{n+1}, he has nothing to offer in trade except x. We also think x plays much the same role here that currency plays in monetary theory outside of the search literature, as discussed in Section 7. Therefore, we come down on the side of saying that x is money and y is a good. In the cases analyzed above, x happens to be a commodity money, as opposed to fiat money, since someone ultimately ends up consuming it, but it still acts like money (more on this later).

middlemen value the good (in monetary terms)." In search theory outside the middleman literature consider this: Butters (1977, p. 466) says "A single homogeneous good is being traded for money." Burdett and Judd (1983, p. 955) say consumers search "to lower the expected costs of acquiring, a desired commodity, balancing the monetary cost of search against its monetary benefit." And in several places Osborne and Rubinstein (1990) describe models where "a single indivisible good is traded for some quantity of a divisible good ('money')." We could go on, but we think the point has been made.

[12] One reason is that many contributions to the search-based monetary literature have both x and y indivisible, whereas others have both divisible. Usually indivisibility is an assumption of mathematical convenience, not substance.

One might ask if this issue—whether we call x or y money—matters for anything. We think it does. First, it determines who we call the buyer or seller and what we mean by the price. To make the point, we begin by suggesting that in nonmonetary exchange—say, when A gives B apples for bananas—it is not meaningful to call either a buyer or seller. Of course, one can call them what one likes, but then the labels "buyer" and "seller" mean nothing more than calling them A and B. However, when A gives B apples in exchange for money, say for dollars or euros, everyone should agree that A is the seller and B the buyer. As any good dictionary says, it is standard usage to identify those who pay money as buyers and those who receive it as sellers. Again, one can use labels as one likes, but would anyone want to reverse the labels in, say, the Mortensen–Pissarides (1994) labor-market model, taking the agents we normally think of as workers and calling them firms and vice versa? One could prove the same theorems, but it makes a difference for substantive questions, e.g., should we tax/subsidize search by workers or firms? If it makes a difference who we call workers and firms in labor markets, it similarly makes a difference who we call buyers and sellers in goods markets, e.g., for questions like should we tax/subsidize shoppers or retailers?

Moreover, the two interpretations give the opposite predictions for price behavior. If we normalize the size of the indivisible object x to 1, then under the interpretation that y is money and x is a good the price is y. But under the interpretation that y is the good and x is money the price is $1/y$, since a normalized unit of cash buys y units of the good. To see how this matters, recall the result in the previous section that as we move forward in time y accelerates as x gets closer to the end user. Using the first interpretation, one would say the price level is increasing—it looks like inflation—because more and more money is required to buy the same amount of x. Using the second interpretation, the inverse of the price level is increasing—it looks like deflation—because more and more consumption can be had for the same amount of money x. Therefore, if one wants to compare a model with the facts, one has to take a stand on whether x or y is money.

We also find it interesting that from a legal standpoint it often makes a difference who is the buyer and seller. It is not uncommon to have laws or conventions that allow buyers to return goods and get a refund, or at least store credit, within a period of time with no questions asked—the principle of *caveat emptor* notwithstanding. Similar laws for sellers apply only in exceptional cases, like buyers passing bad checks, and usually monetary payment entails finality—suggesting a more rigorous principle of *caveat venditor* in markets. Also, modern private trading platforms like eBay have rules and regulations that treat buyers and sellers differently (Beal, 2009). Relatedly, there is a "bias" in law in the following sense: "Buyers are not obligated to disclose what they know about the value of a seller's property, but sellers are under a qualified obligation to disclose material facts about their own property" (Ramsay, 2006). Suppose A transfers his house to B in exchange for cash: If the land if full of radon and A knows this, he is obliged under the law to reveal this; if the land is full of valuable mineral deposits and B knows this he is not so obliged.

