Chapter 1
Measurement

1. SI Units

Length, mass, and time play a fundamental role in describing nature. These are the three quantities on which we base our measurements. Length, mass, and time are called dimensions. We use the International System of Units (also called the metric system or the SI system, for the French term Système International). In mechanics, we need to define only three units,

unit of length	meter (m)
unit of mass	kilogram (kg)
unit of time	second (s)

((Note))

MKS units:	meter, kilogram, second
CGS units:	centimeter, gram, second

(a) Meter

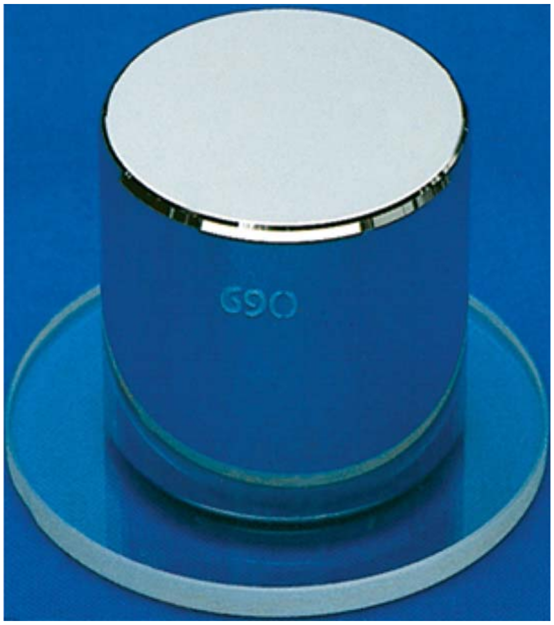
The meter is the length equal to the distance traveled by light in vacuum, in a time of

$$\frac{1}{299{,}792{,}458}.$$

Note that the velocity of light is $c = 2.99792458 \times 10^8$ m/s.

(b) Kilogram (kg)

The SI standard of mass is a platinum-iridium cylinder shown in the figure. The cylinder is kept at the International Bureau of Weights and Measures near Paris and assigned a mass of 1 kilogram. Accurate copies have been sent to other countries.



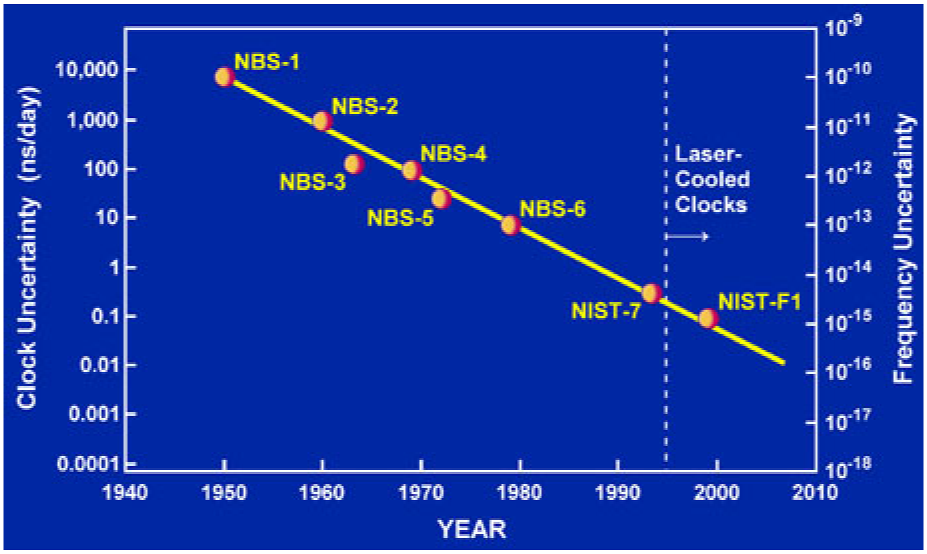
(c) Second (s)

A cesium (-beam) atomic clock (or cesium-beam frequency standard) is a device that uses as a reference the exact frequency of the microwave spectral line emitted by atoms of the metallic element cesium, in particular its isotope of atomic weight 133 ("Cs-133"). The frequency, f_0= 9,192,631,770 hertz (Hz = cycles/second), provides the fundamental unit of time, which may thus be measured by cesium clocks.