There is also asymmetric treatment of buyers and sellers in most illegal markets, including markets for drugs, prostitution, illegal guns, gambling or liquor, and so on, where the ones who get the cash are usually treated much more harshly than the ones who give it. There are exceptions, as in the case of child prostitution, but that can be explained by saying that the distinction between adults and minors takes precedence over the distinction between buyers and sellers, without denying the existence of the latter distinction. On the whole, sellers are clearly the ones subject to more or more severe punishment.[13] It also seems that sellers of stolen merchandise are looked down upon more than buyers, including the "fence" who did not

[13] One rationalization we came across is that there are greater barriers to entry on the sellers' side, so eliminating a seller has a bigger impact on the market. This may be true at the high end, e.g., for drug lords, but the reverse argument can be made at the low end, where a dealer removed from a street corner is easily replaced. A related idea is that sellers have more transactions than buyers, so it is cost effective to go after the former. There is also the view that those who profit more from illegal activity should face higher punishment, as should those that provide bad role models with their financial success. Yet another idea is the principle that we ethically judge actions that put others at risk more harshly

actually steal the merchandise but paid someone for it, and in this sense is just like any other middlemen in our model. Intermediaries in general are often considered less than honorable, of course, since they allegedly "do not themselves produce anything," and simply profit from the labor of others, as evidenced by our epigraph on art dealers. This view of course arises mainly out of ignorance of the idea that getting goods from A to B is a productive activity.

In any case, in terms of interpreting formal models, a typical middlemen paper would call x a consumption good that passes from originator to end user, possibly via intermediaries, trading at each link in the chain for y_n dollars. We prefer to say it trades for y_n units of a different good, produced at disutility y_n by A_{n+1} and consumed for utility $U_n(y_n)$ by A_n, including as a special case $U_n(y_n) = y_n$. To be clear, we have no quibble with linear utility, only with the assertion that agents directly derive linear utility from dollars or the assertion that indirect utility is linear in dollars. Interpreted in this way our framework provides a coherent theory of intermediation chains. At the same time, it is a coherent theory of a medium of exchange x trading for goods produced by A_{n+1} and consumed by A_n. We think it is useful to make a connection between these two applications of search theory—i.e., between search models of middlemen and search models of monetary exchange. One reason is that the connection is natural because intermediaries and media of exchange are alternative institutions that facilitate trade in the presence of frictions. Another reason is that it leads to the results in the next extension.

6. INTERMEDIATION BUBBLES

So far, we can get chains of trade, but equilibrium is always tied down by a determinate end user who ultimately consumes x. Here, we set $N = \infty$ and consider the possibility that *no one ever consumes it*. For instance, imagine the vintage wine market, with individuals continually retrading bottles that no one will ever drink. Another example concerns cigarettes in prisoner of war camps or other prisons that potentially retrade over and over, perhaps until they fully depreciate, without being smoked After seeing the connection to monetary economics in the previous section, this may sound plausible, but to our knowledge the idea has not been developed in the middlemen literature.

For this exercise, assume stationarity: $\alpha_n = \alpha$, $c_n = c$, $\theta_n = \theta$, and $\nu_n = \nu$ for all n. Then, if x were to circulate forever, with no one ever consuming it, the path for y_n would have to satisfy

$$y_{n-1} = (1-\theta)\nu + \theta \frac{\alpha y_n - c}{r+\alpha} = \rho(y_n), \tag{9}$$

if we continue to assume $U_n(y) = y$. The function $y_{n-1} = \rho(y_n)$ was shown above in Figure 4, although here we use it for a different purpose. Instead of constructing equilibrium by working backward from a determinate end user, we are now interested in any solution to (9), starting at any initial condition and moving forward in time as n increase, as long as it does not violate feasibility—i.e., it must respect nonnegativity and the upper bound on production, $y_n \in [0, \bar{y}]$, for all n. Given a path for y_n, we then check the viability condition for A_n to search, $\nu \leq (\alpha y_{n+1} - c) / (r + \alpha)$, for all n.

In the liner case, it is obvious from Figure 4 that there is only one solution to (9) that remains nonnegative and bounded as n increases, the constant path $y_n = y^*$ for all n, where y^* was defined in (7). Any path that begins at $y > y^*$, e.g., starts out looking like a bubble, where A_n is willing to search for A_{n+1} because he expects a high y_n, A_{n+1} is willing to give him a high y_n because he expects an even higher y_{n+1}, and so on. But this is not consistent with equilibrium, since one cannot rationally believe in such explosive paths. The only equilibrium is therefore $y_n = y^*$ for all n. Then, since $y^* < \nu$, we conclude that no one searches. Naturally if $\nu_n = \nu$ for

than those that put oneself at risk, although while this might ring true for drugs, it is less easy to make the case for prostitution. And there is the philosophical position that if B is weak and A exploits that weakness, then A should be judged more harshly. This all seems interesting and worthy of addition research, but we cannot go further into it here.

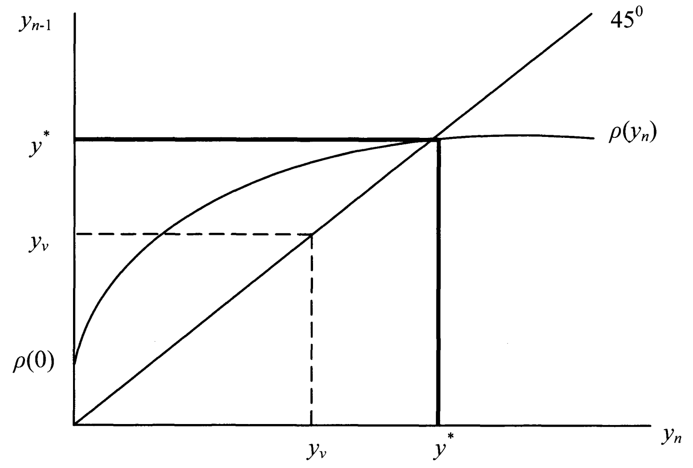

FIGURE 5

BEST-RESPONSE CONDITION (WITH NONLINEAR UTILITY AND $\rho(0) > 0$)

all n there are no gains from trade. If A_n (off the equilibrium path) were to meet and trade with A_{n+1}, he must receive exactly $y_n = \nu$ for x, because he could not get more and would not take less. We call $y_n = \nu$ the *fundamental value* of x and say that a bubble exists when $y_n > \nu$. The above argument verifies that this cannot happen in the model as specified.

PROPOSITION 4. *Let $N = \infty$ and assume fundamentals are stationary. With $U(y) = y$ there is a unique equilibrium and it implies no trade.*

Now suppose the utility of consuming y is $U(y)$, with $U(0) = 0$, $U' > 0$, and $U'' < 0$. Then, $y_\nu = U^{-1}(\nu)$ becomes the cost to A_{n+1} of covering A_n's outside option ν, and if x is going to circulate forever, (9) changes to

$$y_{n-1} = (1-\theta)y_\nu + \theta\frac{\alpha U(y_n) - c}{r+\alpha} = \rho(y_n). \tag{10}$$

Figure 5 shows this for a case where $\rho(0) > 0$. Also, it shows $y^* > y_\nu$, which is equivalent to $U(y^*) > \nu$, which we could not get when $U(y) = y$—recall condition (8). Since $U(y^*) > \nu$ is necessary to satisfy the search viability condition, we at least have a chance for trade with the nonlinear model, something we did not have with $U(y) = y$. Before pursuing this, we verify existence of an equilibrium directly.

PROPOSITION 5. *Let $N = \infty$ and assume fundamentals are stationary. With $U''(y) < 0$ a stationary equilibrium exists.*

PROOF. Under the usual Inada conditions, it is clear from Figure 5 that there is a solution to $y^* = \rho(y^*) > 0$ if and only if $\rho(0) = (1-\theta)y_\nu - \theta c/(r+\alpha)$ is not too negative. If there is no such solution, then a stationary equilibrium exists with no trade, where A_1 consumes x. If there is a solution $y^* = \rho(y^*) > 0$, then we need to check search viability: If $\nu \leq [\alpha U(y^*) - c]/(r+\alpha)$ then there is a stationary equilibrium where x circulates forever and commands $y = t^*$ in every trade; otherwise there is a stationary equilibrium with no trade. ■