Period $T_0 = 1/f_0 = 1/(9{,}192{,}631{,}770)$ s = 1.0878×10^{-11} s = 0.11 ns.

One second is the duration of 9,192,631,770 periods

The time measurement accuracy is 2 nanoseconds per day or one second in 1,400,000 years. *It is the most accurate realization of a unit that mankind has yet achieved.*



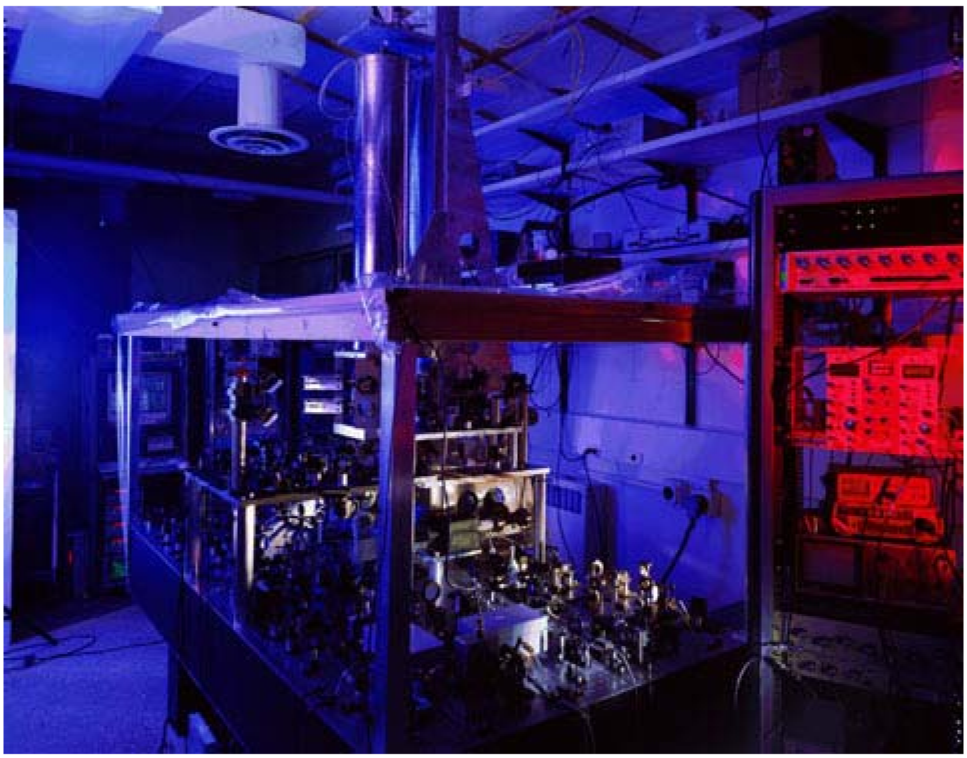
0.1 ns/day = 3.65×10^{-8} sec/1 year = 1 sec/27.4 million years

Link:

NIST-F1 Cesium Fountain Atomic Clock. The Primary Time and Frequency Standard for the United States

http://tf.nist.gov/cesium/fountain.htm

((Atomic clock))



NIST-F1, the nation's primary time and frequency standard, is a cesium fountain atomic clock developed at the NIST laboratories in Boulder, Colorado. NIST-F1 contributes to the international group of atomic clocks that define Coordinated Universal Time (UTC), the official world time. Because NIST-F1 is among the most accurate clocks in the world, it makes UTC more accurate than ever before.

The uncertainty of NIST-F1 is continually improving. In 2000 the uncertainty was about 1×10^{-15}, but as of the summer of 2005, the uncertainty has been reduced to about 5×10^{-16}, which means it would neither gain nor lose a second in more than 60 million years! The graph below shows how NIST-F1 compares to previous atomic clocks built by NIST. It is now approximately ten times more accurate than NIST-7, a cesium beam atomic clock that served as the United State's primary time and frequency standard from 1993-1999.