Our interest is obviously on equilibria with trade, the existence of which depends on parameters. It is not hard to describe conditions guaranteeing there is a fixed point $y^* = \rho(y^*)$ satisfying search viability $\nu \le [\alpha U(y^*) - c]/(r+\alpha)$ (e.g., make ν and c small, naturally). Consider an example with $U(y) = \sqrt{y}$ and $c = 0$. Then,

$$\rho(y) = (1-\theta)\,\nu^2 + \frac{\theta\alpha}{r+\alpha}\sqrt{y}.$$

To find steady state, rewrite this in terms of $U = \sqrt{y}$ and solve for the positive root:

$$U = \frac{1}{2}\left\{\frac{\theta\alpha}{r+\alpha} + \sqrt{\left(\frac{\theta\alpha}{r+\alpha}\right)^2 + 4(1-\theta)\,\nu^2}\right\}.$$

The search viability condition is then

$$\nu \le \frac{\theta\alpha^2}{r^2 + 2r\alpha + \theta\alpha^2} = \nu^*. \tag{11}$$

Since $\nu^* > 0$, for some $\nu > 0$, search is viable.

Hence, at least with $c = 0$ there is an equilibrium where everyone searches and trades for $y_n = y^*$ in each meeting. By continuity this is also true for some $c > 0$.[14] Since $y^* > y_\nu$, the amount of y required to acquire x is above the fundamental value, satisfying our definition of a bubble.

Also by continuity we can generate equilibria where x circulates forever when $\nu = 0$, meaning x is quite like fiat money, in the sense that it has no intrinsic value. We can also generate such equilibria when $\nu < 0$, meaning x is an intrinsically bad asset, like vintage wine that has long since lost its "drinkability." We can also perturb ν_n away from stationarity and generate equilibria where A_n acquires x from A_{n-1} even though $\nu_n < \nu_{n-1}$. Any of these outcomes might constitute a "puzzle" for standard asset-pricing theory. However, in monetary economics, it is standard fare for agents to pay $y > 0$ to acquire an asset with questionable fundamental value, because they rationally expect that others will do the same. And note that such outcomes are efficient.

The equilibrium where x circulates forever and $y_n = y^*$ in every trade is a stationary bubble. Can there be nonstationary bubbles? When $\rho(0) > 0$, the answer is *no*, because all paths satisfying (10), other than $y_n = y^*$ for all n, lead to either $y_n < 0$ or $y_n > \bar{y}$. But suppose $\rho(0) < 0$, as in Figure 6, which occurs whenever $c > y_\nu(r+\alpha)(1-\theta)/\theta$. As long as c is not too big, there are multiple steady states, shown as y_1^* and y_2^*. Suppose $\alpha\left[U(y_1^*) - c\right]/(r+\alpha) > \nu$, so that search is viable when y_n is near y_1^*. Then, as shown, there is a continuum of nonconstant paths for y_n satisfying all the relevant conditions. Starting to the left (right) of y^*, y_n rises (falls) over time, heading toward y_1^*. Although more could be done, this should suffice to make a point: Economies very similar to the standard Rubinstein–Wolinsky model, once extended to allow nonlinear utility, display nontrivial dynamics.[15]

[14] Without the assumption $c = 0$, search is viable iff $Q(y) \ge 0$, where $Q(\cdot)$ is the quadratic

$$Q(y) = -y^2[r^2 + 2r\alpha + \alpha^2\theta] + y[\alpha^2\theta - 2(r+\alpha)c] - c^2.$$

Hence, $\exists\bar{c} > 0$ such that $c < \bar{c}$ implies search is viable for $y \in [y_1, y_2]$, with $0 < y_1 < y_2$, and for $c > \bar{c}$ search is not viable for any $y \ge 0$. As $c \to 0$, $[y_1, y_2] \to [0, \bar{y}]$ consistent with (11). See the Appendix for more details.

[15] More exotic outcomes may be possible, such as cyclic or sunspot equilibria. We leave that to future work, but one conjecture is that there are sunspot equilibria in at least in some special cases. If $\nu = 0$, e.g., there may be equilibria where x circulates in exchange for $y > 0$ for a while, before crashing at some random date to $y = 0$, at which point x gets consumed. This and other more or less complicated equilibria are worth further attention. Note also that there is precedent for generating interesting dynamics once linearity is relaxed, in standard models that have a unique equilibrium with linear utility: Trejos and Wright (2012), e.g., do this in the model of over-the-counter markets for financial assets by Duffie et al. (2005).

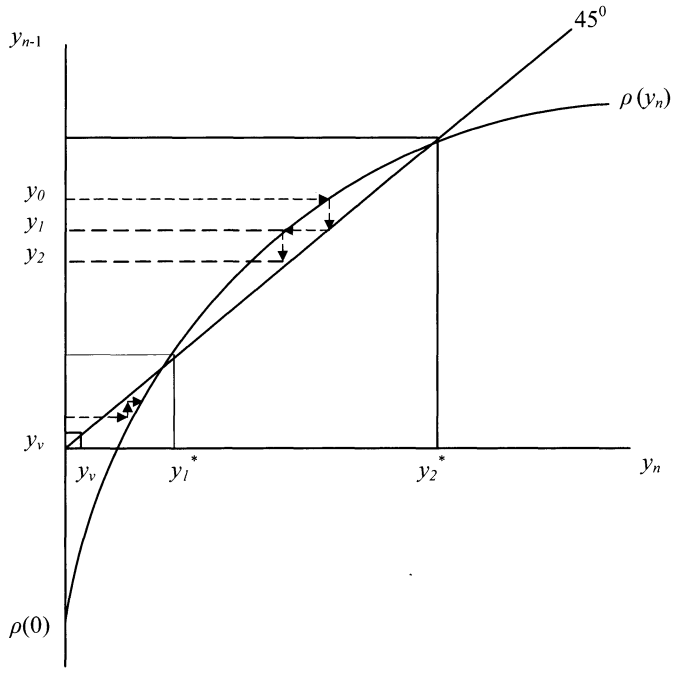

FIGURE 6

BEST-RESPONSE CONDITION (WITH NONLINEAR UTILITY AND $\rho(0) < 0$)

At this stage, once we are using nonlinear utility, it is time to revisit bargaining. Consider the generalized Nash solution:

$$y_{n-1} = \arg\max_y [U(y) - \nu]^\theta (V_n - y)^{1-\theta}.$$

The FOC gives y_{n-1} as a function of V_n. After inserting $V_n = \alpha [U(y_n) - c] / (r + \alpha)$, we get a difference equation $y_{n-1} = \hat{\rho}(y_n)$, analogous to the $y_{n-1} = \rho(y_n)$ we got from our game. Similarly, for Kalai's solution $\theta(V_n - y_{n-1}) = (1 - \theta)[U(y_{n-1}) - \nu]$, after inserting V_n we get $y_{n-1} = \tilde{\rho}(y_n)$. The different bargaining solutions thus lead to the different dynamical systems ρ, $\hat{\rho}$, and $\tilde{\rho}$, but all three can generate multiple steady states and nonstationary equilibria, as demonstrated above for our solution.

We like our bargaining solution because it has a simple strategic foundation. This is also thought to be a nice property of Nash bargaining: One can write down the standard strategic model with repeated counteroffers, let the time between counteroffers go to zero, and in the limit one gets Nash (see, e.g., Osborne and Rubinstein, 1990). But as demonstrated in Coles and Wright (1998) and Coles and Muthoo (2003), this is only strictly correct in stationary environments unless one makes additional assumptions. Thus, if one writes down the same strategic game in a nonstationary setting and takes the same limit, one gets a differential equation that coincides with a Nash solution in but not out of steady state, except in special cases like $U(y) = y$ or $\theta = 1$. Those papers also show how the set of dynamic equilibria can be qualitatively different if one uses the limit of the game instead of the Nash solution. Furthermore, they show that using Nash out of steady state is equivalent to using the original game but imposing myopic (incorrect) expectations. To the extent that one wants strategic foundations and one wants to analyze models out of steady state and one wants to use nonlinear utility and $\theta < 1$, this is an issue.