Current time: if you want to know the current time of New York, go to the Web site

http://www.timeanddate.com/worldclock/city.html?n=179

2. SI base units and SI derived units

There are two kinds of SI units; (a) the SI based units, and (b) the SI derived units

(a) **The SI base units**

Meter (m), kilogram (kg), second (s), ampere (A), kelvin (K), mole (mol), and candela (cd).

(b) **The SI derived units**

Units for other quantities, such as velocity, acceleration, force, energy, and power, are derived from these basic units.

Area	m^2	
volume	m^3	
Velocity:	m/s	
Angular velocity	rad/s	
Acceleration:	m/s^2	
Force	kg m/s^2	(=N, Newton)
Energy	kg m^2/s^2	(= J, Joule)
Power	kg m^2/s^3	(= W, Watt)
Pressure	N/m^2	(= Pa, Pascal)

((Note))
The international system of units (from the NIST Web site)
http://physics.nist.gov/cuu/Units/units.html

Quantity	Name of unit	Symbol	
	SI base units		
length	meter	m	
mass	kilogram	kg	
time	second	s	
electric current	ampere	A	
thermodynamic temperature	kelvin	K	
amount of substance	mole	mol	
luminous intensity	candela	cd	
	SI derived units		**Equivalent units**
area	square meter	m^2	
volume	cubic meter	m^3	
frequency	hertz	Hz	s^{-1}
mass density (density)	kilogram per cubic meter	kg/m^3	
speed, velocity	meter per second	m/s	
angular velocity	radian per second	rad/s	
acceleration	meter per second squared	m/s^2	
angular acceleration	radian per second squared	rad/s^2	
force	newton	N	$kg \cdot m/s^2$
pressure (mechanical stress)	pascal	Pa	N/m^2
kinematic viscosity	square meter per second	m^2/s	
dynamic viscosity	newton-second per square meter	$N \cdot s/m^2$	
work, energy, quantity of heat	joule	J	$N \cdot m$
power	watt	W	J/s
quantity of electricity	coulomb	C	$A \cdot s$
potential difference, electromotive force	volt	V	J/C, W/A
electric field strength	volt per meter	V/m	N/C
electric resistance	ohm	Ω	V/A
capacitance	farad	F	$A \cdot s/V$
magnetic flux	weber	Wb	$V \cdot s$
inductance	henry	H	$V \cdot s/A$
magnetic flux density	tesla	T	Wb/m^2
magnetic field strength	ampere per meter	A/m	
magnetomotive force	ampere	A	
luminous flux	lumen	lm	$cd \cdot sr$
luminance	candela per square meter	cd/m^2	
illuminance	lux	lx	lm/m^2
wave number	1 per meter	m^{-1}	
entropy	joule per kelvin	J/K	
specific heat capacity	joule per kilogram-kelvin	$J/kg \cdot K$	
thermal conductivity	watt per meter-kelvin	$W/m \cdot K$	
radiant intensity	watt per steradian	W/sr	
activity (of a radioactive source)	becquerel	Bq	s^{-1}
radiation dose	gray	Gy	J/kg
radiation dose equivalent	sievert	Sv	J/kg
	SI supplementary units		
plane angle	radian	rad	
solid angle	steradian	sr	

3. Conventional units

We frequently use prefixes to obtain units of a more convenient size. Here are examples of commonly encountered prefixes.

1 nanometer = 1 nm = 10^{-9} m (a little bigger than an atom diameter)
1 micrometer = 1 μm = 10^{-6} m (a human blood cell is about 7μm)
1 millimeter = 1 mm = 10^{-3} m (a pencil lead is about 0.5 mm in diameter)
1 centimeter = 1 cm = 10^{-2} m (the diameter of a ballpoint pen)
1 kilometer = 1 km = 10^{3} m (about 0.6 mi)

1 microgram = 1 μg = 10^{-6} g = 10^{-9} kg (mass of a small dust particle)
1 milligram = 1 mg = 10^{-3} g = 10^{-6} kg (a raindrop is about 2 mg)
1 gram = 1 g = 10^{-3} kg (the mass of a penny is about 2.5 g)

1 nanosecond = 1 ns = 10^{-9} s (time for light to travel 30 cm)
1 microsecond = 1 μs = 10^{-6} s (time for a rifle bullet to travel about 1 mm)
1 millisecond = 1 ms = 10^{-3} s (about 14 ms between human heart beats)

4. Unit Conversion

We sometimes encounter data in units other than those used in SI system. In this case we need to convert the units to the SI system, using conversion factors.