Even in steady state, with nonlinear utility in our game, Nash and Kalai bargaining generally generates different outcomes. The Appendix solves the model with $U(y) = \sqrt{y}$ for each bargaining solution. There is an upper bound for ν that makes search viable, and when $c = 0$ these bounds are all the same. As $r \to 0$, the equilibrium payoffs go to

$$
\begin{aligned}
U_s^* &= \frac{1}{2}[\theta + \sqrt{\theta^2 + 4(1-\theta)\nu^2}], \\
U_n^* &= \frac{1}{2-\theta}[2\nu(1-\theta) + \theta], \\
U_k^* &= \frac{1}{2\theta}[2\theta - 1 + \sqrt{1 - 4\theta(1-\nu)(1-\theta)}],
\end{aligned} \tag{12}
$$

where the subscript indicates strategic, Nash, or Kalai bargaining. Again, we like our solution because it has strategic foundations, in and out of steady state, with linear or nonlinear utility. We want to look beyond steady state because we are interested in nonstationary bubbles, and we need to use nonlinear utility because, as we showed, that is the only way to get them. But if one just wants bubbles and does not care about strategic foundations for bargaining, one can use the Nash or Kalai solutions and get similarly "bubbly" results.

To summarize the main results of this section, we can conclude that bubbles exist in two situations. First, there can exist a stationary equilibrium where $y = y^* > \nu$, in which case x circulates forever. This exists under standard Inada conditions as long as $\rho(0)$ is not too big of a negative number, which we need for existence of the fixed point $y^* = \rho(y^*) > 0$, and search is viable, which means c and ν are not too big. Second, there can be nonstationary bubble equilibria whenever there are multiple stationary equilibria, which requires $\rho(0) < 0$ but not too big in absolute value. Future work may explore whether there are other equilibria, such as cyclic or sunspot equilibria, as we expect there may be.

7. DISCUSSION

We now provide some comments on comparing our framework with pure monetary theory. Discrete-time monetary models generally have equilibria that can be characterized by a difference equation, as does our model. Given a difference equation, the mathematics are similar, regardless of the economic assumptions underlying the model. Of course, our model is actually specified in continuous time, but with Poisson arrivals this is effectively the same as discrete time, except that the interarrival times are stochastic instead of deterministic.

Here we consider the discrete-time OLG model of money described in Wallace (1980), among other places, but one can do something similar assuming a cash-in-advance constraint or putting money in the utility function, if one is so inclined. A simple specification of an OLG model has two-period-lived agents, with utility $U_{n+1}(y_{n+1}) - y_n$ for an agent born at date $n = 1, 2, \ldots$, where y_n is production while young and y_{n+1} is consumption while old. Assume $U_n(0) = 0$, $U_n'(0) > 0$, and $U_n''(0) < 0$. At the first date $n = 1$, there are some initial old agents endowed with x units of money that they supply inelastically, which means that for this presentation one can take the money to be indivisible if one likes.

All other agents solve

$$\max U_{n+1}(y_{n+1}) - y_n \text{ st } p_{n+1}y_{n+1} = p_n y_n,$$

assuming Walrasian (perfectly competitive) price taking. The first order condition is $U_{n+1}'(y_{n+1}) = p_{n+1}/p_n$. By market clearing, which requires $p_n y_n = x$, each period, this reduces to

$$y_n = U_{n+1}'(y_{n+1})\, y_{n+1} = \omega(y_{n+1}). \tag{13}$$

The difference equation in (13) is not so different from (10). Thus, suppose $U_n(y) = U(y)$ is stationary. Then, (13) admits a monetary steady state with $y_n = y^*$ for all n , where $U'(y) = 1$, plus a nonmonetary steady state with $y_n = 0$ for all n. It also admits equilibria where $y_n \to 0$ as $n \to \infty$.