((**Example**))
We consider a speed of 65 miles/hour.

1 mile = 1610 m.
1 hour = 60 min = 60 x 60=3600 s.

Velocity

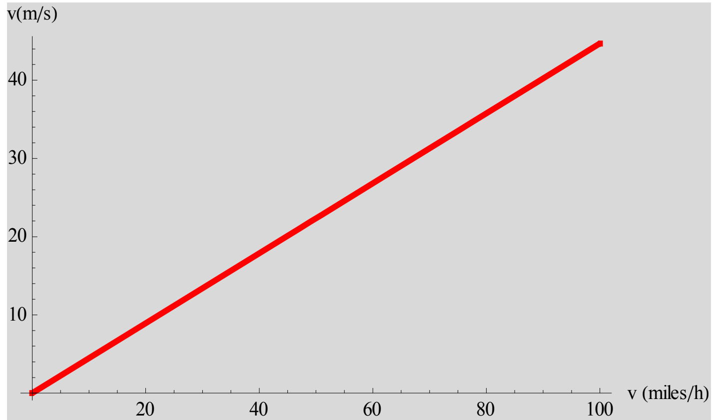
$$65(miles/h) =$$

$$= 65\frac{1.609km}{60\min} = 65\frac{1.609}{60}\frac{1000m}{60s} = \frac{65\times 1.609\times 1000}{60\times 60}m/s = 29(m/s)$$

$$x(miles/h) =$$

$$= x\frac{1.609km}{60\min} = x\frac{1.609}{60}\frac{1000m}{60s}$$

$$= \frac{x\times 1.609\times 1000}{60\times 60}m/s = 0.4469\times x(m/s)$$



Density

$$\rho = \frac{Mass}{Volume}$$ in the units of g/cm^3 or kg/m^3

$$\frac{g}{cm^3} = \frac{10^{-3} kg}{(10^{-2} m)^3} = 10^3 (kg/m^3)$$

((**Table**))

Length
1 inch (in) = 2.54 centimeters (cm)
1 foot (ft) = 0.3048 meter (m)
1 mile (mi) = 5280 ft = 1.609 kilometers (km)
1 m = 3.281 ft
1 km = 0.6214 mi
1 angstrom (Å) = 10^{-10} m
1 light year = 9.461×10^{15} m
1 astronomical unit (AU) = 1.496×10^{11} m
1 parsec (pc) = 3.09×10^{16} m

Mass
1 slug = 14.59 kilograms (kg)
1 kg = 1000 grams = 6.852×10^{-2} slug
1 atomic mass unit (amu) = 1.6605×10^{-27} kg
(1 kg has a weight of 2.205 lb where the acceleration due to gravity is 32.174 ft/s^2)

Time
1 day = 24 h = 1.44×10^3 min = 8.64×10^4 s
1 year (yr) = 365.24 days = 3.156×10^7 s
1 h = 60 min = 3600 s

Speed
1 mi/h = 1.609 km/h = 1.467 ft/s = 0.4470 m/s
1 km/h = 0.6214 mi/h = 0.2778 m/s = 0.9113 ft/s

Volume
1 liter (L) = 10^{-3} m^3 = 1000 cm^3 = 0.03531 ft^3
1 ft^3 = 0.02832 m^3 = 7.481 U.S. gallons (gal)
1 U.S. gal = 3.785×10^{-3} m^3 = 0.1337 ft^3