By comparison, in our model, making x a fiat object by setting $\nu = 0$ and eliminating search costs by setting $c = 0$, we have

$$y_n = \frac{\alpha\theta U_{n+1}(y_{n+1})}{r+\alpha} = \rho(y_{n+1}). \tag{14}$$

The systems $\rho(y)$ and $\omega(y)$ in (13) and (14) are different in terms of economics, because our agents trade according to a Poisson random-matching process instead of once per period and because they use bargaining instead of price taking, but in terms of mathematics they are quite similar. Both have monetary and nonmonetary steady states plus dynamic equilibria where $y_n \to 0$. In fact, there is no reason in principle why one cannot use bargaining in the OLG model or Walrasian pricing in our model. Then, the only difference is that our trades occur according to a stochastic process in continuous time instead of once per period in discrete time. The idea of supporting equilibria where x commands more than its fundamental value is the same.

We can also make a tighter connection to search-based monetary theory. One way to proceed is to use modulo N arithmetic in the description of the population graph, which ties the ends of the chain together to form a circle. Then, A_1 consumes the output y_1 of A_2 who consumes ... of A_N who consumes the output y_N of A_1. This looks very much like the N-good generalization of Kiyotaki and Wright (1989) by Aiyagari and Wallace (1991).[16] Given that the search and OLG models both look similar to our setup, can one say that the search and OLG models look similar to each other? More work would be welcome on this, but we note that long ago Cass and Yaari (1966) noted the resemblance between the OLG structure and a circle like the one that emerges when we tie together the ends of our chain. Another detail is that search models of money typically assume that anyone can meet anyone, at random, as in many middleman papers, whereas our agents meet in a precise pattern given by the network. One can impose a similar structure on those monetary models, and in a sense this has been done in Corbae et al. (2003). Although in that model agents *choose* who meets whom (i.e., search is directed) the endogenous trading pattern looks very much like our network.

Having some goods circulate as a means of payment, and valued above what fundamentals suggest, does not contradict experience. A classic case concerns cigarettes in Radford's (1945) description of a POW camp: "Most trading was for food against cigarettes or other food stuffs, but cigarettes rose from the status of a normal commodity to that of currency.... With this development everyone, including nonsmokers, was willing to sell for cigarettes, using them to buy at another time and place" (see Burdett et al., 2001, for a formal model based on these observations). Similarly, Friedman (1992) reports that "After World War II [in Germany] the Allied occupational authorities exercised sufficiently rigid control over monetary matters, in the course of trying to enforce price and wage controls, that it was difficult to use foreign currency. Nonetheless, the pressure for a substitute currency was so great that cigarettes and cognac emerged as substitute currencies and attained an economic value far in excess of their value purely as goods to be consumed.... Foreigners often expressed surprise that Germans were so addicted to American cigarettes that they would pay a fantastic price for them. The usual reply was 'Those aren't for smoking; they're for trading'."

Finally, we emphasize how nonlinear utility is crucial for understanding certain aspects of the results. For instance, the agents in our model can sometimes be said to be engaging in the dubious practice of buying high and selling low. Consider starting to the right of y_1^* in an

[16] A difference is that those early search models assumed x and y were indivisible, but as mentioned in Section 5 there are many papers relaxing one or both of those restrictions.

equilibrium where $y_n \to y_1^*$ from above. In this situation, A_n gives up y_{n-1} to get x, and later exchanges x for $y_n < y_{n-1}$. This is ostensibly a strange strategy for a middleman, even without accounting for his time and search costs, but it is actually a good deal. The key point is that $U(y_n)$ can exceed y_{n-1} by enough of a margin to make this a viable arrangement, even if $y_n < y_{n+1}$. This obviously requires nonlinear utility.

8. CONCLUSION

This article has developed a model of intermediated trade in markets with frictions. For a finite number of agents N, we proved existence and generic uniqueness. In an application, with $N = 3$, we characterized the trading pattern as a function of parameters. This allows one to determine when the potential intermediary fulfills his role, depending on fundamentals as well as strategic considerations (holdup problems). In another application, with a general $N < \infty$, we showed how to compute the maximum length of a middlemen chain. Market outcomes with these chains look something like bubbles—trade comes in clusters with accelerating paths for y_n—but we argued this is not really a bubble. With $N = \infty$ we showed how to construct equilibria with genuine bubbles, where y_n differs from its fundamental value, and can increase or decrease over time, in perfect-foresight equilibria. We also discussed the general concepts of buyers, sellers, money, and prices and made some connections between different models. More can be done with this model, perhaps in applications, say, to finance or real estate. This is left for future work.