Force
1 pound (lb) = 4.448 newtons (N)
1 N = 10^5 dynes (dyn) = 0.2248 lb

Work and Energy
1 joule (J) = 0.7376 ft · lb = 10^7 ergs
1 kilogram-calorie (kcal) = 4186 J
1 Btu (60°F) = 1055 J
1 kilowatt-hour (kWh) = 3.600×10^6 J
1 electron volt (eV) = 1.602×10^{-19} J

Angle
1 radian (rad) = 57.30°
1° = 0.01745 rad

Pressure
1 pascal (Pa) = 1 N/m^2 = 1.450×10^{-4} lb/in^2
1 lb/in^2 = 6.895×10^3 Pa
1 atmosphere (atm) = 1.013×10^5 Pa = 1.013 bar
= 14.70 lb/in^2 = 760 torr

Power
1 horsepower (hp) = 550 ft · lb/s = 745.7 W
1 watt (W) = 0.7376 ft · lb/s

5. The use of Mathematica for the evaluation of physical quantities

It is very convenient to use the Mathematica for the evaluation of physical quantities. The numerical values can be obtained from the Web-site of NIST.

Link: Fundamental Physical constant
http://physics.nist.gov/cuu/

((Matematica))

```
Physconst = {u → 1.66053886 10^-27, cal → 4.19, atm → 101.3,
  g → 9.80665, G → 6.6742867 10^-11, NA → 6.02214179 10^23,
  R → 8.314472, me → 9.1093821545 10^-31, u → 1.660538782 10^-27,
  eV → 1.602176487 10^-19, qe → 1.602176487 10^-19,
  ge → 2.0023193043622, kB → 1.3806504 10^-23,
  rB → 0.52917720859 10^-10, μB → 927.40091523 10^-26,
  μN → 5.05078324 10^-27, λc → 2.4263102175 10^-12,
  c → 2.99792458 10^8, μ0 → 12.566370614 10^-7,
  ϵ0 → 8.854187817 10^-12, mn → 1.674927211 10^-27,
  mp → 1.672621637 10^-27, h → 6.62606896 10^-34,
  ℏ → 1.05457162853 10^-34, σSB → 5.670400 10^-8,
  z0 → 376.730313461, Φ0 → 2.06783366752 10^-15,
  Rk → 25812.80755718, Mea → 5.9736 10^24, Rea → 6.372 10^6,
  Msun → 1.988435 10^30, Rsun → 6.9599 10^8, Mmoon → 7.3483 10^22,
  Rmoon → 1.783 10^6, ly → 9.4605 10^15, pc → 30.857 10^15,
  AU → 1.49597870 10^11, mile → 1.609344 10^3, hour → 3600,
  min → 60, gram → 10^-3, cm → 10^-2}
```

```
{u → 1.66054×10^-27, cal → 4.19, atm → 101.3, g → 9.80665,
 G → 6.67429×10^-11, NA → 6.02214×10^23, R → 8.31447,
 me → 9.10938×10^-31, u → 1.66054×10^-27, eV → 1.60218×10^-19,
 qe → 1.60218×10^-19, ge → 2.00232, kB → 1.38065×10^-23,
 rB → 5.29177×10^-11, μB → 9.27401×10^-24, μN → 5.05078×10^-27,
 λc → 2.42631×10^-12, c → 2.99792×10^8, μ0 → 1.25664×10^-6,
 ϵ0 → 8.85419×10^-12, mn → 1.67493×10^-27, mp → 1.67262×10^-27,
 h → 6.62607×10^-34, ℏ → 1.05457×10^-34, σSB → 5.6704×10^-8,
 z0 → 376.73, Φ0 → 2.06783×10^-15, Rk → 25812.8,
 Mea → 5.9736×10^24, Rea → 6.372×10^6, Msun → 1.98844×10^30,
 Rsun → 6.9599×10^8, Mmoon → 7.3483×10^22, Rmoon → 1.783×10^6,
 ly → 9.4605×10^15, pc → 3.0857×10^16, AU → 1.49598×10^11,
 mile → 1609.34, hour → 3600, min → 60, gram → 1/1000, cm → 1/100}
```

u=atomic mass unit

cal=4.19 J, 1 atm=101.3 kPa, g=Acceleration due to gravity (m/s^2)

G=gravitational constant (N m^2/kg^2),

NA=Avogadro number

R=Gas constant (J/mol K),

me=mass of electron (kg)

qe=Charge of electron (C),

ge=electron g factor

kB=Boltzmann constant (J/K)

rB=Bohr radius (m),

μN=Nuclear magneton (J/T)

λc=Compton wavelength (m)

c=velocity of light (m),

μ0=Magnetic constant

ϵ0=electric constant,

mn=mass of neutron (kg)

mp=mass of proton (kg),

h=Planck constant

$\hbar$=Dirac constant,

Planck mass=mpl $= \sqrt{\frac{\hbar c}{G}}$,

Planck time = τpl $= \left(\frac{\hbar G}{c^5}\right)^{1/2}$,

Planck length=lpl $= \left(\frac{\hbar G}{c^3}\right)^{1/2}$,

σSB=Stefan-Boltzmann constant (W/m^2 K^4),

z0 =impedance of free space (Ω),

Φ0=magnetic flux quantum (T m^2)

Rk=von Klitzing constant (Ω),

Mea = 5.9736×10^{24} kg; Mass of the earth,

Rea=6372.797 km, radius of the earth,

Msun=mass of sun (kg) =Solar mass

Rsun=radius of Sun (m)=Solar radius

Mmoon=Mass of moon

Rmoon=radius of moon

light year=a distance light travels in a vacuum in one year=9.4605×10^{15} m,

Parsec (pc) = a unit of distance = 3.26 light yeras = 30.857×10^{15} m,

AU = astronomical unit = average distance between the Earth and the Sun = $1.49597870 \times 10^{11}$ m

u=atomic mass constant

mile=1.609344×10^3 m

hour=3600 sec

Gravity of Moon

$$G \frac{Mmoon}{Rmoon^2} \text{ /. } Physconst$$

1.54273

Gravity of Earth

$$G \frac{Mea}{Rea^2} \text{ /. } Physconst$$

9.8195

Gravity of Sun

$$G \frac{Msun}{Rsun^2} \text{ /. } Physconst$$

273.975

Planck' s time

$$\sqrt{\frac{\hbar G}{c^5}} \text{ /. } Physconst$$

5.39124×10^{-44}

Escape velocity from the Earth

$$\sqrt{\frac{2 G Mea}{Rea}} \text{ /. } Physconst$$

11186.6

Escape velocity from the Sun

$$\sqrt{\frac{2 G Msun}{Rsun}} \text{ /. } Physconst$$

617549.

Escape velocity from the Moon

$$\sqrt{\frac{2\,G\,\mathrm{Mmoon}}{\mathrm{Rmoon}}}\ \texttt{/.}\ \mathrm{Physconst}$$

2345.5

Velocity

Velocity

65 mile / hour /. Physconst

29.0576

80 mile / hour /. Physconst

35.7632

95 mile / hour /. Physconst

42.4688

AU/min

AU / min /. Physconst

2.4933×10^9

c / (AU / min) /. Physconst

0.120239

AU / (c min) /. Physconst

8.31675

Density

$1\ \mathrm{gram} \big/ \mathrm{cm}^3$ /. Physconst

1000

6. Uncertainty and significant figures

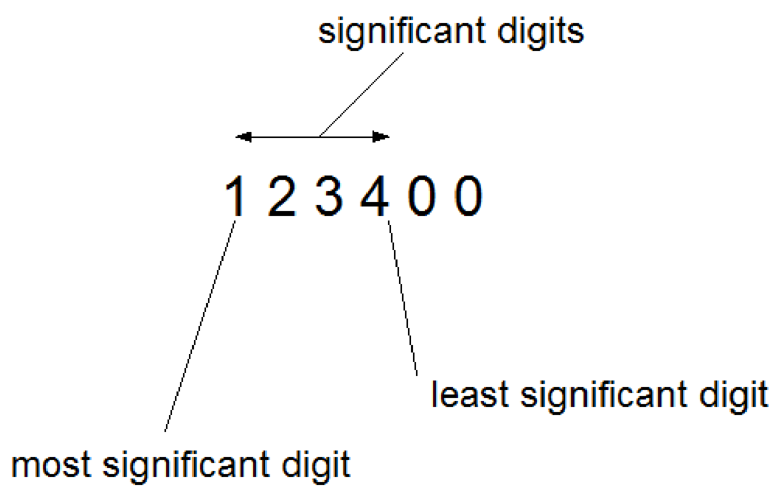
The uncertainty is indicated by the number of meaningful digits, or significant figures, in the measured value.