APPENDIX

Here, we give some more details concerning the example with $U(y) = \sqrt{y}$. First, we can extend the case of our bargaining solution by relaxing the assumption $c = 0$ made in the text. One can show search is viable if and only if $Q(\nu) \geq 0$, where $Q(\cdot)$ is the quadratic

$$Q(\nu) = -\nu^2 \left(r^2 + 2r\alpha + \alpha^2\theta\right) + \nu[\alpha^2\theta - 2\,(r+\alpha)\,c] - c^2.$$

Hence, $\exists \bar{c} > 0$ such that $c < \bar{c}$ implies search is viable for $\nu \in [\nu_1, \nu_2]$, with $0 < \nu_1 < \nu_2$, and for $c > \bar{c}$ search is not viable for any $\nu \geq 0$. As $c \to 0$, $[\nu_1, \nu_2] \to [0, \nu]$ consistent with (11).

Now consider the first-order condition from generalized Nash bargaining, $\theta y = \theta V - (1 - \theta)\left(\sqrt{y} - \nu\right) 2\sqrt{y}$. Substituting V and rearranging terms, the steady state y solves

$$(2-\theta)\,(r+\alpha)\,y - [2\nu(1-\theta)\,(r+\alpha) + \alpha\theta]\sqrt{y} + c\theta = 0.$$

The solution satisfies

$$\sqrt{y} = \frac{[2\nu(1-\theta)\,(r+\alpha) + \alpha\theta] + \sqrt{[2\nu(1-\theta)\,(r+\alpha) + \alpha\theta]^2 - 4(2-\theta)\,(r+\alpha)\,c\theta}}{2(2-\theta)\,(r+\alpha)}.$$

Inserting $U = \sqrt{y}$ into the viability condition $\nu \leq (\alpha U - c)\,/\,(r+\alpha)$ and simplifying, we have

$$\nu^2[r^2(2-\theta) + 2r\alpha + \alpha^2\theta] + \nu\{[r(2-\theta) + \alpha]2c - \alpha^2\theta\} + (2-\theta)c^2 \leq 0.$$

Again, there exists $\bar{c} > 0$ such that $c < \bar{c}$ implies search is viable for $\nu \in [\nu_1, \nu_2]$, with $0 < \nu_1 < \nu_2$, and for $c > \bar{c}$ search is not viable for $\nu \geq 0$.

One can do the same for proportional bargaining. At steady state, we have

$$\theta y + \frac{[(1-\theta)r + \alpha]}{r+\alpha}\sqrt{y} + \frac{c\theta}{r+\alpha} - \nu(1-\theta) = 0.$$

The solution satisfies

$$\sqrt{y} = \frac{1}{2\theta}\left\{-\frac{(1-\theta)r+\alpha}{r+\alpha} + \sqrt{\frac{[(1-\theta)r+\alpha]^2}{(r+\alpha)^2} - 4\theta[c\theta/(r+\alpha) - \nu(1-\theta)]}\right\},$$

and the viability condition is

$$\nu^2(r+\alpha)^2\theta + \nu\{2\theta c(r+\alpha) + \alpha[r(1-\theta) - \theta\alpha]\} + c[\theta c + \alpha(1-\theta)] \leq 0.$$

Again, search is viable if and only if c is small. When $c = 0$, the upper bounds for ν that allow search are

$$\overline{\nu}_s = \frac{\theta\alpha^2}{r^2 + 2r\alpha + \theta\alpha^2}, \overline{\nu}_n = \frac{\theta\alpha^2}{r^2(2-\theta) + 2r\alpha + \theta\alpha^2}, \text{ and } \overline{\nu}_k = \frac{\alpha[\theta\alpha - r(1-\theta)]}{\theta(r+\alpha)^2}.$$

When $c = 0$ and $r \to 0$, all $\overline{\nu}_n = \overline{\nu}_s = \overline{\nu}_k = 1$ independent of θ, and in equilibrium U is given by (12).

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