6.1. Definition of the number of significant figures

(a) The leftmost nonzero digit is the most significant digit.

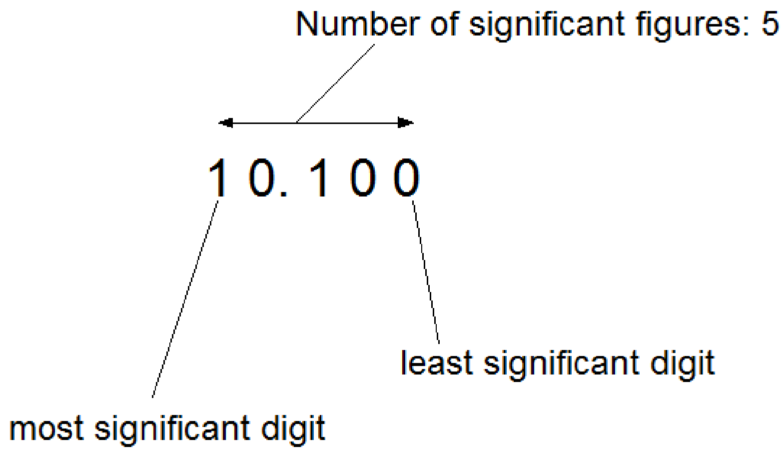
(b) If there is no decimal point, the rightmost nonzero digit is the least significant digit.

1234	123,400	123	
470,000,000	130,000,	1500,	30
1001	10010		



(c) If there is a decimal point, the rightmost digit is the least significant digit, even if it is a 0.

1000.	10.10	1.010	100.0
4.00	0.01	0.001	
0.58	0.047	0.00590	



(d) All digits between the least and most significant digits are counted as significant digits.

((**Note**))

1. The number 10010 is considered to only four significant digits even though the last digit might be physically significant.

2. If you are not sure whether a digit is significant, assume that it is not. For example, if the directions for an experimental read: add the sample to 400 *mL* of water," assume that volume of water is known to one significant digit.

6.2 Multiplication and division

When multiplying or dividing, or taking roots, the number of significant figures in the results should match the number of significant figures of the number with the fewest significant figures.

((Example))

3.923 x 2.34 x 0.58 = 5.3; round off 5.32

3.149 x 2.12 x 0.12 = 0.80; round off 0.801

4.832 ÷ 2.5 = 1.9 round off 1.933

((Note))

The number of significant figures is the number of digits when written in scientific notation.

$0.58 = 5.8 \times 10^{-1}$ (number of significant figures: 2)

$231.300 = 2.31300 \times 10^{2}$ (number of significant figures: 6)

6.3 Addition and subtraction

When we add and subtract numbers, it is the location of the decimal point that matters, not the number of significant figures.

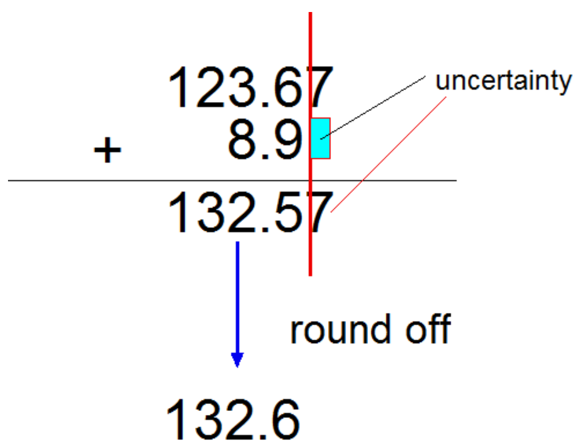
When adding or subtracting, the number of decimal places in the result should equal the smallest number of decimal places in any term in the sum.

((Example))

123.62 + 8.9 = 132.5

6.47 + 1.2 = 7.7

150.0 + 0.507 = 150.5



6.4 Roundoff

When you reduce an answer to the appropriate number of significant figures, you must reduce, not truncate.

((Example))

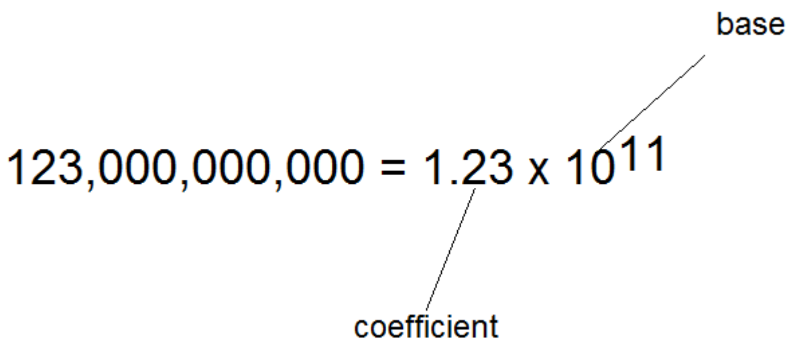
525 ÷ 311 = 1.688102894→1.69, not 1.68.

6.5 Scientific notation

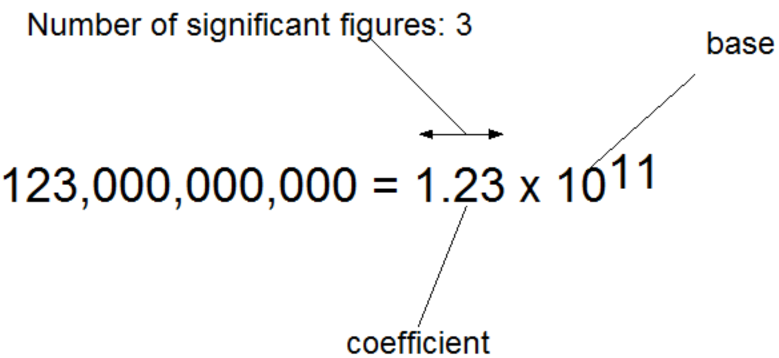
Scientists have developed a shorter method to express very large numbers. This method is called **scientific notation**. Scientific Notation is based on powers of the base number 10.

When you calculate with very large or very small numbers, you can show significant figures much more easily by using scientific notation.

The number 123,000,000,000 in scientific notation is written as 1.23×10^{11}. The first number 1.23 is called the coefficient. It must be greater than or equal to 1 and less than 10. The second number is called the base. It must always be 10 in scientific notation. The base number 10 is always written in exponent form. In the number 1.23×10^{11}, the number 11 is referred to as the exponent or power of 10.



The number of significant figures is the number of digits when written in scientific notation.



((Example))

$384,000,000 = 3.84 \times 10^8$: the number of significant figures is 3.

$0.00620 = 6.20 \times 10^{-3}$: the number of significant figures is 3.

4.00×10^7: the number of significant figures is 3, even though two of them are zeros.

$(4.44 \times 10^{-4}) \times (2.7 \times 10^3) = 1.1988$

1.2: the number of significant figures is 2.

((**Mathematica**))

```
0.0001305 // ScientificForm
```

1.305×10^{-4}

7. Integer, fraction, and π

We treat that number as having no uncertainty at all.

((Example))
In the equation

$$v^2 - v_0{}^2 = 2a(x - x_0),$$

The coefficient 2 is exactly 2. We can consider this coefficient as having an infinite number of significant figures (2.00000000….).

π
3.141592653589793238462643383279502884197169399375105820974944592307816406286208998628034825342117068